HYPERSONIC FLOW THEORY

Session -01: Course Introduction

Dr. S.R. Srikrishnan

2024

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COURSE OBJECTIVE & SYLLABUS

- **Course Objective:** [illegible]
- **Syllabus:**

UNIT I	UNIT II	UNIT III
[illegible]		
Introduction to hypersonic flow and flight paths	Inviscid analysis of hypersonic flows	[illegible]

COURSE PLAN: *OVERVIEW OF THE COURSE*

- The Course plan will follow this sequence:
  - Introduction to hypersonic flight. Complex phenomena related to hypersonic aerodynamics, flight path
  - Analysis of hypersonic flow using inviscid theoretical models
  - Study of the application of the Method of Characteristics
  - Analysis of viscous phenomena

TEXT BOOK & REFERENCES

- **Text Book**
  - *John D. Anderson, "Hypersonic and High Temperature Gas Dynamics," McGraw Hill, [illegible].*
- **References**
  - *Wallace D. Hayes and Ronald F. Probstein, "Hypersonic Flow Theory," 2nd edition, Academic Press*
  - *Fundamentals of Hypersonic Aerodynamics, Fletcher, [illegible]*

SYLLABUS

HYPERSONIC FLOW THEORY

Session-02: Hypersonic Flight

[illegible], Department of Aerospace engineering

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SUPERSONIC (M>1) TO HYPERSONIC (M>5)

- Not a step-change !
  - Unlike transonic to supersonic
  - [illegible]
- Mach 5 conventionally used as the bordering point between supersonic & hypersonic regimes
- Nothing dramatically happens at Mach – The changes are gradual and increasingly become prominent at higher Mach numbers
- A hypersonic vehicle can be an airplane, missile, or spacecraft.

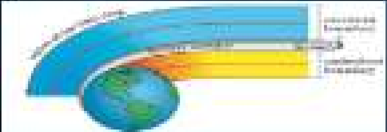
TWO INSTANCES OF HYPERSONIC FLOW

- An atmospheric re-entry vehicle
  - Soyuz Re-entry Video [illegible]
  - [illegible] while re-entering the atmosphere after an orbital mission

- An air-breathing engine operating within earth's atmosphere
  - The only propulsion option is SCRAMJET
  - Why ?

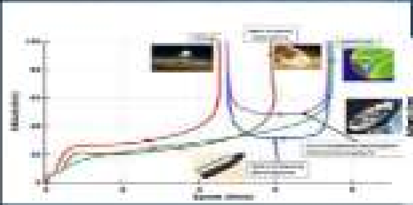
ATMOSPHERIC RE-ENTRY

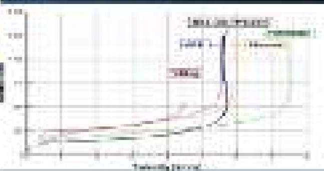
THE RE-ENTRY DILEMMA: MISS THE ATMOSPHERE OR GET BURNT IN IT !



THE RE-ENTRY PROFILE (FOR SPACE SHUTTLE)

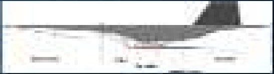
HYPERSONIC FLIGHT TRAJECTORY





HYPERSONIC AIRBREATHING "*CONCEPT*"

- Flow deceleration from hypersonic Mach number
- Complexity of oblique shock structure
- Research as from 1960's



CONSIDER THIS

- An aircraft takes off horizontally from a runway
- Accelerates into orbit around the Earth
  - [illegible]
- [illegible] upper regions of the atmosphere
- Then reenters the atmosphere at Mach [illegible]
- Finally lands under power on a conventional runway

WHAT DOES THIS MISSION ACCOMPLISH OVER THE CONVENTIONAL ROCKET LAUNCHS ?

- No expendable parts – like a rocket, for which each stage is thrown away after its burnout
  - Lesser propellant mass, higher payload
- Use of atmospheric oxygen as much as possible for combustion
- REUSABILITY

SINGLE/DOUBLE STAGE TO ORBIT CONCEPT (SSTO/TSTO)

WHAT MAKES IT TOUGH ?!

- History: Conceptualization
- Global efforts
- Present status
- Main issues converging on to the problems associated with hypersonic aerodynamics.
- Complexity & Diversity of the issues

DIDN'T WORK !

WELCOME TO
HYPERSONIC AERRODYANIMCS

HYPERSONIC FLOW THEORY

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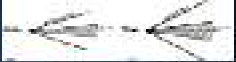
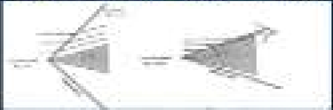
CHARACTERISTICS OF HYPERSONIC FLOWS

- Thin shock layers
- Entropy layer
- High temperature effects
  - Deviation from perfect gas behaviour
  - Ionization

THIN SHOCK LAYER



- [illegible]
- With increase in Mach number, the wave angle (β) keeps decreasing
- In hypersonic Mach number range, the angle is so low that the distance between between the body and the shock wave is very small
  - Hypersonic flows are characterized by very thin shock layers



THIN SHOCK LAYER

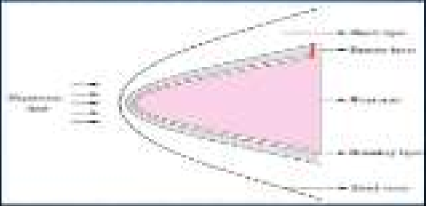
- Too thin?
  - Shock layer is so thin that i [illegible]
  - Fully viscous shock layer

VISCOUS INTERACTION

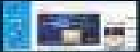
- Interaction between the boundary layer and the outer inviscid flow
  - The flow at high speeds [illegible] the temperature rise in the [illegible] near the body surface causes the boundary layer to become thicker
  - [illegible]
- The [illegible] in the boundary layer thickness varies as the square of the Mach number: [illegible]
  - Thus, as Mach number increases, the boundary layer can grow rapidly resulting in very high drag
    - [illegible] outward from the body
    - Viscous interaction can have a great influence on the surface pressure distribution and also friction on the body
    - Impacts the lift, drag, stability and heating characteristics of the body

ENTROPY LAYER

- Close to the blunt leading edge, the oblique shock becomes highly curved.
- Entropy increases across a shock, and the entropy increase becomes greater as the shock strength increases.
- Since flow near the nose passes through a nearly normal shock, it will experience a much greater change in entropy compared to flow passing through the weak oblique shock angle further from the body centerline.
- Thus, strong entropy gradients exist near the leading edge generating an "entropy layer" that flows downstream along the body surface.
- This [illegible]



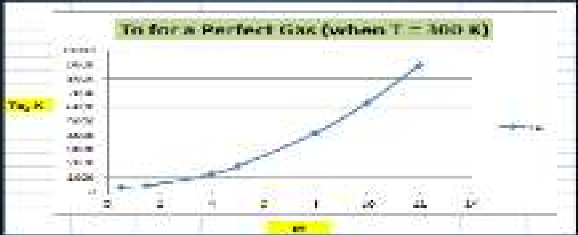
HIGH TEMPERATURE EFFECTS



- Hypersonic vehicles entail dissipation of kinetic energy to thermal energy
  - [illegible]
- The most notable changes air undergoes as temperature increases:
- Perfect gas assumptions do not hold at High temperatures
- Chemically reacting boundary layer
- The aerodynamics & heat transfer change drastically

