26CS803 Applied Linear Algebra and Optimization for Computational Problems 3 0 1 4

Course Objective

To develop rigorous theoretical and algorithmic foundations in applied linear algebra and optimization, enabling PhD scholars to model, analyze, and solve large-scale computational problems arising in advanced computer science research.

Course Outcomes (COs)

- **CO1:** Analyze advanced vector spaces, matrix structures, and spectral properties used in computational models.
- **CO2:** Design and analyze numerical linear algebra algorithms for large-scale and high-dimensional data.
- **CO3:** Apply eigenvalue-based methods and matrix factorizations in machine learning and data analysis.
- **CO4:** Formulate and solve convex optimization problems with rigorous theoretical guarantees.
- **CO5:** Apply large-scale and non-convex optimization techniques to contemporary computer science research problems.

UNIT I: Advanced Linear Algebra Foundations

- Normed and inner product spaces
- Orthogonality and projections
- Matrix norms and operator norms
- Positive definite and semidefinite matrices
- Kronecker products and block matrices
- Sparse and structured matrices

Research Relevance: Feature spaces, kernel methods, high-dimensional representations

UNIT II: Numerical Linear Algebra and Matrix Computations

- Conditioning and numerical stability
- Direct and iterative methods for linear systems
- Krylov subspace methods (CG, GMRES)
- Randomized numerical linear algebra
- Low-rank matrix approximation

Research Relevance: Large-scale data analytics, recommender systems

UNIT III: Eigenvalue Problems and Matrix Factorizations

- Eigenvalue algorithms and spectral theory
- Singular Value Decomposition (SVD)
- Generalized eigenvalue problems
- Non-negative Matrix Factorization (NMF)
- Tensor decompositions (CP, Tucker)

Research Relevance: Dimensionality reduction, spectral clustering, graph analytics

UNIT IV: Convex Optimization Theory

- Convex sets and convex functions
- First and second-order optimality conditions
- Lagrangian duality and KKT conditions
- Sensitivity and perturbation analysis
- Interior-point methods

Research Relevance: Convex relaxations, sparse optimization

UNIT V: Large-Scale and Non-Convex Optimization

- Gradient descent and accelerated methods
- Stochastic optimization (SGD, mini-batch methods)
- Proximal and coordinate descent methods
- Optimization in deep learning
- Distributed and parallel optimization

Research Relevance: Deep learning, scalable AI systems

Reference Textbook (Primary)

Boyd, S. and Vandenberghe, L., *Convex Optimization*, Cambridge University Press.

Additional References

- Golub, G. H. and Van Loan, C. F., *Matrix Computations*
- Trefethen, L. N. and Bau, D., *Numerical Linear Algebra*
- Nocedal, J. and Wright, S., *Numerical Optimization*

Evaluation Pattern

Internal	External	Weightage	CO Mapping
Midterm Examination		20	1,2,3
Assignment-Journal/conference paper submission		50	1,2,3,4,5
	End Semester Examination	30	1,2,3,4,5

CO–PO Mapping Matrix

CO \ PO	PO1	PO2	PO3	PO4	PO5
CO1	3	2	2	–	–
CO2	2	3	3	2	–
CO3	2	3	3	2	1
CO4	3	3	2	1	2
CO5	2	3	3	3	3