High Temperature Effects on Air

Temperature (K)	Chemical Change
[illegible]	[illegible]
[illegible]	[illegible]
[illegible]	[illegible]
[illegible]	[illegible]



Critical Aerothermodynamics


INFRARED
SPECTROPHOTOMETER

HIGH TEMPERATURE REACTIONS

- Dissociation

[illegible] → [illegible] – [illegible]

[illegible] → [illegible] – [illegible]

- Ionization (Above [illegible] K) : Formation of charged ions

$O \rightarrow O^+$ Releasing electron

$N \rightarrow N^+$ Releasing electron

HYPERSONIC FLOW THEORY

Session 01 05

Hypersonic Limit Approximations

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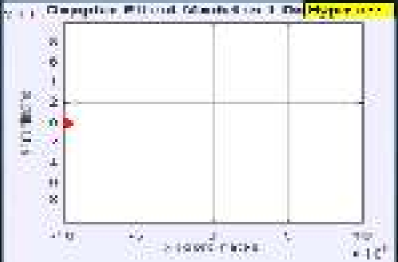
INVISCID HYPERSONIC FLOW THEORY



SHOCK-EXPANSION THEORY

THE APPROACH

- Oblique shock and expansion waves
- Basic assumptions & Governing Equations
- Property Relations as $M \to \infty$
- Applications

REVIEW: OBLIQUE SHOCK RELATIONS

- Review of the formulation
- Compression corner in supersonic flow

REVIEW: OBLIQUE SHOCK RELATIONS – CTD.

- Governing equations
  - Continuity Equation
    - Resolved for normal and tangential components of velocity
    - Tangential component remains unchanged across the shock
  - [illegible]
    - Resolved for normal and tangential components of velocity
  - One-dimensional energy equation
- Assumptions: Steady, inviscid, adiabatic flow, one-dimensional except across the shock; Perfect gas

$$M_{n_1} = M_1 \sin\beta$$

OBLIQUE SHOCK FORMULATION

$$M_{n_1} = M_1 \sin \beta$$

REVIEW: OBLIQUE SHOCK RELATIONS – CTD.

- Mach Number, based on normal component of the velocity, downstream of the shock:

$$M_{n_2}^2 = \frac{M_{n_1}^2 + [2/(\gamma - 1)]}{[2\gamma/(\gamma - 1)]M_{n_1}^2 - 1}$$

REVIEW: OBLIQUE SHOCK RELATIONS – CTD.

- Static pressure ratio
- Density ratio
- Static temperature ratio

$$\frac{p_2}{p_1} = 1 + \frac{2\gamma}{\gamma + 1}(M_{n1}^2 - 1)$$

$$\frac{\rho_2}{\rho_1} = \frac{(\gamma + 1)M_{n1}^2}{(\gamma - 1)M_{n1}^2 + 2}$$

$$\frac{T_2}{T_1} = \frac{p_2}{p_1}\frac{\rho_1}{\rho_2}$$

RELATION BETWEEN DEFLECTION ANGLE & SHOCK ANGLE

$$\tan\theta = \frac{2\cot\beta\left(M_1^2\sin^2\beta - 1\right)}{M_1^2(k + \cos 2\beta) + 2}$$

θ-β-M Relation

$$\tan\theta = \frac{2\cot\beta\,(M_1^2\sin^2\beta - 1)}{M_1^2(\gamma + \cos 2\beta) + 2}$$

- **Key Observations:**
  - Two solutions for each M, θ combination
  - The Weak shock solution & the Strong shock solution
  - Maximum deflection angle for attached shock solution

Shocks in hypersonic flow: relations as $M_1 \to \infty$

APPROXIMATIONS FOR HYPERSONIC LIMIT

- Static pressure ratio across oblique shock: $\frac{p_2}{p_1} = 1 + \frac{2\gamma}{\gamma+1}(M_1^2 \sin^2\beta - 1)$
- As $M_1 \to \infty$: $\frac{p_2}{p_1} \to \frac{2\gamma}{\gamma+1} M_1^2 \sin^2\beta$
- *Density ratio across oblique shock:* $\frac{\rho_2}{\rho_1} = \frac{(\gamma+1) M_1^2 \sin^2\beta}{(\gamma-1) M_1^2 \sin^2\beta + 2}$
  - As $M_1 \to \infty$: $\frac{\rho_2}{\rho_1} \to \frac{\gamma+1}{\gamma-1}$

THE θ-β-M RELATION

- Basic relation for supersonic flow:

$$\tan\theta = 2\cot\beta\left[\frac{M_1^2\sin^2\beta - 1}{M_1^2(\gamma + \cos 2\beta) + 2}\right]$$

- For hypersonic flow, as θ -> 0, β also -> 0
  - Hence:

$$\theta = \frac{2}{\beta}\left[\frac{M_1^2\beta^2 - 1}{M_1^2(\gamma + 1) + 2}\right]$$

  - Neglecting 1 and 2 for high values of M:

$$\theta = \frac{2}{\beta}\left[\frac{M_1^2\beta^2}{M_1^2(\gamma + 1)}\right]$$

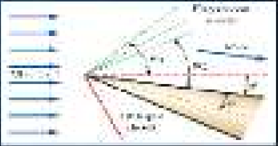
  - Or

$$\frac{\beta}{\theta} = \frac{\gamma + 1}{2}$$

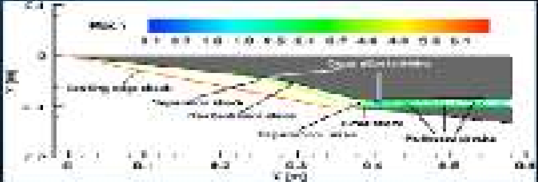
EXPANSION WAVE

REVIEW: PM EXPANSION

P-M EXPANSION..



APPLICATIONS



HYPERSONIC LIMIT

- As $M \rightarrow \infty$: $\sqrt{M^2 - 1} \approx M$

$$\nu(M) = \sqrt{\frac{\gamma+1}{\gamma-1}} \tan^{-1} \sqrt{\frac{\gamma-1}{\gamma+1}} M - \tan^{-1} M$$

$$\tan^{-1} x = \frac{\pi}{2} - \tan^{-1}\left(\frac{1}{x}\right)$$

$$\tan^{-1}\left(\frac{1}{x}\right) = \frac{1}{x} - \frac{1}{3x^3} + \frac{1}{5x^5} - \frac{1}{7x^7} + \cdots$$

P-M FUNCTION

$$\nu(M) = \sqrt{\frac{\gamma+1}{\gamma-1}}\left(\frac{\pi}{2} - \sqrt{\frac{\gamma-1}{\gamma+1}}\frac{1}{M} + \cdots\right) - \left(\frac{\pi}{2} - \frac{1}{M} + \cdots\right)$$

MACH NUMBER – DEFLECTION ANGLE RELATION

$$\theta = \nu(M_2) - \nu(M_1)$$

$$\theta = \frac{1}{M_2} - \left(\frac{\gamma+1}{\gamma-1}\right)\frac{1}{M_2} - \frac{1}{M_1} + \left(\frac{\gamma+1}{\gamma-1}\right)\frac{1}{M_1}$$

$$\theta = \frac{2}{\gamma-1}\left(\frac{1}{M_1} - \frac{1}{M_2}\right)$$

STATIC PRESSURE RATIO

- Since the flow is isentropic & stagnation pressure remains constant:

$$\frac{p_2}{p_1} = \left[\frac{1 + (\gamma - 1)/2\, M_1^2}{1 + (\gamma - 1)/2\, M_2^2}\right]^{\gamma/(\gamma - 1)}$$

- For M >> 1 (Hypersonic range)

$$\frac{p_2}{p_1} = \left(\frac{M_1}{M_2}\right)^{2\gamma/(\gamma - 1)}$$

PRESSURE COEFFICIENT

- C_p, a convenient term for normalizing pressure $C_p = \frac{p - p_\infty}{q_\infty}$
- For incompressible flows, q is defined as: $q_\infty = \frac{1}{2}\rho_\infty V_\infty^2$
    - The significance
- This is not suitable for compressible flows
- In hypersonic aerodynamics we express q in terms of Mach number:

$$q_\infty = \frac{1}{2}\rho_\infty V_\infty^2 = \frac{1}{2}\frac{\gamma p_\infty}{\gamma p_\infty}\rho_\infty V_\infty^2 = \frac{\gamma}{2} p_\infty \frac{V_\infty^2}{a_\infty^2}$$

$$q_\infty = \frac{\gamma}{2} p_\infty M_\infty^2$$

$$C_p = \frac{p - p_\infty}{q_\infty} = \frac{2}{\gamma M_\infty^2}\left(\frac{p}{p_\infty} - 1\right)$$

COMING NEXT…

- Specific theoretical formulations for pressure distribution over hypersonic bodies
- Calculation of aerodynamic forces
- Evaluation & applicability of the models

HYPERSONIC FLOW THEORY

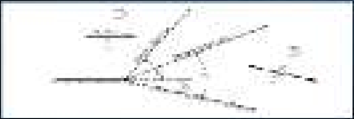
Session 06- Expansion Waves: Hypersonic Approximation

Dr. R.R. Srikrishnan

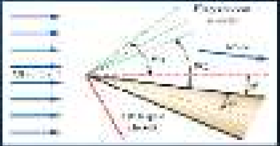
EXPANSION WAVE

Prandtl-Meyer Expansion

REVIEW: PM EXPANSION



P-M EXPANSION..



HYPERSONIC LIMIT

- As $M \to \infty$, $\sqrt{M^2 - 1} \approx M$

$$\nu(M) = \sqrt{\frac{\gamma+1}{\gamma-1}}\tan^{-1}\sqrt{\frac{\gamma-1}{\gamma+1}}M - \tan^{-1}M$$

$$\tan^{-1}x = \frac{\pi}{2} - \tan^{-1}\left(\frac{1}{x}\right)$$

$$\tan^{-1}\left(\frac{1}{x}\right) = \frac{1}{x} - \frac{1}{3x^3} + \frac{1}{5x^5} - \frac{1}{7x^7} + \cdots$$

P-M FUNCTION

$$\nu(M) = \sqrt{\frac{\gamma+1}{\gamma-1}}\left(\frac{\pi}{2} - \sqrt{\frac{\gamma-1}{\gamma+1}}\frac{1}{M} + \cdots\right) - \left(\frac{\pi}{2} - \frac{1}{M} + \cdots\right)$$

MACH NUMBER – DEFLECTION ANGLE RELATION

$$\theta = \nu(M_2) - \nu(M_1)$$

$$\theta = \frac{1}{M_2} - \left(\frac{\gamma+1}{\gamma-1}\right)\frac{1}{M_2} - \frac{1}{M_1} + \left(\frac{\gamma+1}{\gamma-1}\right)\frac{1}{M_1}$$

$$\theta = \frac{2}{\gamma-1}\left(\frac{1}{M_1} - \frac{1}{M_2}\right)$$

STATIC PRESSURE RATIO

- Since the flow is isentropic & stagnation pressure remains constant:

$$\frac{p_2}{p_1} = \left[\frac{1 + (\gamma - 1)/2\, M_1^2}{1 + (\gamma - 1)/2\, M_2^2}\right]^{\gamma/(\gamma-1)}$$

- For M >> 1 (Hypersonic range)

$$\frac{p_2}{p_1} = \left(\frac{M_1}{M_2}\right)^{2\gamma/(\gamma-1)}$$

PRESSURE COEFFICIENT

- C_p, a convenient term for normalizing pressure: $C_p = \frac{p - p_\infty}{q_\infty}$
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$$q_\infty = \frac{1}{2}\rho_\infty V_\infty^2 = \frac{1}{2}\frac{\gamma p_\infty}{\gamma p_\infty}\rho_\infty V_\infty^2 = \frac{\gamma p_\infty}{2}\frac{V_\infty^2}{a_\infty^2}$$

$$q_\infty = \frac{\gamma}{2} p_\infty M_\infty^2$$

$$C_p = \frac{p - p_\infty}{q_\infty} = \frac{2}{\gamma M_\infty^2}\left(\frac{p}{p_\infty} - 1\right)$$

COMING NEXT…

- Specific theoretical formulations for pressure distribution over hypersonic bodies
- Calculation of aerodynamic forces
- Evaluation & applicability of the models

HYPERSONIC FLOW THEORY

Session: 07-08
Newtonian Theory for Hypersonic Flows

Dr. S.R. Srikrishnan

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CALCULATING SURFACE PRESSURE DISTRIBUTION

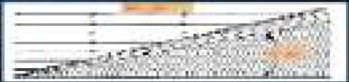
- An integral part of aerodynamic design
- A surprising match with an analytical approach proposed by Newton for low speed flow
- Useful in some cases!
- Further developments improve the utility of the formulation

NEWTON'S THEORY

- Flow impacting a solid surface at an angle of attack
- Newton: Particles of the oncoming flow that will lose their momentum in the normal direction and flow tangentially along

RELEVANCE OF THE MODEL IN HYPERSONIC FLOW

- Oblique shock solutions give
- Shock wave nearly parallel to the surface



BASIC FORMULATION

- Mass flow rate is $\rho_\infty V_\infty A \cos\theta$
- Rate of change of momentum [illegible]

$$\rho_\infty V_\infty A \sin\theta (V_\infty \sin\theta) \qquad \rho_\infty V_\infty^2 A \sin^2\theta$$

- Force = Rate of change of momentum
- Hence the normal force $F = \rho_\infty V_\infty^2 A \sin^2\theta$

$$\frac{F}{A} = \rho_\infty V_\infty^2 \sin^2\theta$$

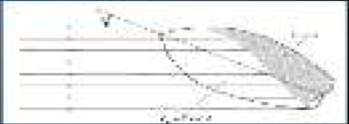
PRESSURE COEFFICIENT

- [illegible]

[illegible]

PRESSURE DISTRIBUTION AS PER N'S THEORY

- Can be computed based on the local deflection.
- Since the flow does not [illegible] around on the surface (as per this theory) the side as opposed to the flow has $C_p = 0$ (i.e. at free stream static pressure)



WHY NEWTON'S THEORY FOR HYPERSONIC FLOWS ?

- No upstream stream-line turning in $M \gg 1$ flow
  - Flow strikes the surface in the same way as conceived by Newton
- Mach number independence of C_p in hypersonic flow
  - In accordance with more accurate models
- Near tangential flow over the surface due to low wave angle
  - Impinges and turns "parallel" (almost) to the surface – as Newton conceptualized!

AERODYNAMIC FORCES AS PER N'S THEORY

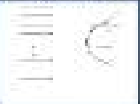
- Normal force coefficient:
- For unit length (normal to the plane

MODIFIED N'S THEORY

- Lees's modification
- An [illegible]
  - This was for a blunt body case with detached shock
  - $C_{p,max}$ corresponds to the total pressure behind the normal shock

$$C_p = C_{p,max}\sin^2\theta$$

$$C_{p,max} = \frac{p_{0_2} - p_\infty}{\frac{1}{2}\rho_\infty V_\infty^2}$$



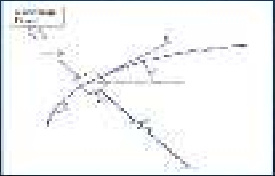
THE EFFECT OF LEE'S MODIFIED FORMULATION

SURFACE PRESSURE DISTRIBUTION

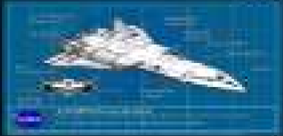
NOTE ON NEWTONIAN APPROACH – WHY HYPERSONIC ?

MODIFICATION TO INCLUDE CURVATURE EFFECT OF BLUNT BODIES

- [illegible]
- Incorporates additional terms to reflect the effect of centrifugal forces that arise due to the curvature
- [illegible]



HYPERSONIC FLOW THEORY



Session 09 Shock Expansion Method

Dr. A.R. Srikrishnan

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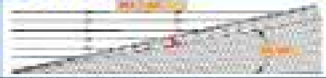
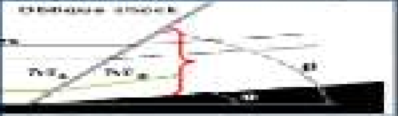
NOTE ON NEWTONIAN APPROACH – WHY HYPERSONIC ?



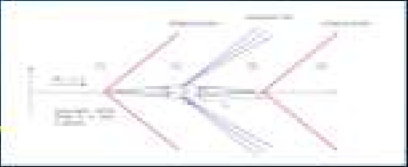


MODIFICATION TO INCLUDE CURVATURE EFFECT OF BLUNT BODIES

- the effect of centrifugal forces that arise due to the curvature

SHOCK EXPANSION "THEORY" [METHOD]

- An approach for calculation of aerodynamic forces on two dimensional supersonic/hypersonic bodies whose surfaces can be approximated as formed by straight lines
  - Flat plate at an angle of attack
  - Diamond airfoils
- Using the oblique shock relations and the Prandtl-Meyer function
  - obtain pressures on all surfaces of the diamond wedge and flat plate and then lift and drag forces can be calculated.
- Applicable to sharp nosed bodies with an attached shock wave



WAVE DRAG

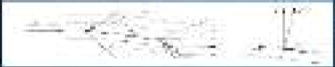
- [illegible] ZERO drag [illegible]
- **Not so in supersonic flow**
  - [illegible]
- [illegible]

[illegible]

[illegible]

WAVE DRAG – TERM FROM SHIP HYDRADYNAMICS ?

WAVE DRAG – ON FLAT PLATE



SHOCK EXPANSION CALCULATIONS

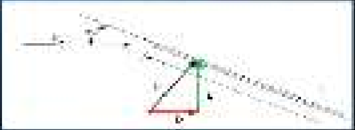
- Calculate P2 using PM expansion method
- Calculate P3 using Oblique shock relations
- Lift & Drag from the pressure distributions

ACCURACY OF SHOCK-EXPANSION MODEL

[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]

EXAMPLE: APPLICATION OF NEWTONIAN THEORY

- Calculate the L/D Ratio for a flat plate at Mach 10 with an angle of attack = 10deg
- Normal force: $F = \rho_\infty V_\infty^2 A \sin^2\theta$
- Lift = F cos θ
- Drag = F sin θ
- **L/D = cot θ = 5.67**



EXAMPLE: SHOCK-EXPANSION METHOD

[illegible]

- Region 2: [illegible]
- Region 3: [illegible]

HYPERSONIC FLOW THEORY

Session 10-Shock-Expansion Method for HYPERSONIC FLOWS

Dr. [illegible]

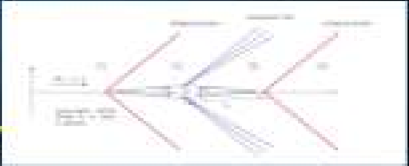
[illegible]

AMRITA

RECAP:

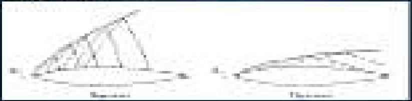
Shock-Expansion Method

- An approach for calculation of aerodynamic forces on two-dimensional supersonic/hypersonic bodies whose surfaces can be approximated as formed by straight lines
    - Flat plate at an angle of attack
    - Diamond airfoils
- Using the oblique shock relations and the Prandtl-Meyer expansion
    - obtain pressures on all surfaces of the diamond wedge and flat plate and then lift and drag forces can be calculated.
- Applicable to sharp nosed bodies with an attached shock wave



SUPERSONIC VS HYPERSONIC APPLICATION

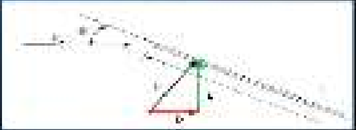
- Relatively large wave angle results in reflected waves in supersonic flows
- [illegible]
- [illegible] results in hypersonic flows



COMPARISON

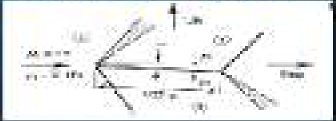
EXAMPLE: APPLICATION OF NEWTONIAN THEORY

- Calculate the L/D Ratio for a flat plate at Mach 10 with an angle of attack =10deg
- Normal force: $N = \rho_\infty V_\infty^2 A \sin^2 \theta$



- lift = F cos θ
- drag = F sin θ
- **L/D = cot θ = 5.67**

EXAMPLE: SHOCK-EXPANSION METHOD



NUMERICAL PROBLEM: PRESSURE DISTRIBUTION ON A DIAMOND AF IN HYPERSONIC FLOW

- [illegible]

[illegible]

[illegible]

[illegible]

[illegible]

[illegible]

[illegible]

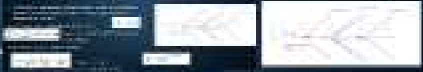
HYPERSONIC FLOW THEORY

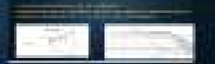
Session 11: On Surface Inclination Methods

Dr. R.B. Srikrishnan

AMRITA

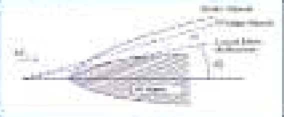
RECAP



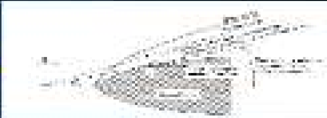


TANGENT WEDGE METHOD

- Suitable for sharp-nosed bodies with sufficiently high Mach numbers to maintain the shock attached to the nose
- Can be used to estimate pressure at any point on the surface of the object
    - Based on the local surface inclination or flow deflection angle



THE PROCEDURE..



- [illegible]
- Using this angle and freestream Mach number calculate the probable oblique shock angle.
- [illegible]
    - [illegible]
    - [illegible]
- Follow this procedure at all points on the wedge surface and calculate the pressure on it.

THE IMPORTANCE OF SURFACE INCLINATION APPROACH

- Local inclination methods for aerodynamic analysis have proven sufficiently accurate for use in early design stages
- Limitations of experimental facilities:
    - Issues related to flow quality
    - Test time
    - Instrumentation, etc.

EXPERIMENTAL CHALLENGES

- Low stagnation enthalpy facilities provide longer running times, but exclude high temperature effects
- High enthalpy facilities: Shock tunnels, detonation driven
  - Limited run time
  - Flow/test condition measurements are challenging

Table 1

[illegible]

HYPERSONIC FLOW THEORY

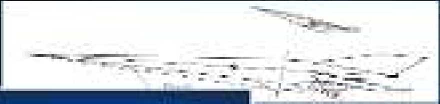
Session 13- Hypersonic Small Perturbation Theory

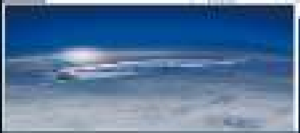
SMALL PERTURBATIONS – THE GENERAL APPROACH

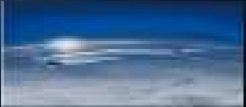
- [illegible]
  - [illegible]
  - [illegible]
- [illegible]
- [illegible]
- [illegible]
- [illegible]
- [illegible]

STATIC PRESSURE RATIO ACROSS SHOCKS AT SLENDER AND NON-SLENDER BODIES

SLENDER DESIGNS: CHARACTERIZE HYPERSONIC AIR-BREATHING CONCEPTS





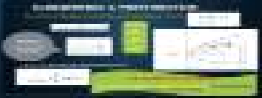


SLENDERNESS & PERTURBATION

GOVERNING EQUATIONS IN PERTURBATION FORMS

[illegible]

NON-DIMENSIONALIZATION OF PERTURBATION EQUATIONS



ORDERS OF MAGNITUDE

- For slender bodies, as $M_\infty \to \infty$

$$\sin\beta \approx \sin\theta \approx \theta \approx \frac{d_b}{l_b} \approx \tau$$

- Across the shock, $\frac{p_2}{p_\infty} \to \frac{2\gamma}{\gamma+1} M_\infty^2 \sin^2\beta$

$$\frac{p_2}{p_\infty} = O(M_\infty^2\tau^2)$$

- Hence the pressure inside shock layer can be non-dimensionalized as,

$$\bar{p} = p/\gamma M_\infty^2 \tau^2 p_\infty$$

ORDERS OF MAGNITUDE - 2

- Similarly:

$$\frac{\rho_2}{\rho_\infty} = \frac{\gamma+1}{\gamma-1}$$

Hence: $\rho' = \rho_2/\rho_\infty$ is $O(1)$

Also:

$$\frac{v_2}{V_\infty} = 1 - \frac{2\sin^2\beta}{\gamma+1}$$

$$\frac{\Delta u}{V_\infty} = \frac{V_\infty - u_2}{V_\infty} = \frac{2\sin^2\beta}{\gamma+1} = O(\tau^2)$$

$u' = u/V_\infty\tau^2$ is $O(1)$

- Similarly:

[illegible]

[illegible]

THE NON-DIMENSIONALIZED PARAMETERS

HYPERSONIC PERTURBATION EQUATIONS

THE HYPERSONIC SMALL-DISTURBANCE EQUATIONS

- A set of coupled, nonlinear partial differential equations
  - No general analytical solution
- Reflects the inherently non-linear nature of hypersonic flow physics
- Leads to hypersonic similarity analysis

SIMILARITY ANALYSIS

SIMILARITY APPROACH

THE EQUATIONS

THE BOUNDARY CONDITIONS

- As $\beta \to 0$ & $\theta \to 0$ (slender body with low angle of attack)
- $\sin\beta \approx \sin\theta \approx \theta$
- By geometry of the body, at surface: $dy/dx = \theta = \tau$

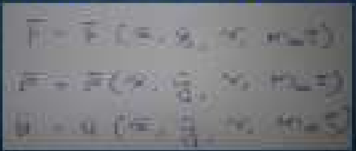
WITH SLENDER BODY ASSUMPTIONS...

[illegible]

M_τ

Referred to as the Hypersonic Similarity Parameter

THE SOLUTIONS ARE EXPECTED AS:



IMPLICATIONS OF *HYPERSONIC SIMILARITY*

$K = M_\infty \tau$

- Solutions for two different flows over two different bodies with similar geometry but different scale in one dimension but affinely related bodies (bodies that have essentially the same mathematical shape but that differ by a scale factor on one direction, such as different values of thickness) will lead to the same values of non dimensional perturbed components v'' and u'' if they have the same value of $K = M_\infty \tau$
-

SIMILARITY RELATIONS FOR *SMALL ANGLES OF ATTACK*

- For small values of angle of attack α the functional relations are modified as:

[illegible]

[illegible]

PRESSURE COEFFICIENT

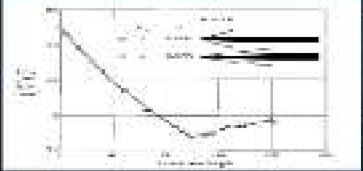
[illegible]

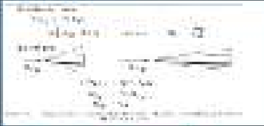
[illegible]

[illegible]

[illegible]

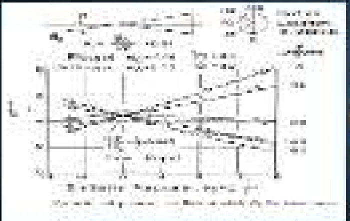
VALIDATION OF HYPERSONIC SIMILARITY – USING EXACT METHODS





EXPERIMENTAL VALIDATION

- [illegible]
- [illegible]



EXAMPLE: VALIDITY OF HYPERSONIC SIMILARITY

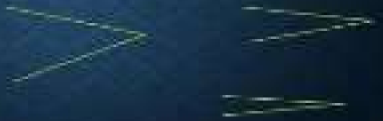
- Consider flow across a wedge-shaped body. Use shock theory [illegible]

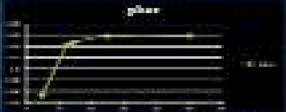
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]

[illegible]

[illegible]

CORNERS WITH VARIOUS ANGLES





HYPERSONIC FLOW THEORY

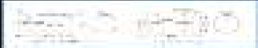
Sessions 18-: Hypersonic Equivalence

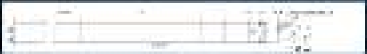
AMRITA

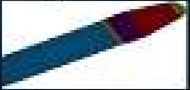
SIMILARITY TO *EQUIVALENCE*

- Hypersonic similarity: For Slender Bodies at small angles of attack–
  - *Flows over affinely related bodies with the same values of γ, $M_\infty\tau$ and α/τ will have the same values of C_p/τ^2 at the corresponding normalized locations*
- Based on small perturbation analysis
- *Hypersonic Equivalence:* Extension of approximate analysis to *blunt-nosed* slender bodies

BLUNT-NOSED SLENDER BODIES







THE COMPARISON OF INVISCID GOVERNING EQUATIONS

CYLINDRICAL (BLAST) WAVES

- [illegible]
- [illegible]
- [illegible]
  watch from [illegible] to [illegible]

HYPERSONIC EQUIVALENCE

The hypersonic equivalence principle is essentially the statement that the steady hypersonic flow over a slender body is equivalent to an unsteady hypersonic flow in one less spatial dimension.

THE ANALOGY

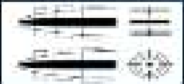
- This analogy establishes the relationship between an unsteady flow and a steady motion with one more spatial dimension.
- Based on this analogy, blast wave results have been successfully used to estimate the pressure distribution for blunt-nosed bodies at hypersonic speeds.
- The analogy theoretically obtains the general trends observed with experimental data
  - Only holds for slender configurations where the small perturbation equations are valid.

BLAST WAVE ANALOGY

- ***Analogy between***
  - Steady flow past a blunt-nosed body of revolution or two-dimensional slab

AND

- An instantaneous energy release in the form of a concentrated explosion in the plane perpendicular to the axis of motion.



propagation of shock wave due to energy release is called blast wave

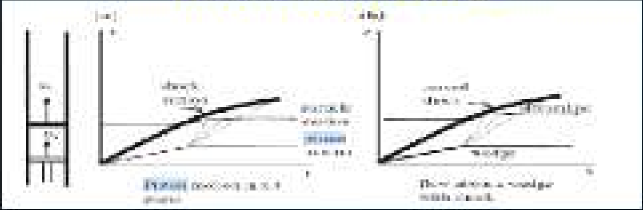
SO ?

Study of three dimensional steady hypersonic flow over a slender object is equivalent to study two dimensional unsteady flow over the same

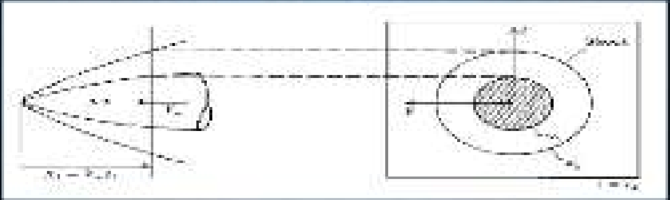
THE COMPARISON OF INVISCID GOVERNING EQUATIONS

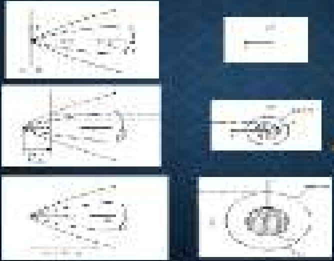
The hypersonic equivalence principle: the steady hypersonic flow over a slender body is equivalent to an unsteady flow in one less space dimension

PISTON-EQUIVALENCE

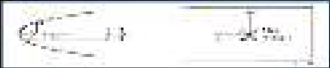



Shock





FROM CLASSIC THEORY OF BLAST WAVES – CYLINDRICAL BLAST WAVE



DRAG VS ENERGY RELEASE

The shock wave generated by this energy release is a cylindrical blast wave, propagating outward in the radial direction

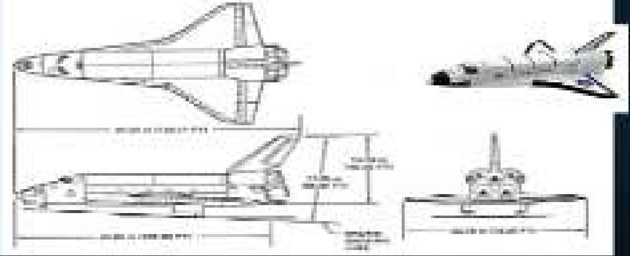
RELATING THE PHENOMENA

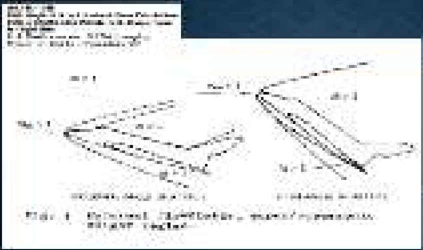
CYLINDRICAL (BLAST) WAVES

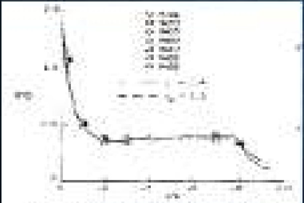
- watch from to

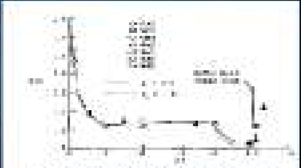
PRACTICAL CONSIDERATIONS

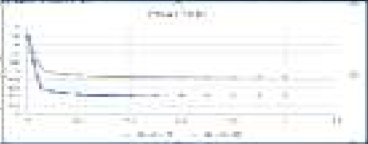
- **Limitation of the Approach:**
  - [illegible]
  - [illegible]
- **Utility**
- Physical meaning behind the analogy between the steady flow over a hypersonic slender body and the unsteady flow in one less space dimension











HYPERSONIC FLOW THEORY

Session 27-28-: Viscous Flow: Introduction

Dr. R.R. Srikrishnan

AMRITA

CAPABILITIES OF THE INVISCID APPROACH

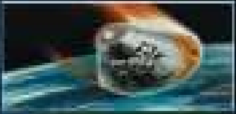
- Shock angle
- Pressure distribution
- Wave drag

Not significantly affected by viscosity

WHY VISCOUS ??

- Friction Drag
- Heat Transfer

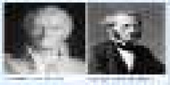
VELOCITY-ALTITUDE MAP FOR RE-ENTRY TRAJECTORIES



HOW DO WE INCLUDE VISCOUS EFFECTS IN ANALYSIS ?

- Consideration of the full Navier-Stokes equations
  - Computational application of NS equations…
- Incorporating boundary layer theory
  - "……the most important intellectual advancement in fluid dynamics since the beginning of the 20th century…."
- Formulation of boundary layer equations

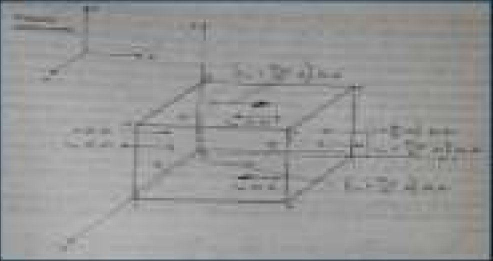
NAVIER STOKES EQUATIONS



- The approach to derivation of:
  - Continuity
  - Momentum &
  - Energy Equations
- Note the terminology difference: Conventionally "N-S" refers to momentum equations in fluid mechanics
- CFD: All governing equations of viscous flow
- Mathematical nature of the equations

CONTINUITY EQUATION

MOMENTUM EQUATIONS



ENERGY EQUATIONS

$$\rho \frac{D(e + V^2/2)}{Dt} = \rho \dot{q} + \frac{\partial}{\partial x}\left(k \frac{\partial T}{\partial x}\right) + \frac{\partial}{\partial y}\left(k \frac{\partial T}{\partial y}\right) + \frac{\partial}{\partial z}\left(k \frac{\partial T}{\partial z}\right) - \nabla \cdot p\mathbf{V}$$
$$+ \frac{\partial (u\tau_{xx})}{\partial x} + \frac{\partial (u\tau_{yx})}{\partial y} + \frac{\partial (u\tau_{zx})}{\partial z} + \frac{\partial (v\tau_{xy})}{\partial x} + \frac{\partial (v\tau_{yy})}{\partial y}$$
$$+ \frac{\partial (v\tau_{zy})}{\partial z} + \frac{\partial (w\tau_{xz})}{\partial x} + \frac{\partial (w\tau_{yz})}{\partial y} + \frac{\partial (w\tau_{zz})}{\partial z}$$

SOLUTION ?

- Approximate Boundary layer equations
- Direct (DT)

NS EQUATIONS IN NON-DIMENSIONAL FORM

- Non-dimensional forms of the variables

[illegible]

...FOR STEADY 2-DIMENSIONAL FLOW

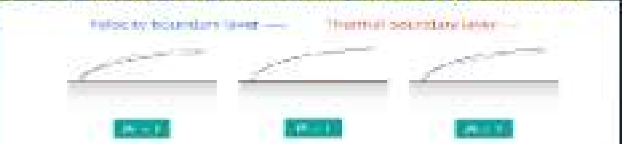
DISCUSSION OF NON-DIMENSIONAL PARAMETERS

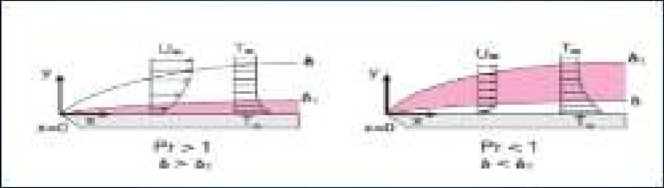
PRANDTL NUMBER: WHAT AND WHY ?

- A non-dimensional parameter prominently used in heat transfer
- Used in calculation of heat transfer between a moving fluid and a solid body
- Defined as $Pr = \frac{\mu C_p}{k}$
- A property of the fluid
- ~ 0.71 for air at standard conditions
- Ratio of momentum diffusivity to thermal diffusivity

PRANDTL NUMBER: WHAT AND WHY ?-CTD.

- Pr << 1 shows that thermal diffusivity dominates
- Pr >> 1 shows that momentum diffusivity dominates
- (




Pr > 1
Pr < 1

BOUNDARY CONDITIONS FOR N-S EQUATIONS (2D)

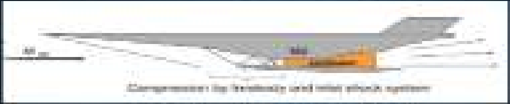
- No slip condition at the wall (without mass transfer): $u = v = w = 0$
- Thermal boundary condition if wall is at constant specified temperature:

$$T = T_w$$

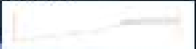
- Thermal boundary condition if wall temperature varies along: $T = T(x)$
- Thermal boundary condition if wall heat flux is prescribed:

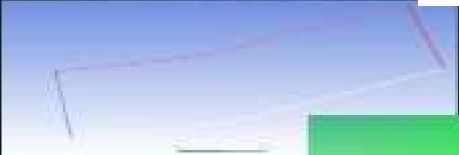
$$q = -k\left(\frac{\partial T}{\partial n}\right)_w$$

SCRAMJET INTAKE

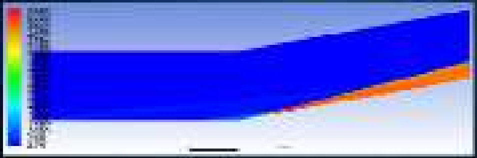


DOUBLE COMPRESSION RAMP

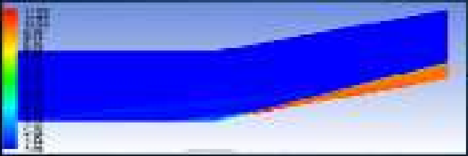




STATIC PRESSURE FIELD



NON-DIMENSIONALIZED STATIC PRESSURE FIELD



$$\bar{p} = \frac{p}{p_\infty}$$

SUBSTANTIAL/MATERIAL DERIVATIVE

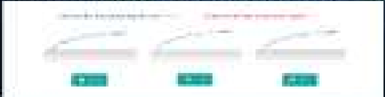
HYPERSONIC FLOW THEORY

Session 29: Boundary Layer Equations

Dr. R.R. Srikrishnan

PRANDTL NUMBER: WHAT AND WHY ?-CTD.

- Pr << 1 shows that thermal diffusivity dominates
- Pr >> 1 shows that momentum diffusivity dominates
- [Pr is around 7.6 for water & > 400 for engine oil]



$Pr > 1$
$\delta > \delta_T$

$Pr < 1$
$\delta < \delta_T$

BOUNDARY CONDITIONS FOR N-S EQUATIONS (2D)

- No slip condition at the wall (without mass transfer): [illegible]
- Thermal boundary conditions if wall is at constant specified temperature:

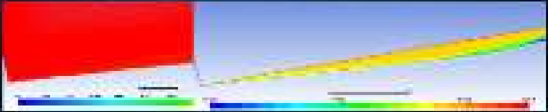
$$T = T_w$$

- Thermal boundary conditions if wall temperature varies along: $T = f(x,t)$
- Thermal boundary conditions if wall heat flux is provided:

$$q = -k \left(\frac{\partial T}{\partial n} \right)_w$$

PRANDTL'S ORDER OF MAGNITUDE ANALYSIS

- Viscous effects are confined to a thin region near the solid surface
- This region is called boundary layer
- The thickness of boundary layer is much smaller than the scale of the body



ORDERS OF MAGNITUDE

- Continuity Equation
- u varies from 0 (at wall) to 1 (Free stream – where $u = V_{inf}$)
- ∂u is of order of magnitude 1
- x varies from 0 to 1: Hence $O(x) = 1$
- y varies from 0 to δ/c (which is δ): Hence $O(y) = \delta/c$
- Considering unit length ($x=1$), the order of magnitude form of continuity eqn becomes: $= 0$ → THIS IMPLIES THAT v is $O(\delta)$

ORDERS OF MAGNITUDE-CTD.

- Similar analysis leads to: [illegible]
- Also Re >> 1
- [illegible] [illegible]
- [illegible]

THE BOUNDARY LAYER EQUATIONS

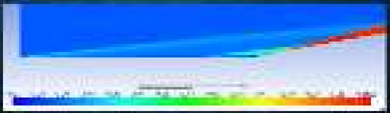
- Using observations based on the order of magnitude analysis, applying across the BL, the dimensional forms of steady, 2D governing equations within boundary layer become:

THE BOUNDARY CONDITIONS

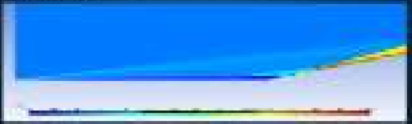
- At the wall: [illegible] [illegible] [illegible]
- At the edge of boundary layer: [illegible]

$$\frac{\partial \varphi}{\partial x} = 0$$

EXAMINING THE ORDERS OF MAGNITUDE...



- β is of order of magnitude 1



APPENDIX

CONTINUITY EQUATION

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \mathbf{v}) = 0$$

MOMENTUM EQUATIONS

[illegible]

[illegible]

ENERGY EQUATIONS

$$\rho \frac{D(e + V^2/2)}{Dt} = \rho \dot{q} + \frac{\partial}{\partial x}\left(k \frac{\partial T}{\partial x}\right) + \frac{\partial}{\partial y}\left(k \frac{\partial T}{\partial y}\right) + \frac{\partial}{\partial z}\left(k \frac{\partial T}{\partial z}\right) - \nabla \cdot p\mathbf{V}$$

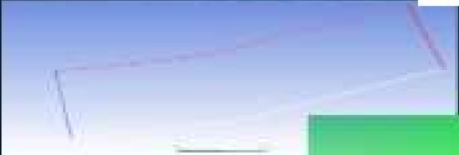
[illegible]

[illegible]

SCRAMJET INTAKE

Compression by forebody and inlet shock system

DOUBLE COMPRESSION RAMP



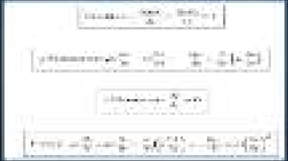
HYPERSONIC FLOW THEORY

Session 31-33: Self Similarity Transformation in Laminar Boundary Layer

Dr. S.R. Srikrishnan

AMRITA

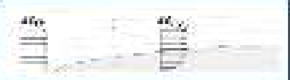
NS EQUATIONS AS APPLIED TO BL – 2D, STEADY



THE TRANSFORMATION: OBJECTIVE

- Transform the governing PDE to ODE which are easier to solve
- Such transformations lead to a similarity solution: the (velocity) profile becomes independent of location along the surface

THE TRANSFORMATION



- In order to transform the dimensionless variables, define t and g such that

TRANSFORMED GOVERNING EQUATIONS

[illegible]

[illegible]

[illegible]

[illegible]

[illegible]

BC'S IN TERMS OF THE TRANSFORMED VARIABLES

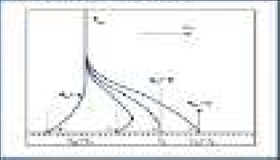
THERMAL BOUNDARY CONDITIONS: ADIABATIC WALL TEMPERATURE

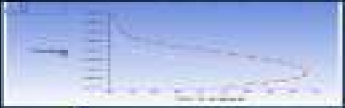
- T_{aw} is the temperature of the wall when heat transfer into the wall is zero.

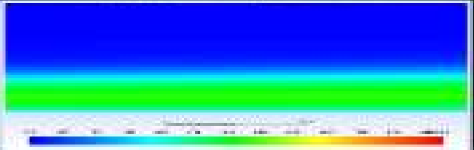
Curve 1: Adiabatic wall, q = 0

Curve 2: Heat transfer from the wall into the fluid

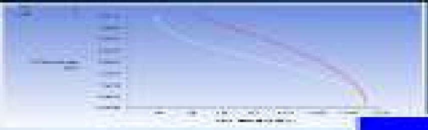
Curves 3, 4: Heat transfer from the fluid to the wall

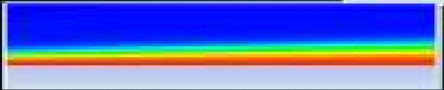






ADIABATIC WALL

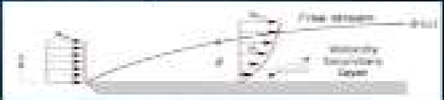




FLOW OVER A FLAT PLATE

- For flat plate: [illegible]

[illegible]



[illegible]

$$C = \frac{\rho_w \mu_w}{\rho_e \mu_e}$$

$$C_f = \frac{\tau_w}{\frac{1}{2}\rho_e u_e^2}$$

$$\tau_w = \left[\mu\left(\frac{\partial u}{\partial y}\right)\right]_w$$

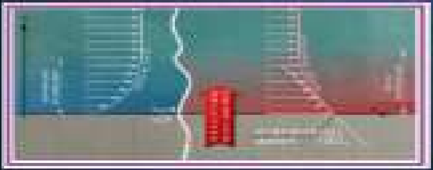
$$q_w = \left[k\frac{\partial T}{\partial y}\right]_w$$

NUSSELT NUMBER & STANTON NUMBER

$$Nu = \frac{q_w x}{k_w (T_{aw} - T_w)}$$

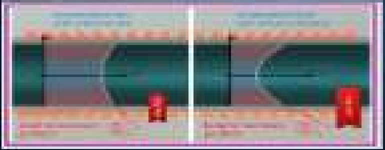
$$C_H = \frac{q_w}{\rho_e u_e (h_{aw} - h_w)}$$

CONDUCTION VS CONVECTION



CONDUCTION VS CONVECTION – CTD.

- Left: When the fluid is at rest; Right: When the fluid is flowing



NUSSELT NUMBER

- Rate of [illegible] heat transfer:
- Rate of Conduction heat transfer:
- Nusselt number is the ratio of convection heat transfer to conduction heat transfer under identical conditions.

$$Nu_L = \frac{\dot{Q}_L(convection)}{\dot{Q}_L(conduction)} = hL/k$$

SKIN FRICTION COEFFICIENT

- Cf: The associated wall shear stress [illegible]

  [illegible]

- Similarly analysis for Blasius flatplate leads to Cf: [illegible]

STANTON NUMBER (S_T OR C_H)

- Consider the grouping: [illegible] [illegible] [illegible]
- This non-dimensional parameter is referred to as Stanton number
- St (C_H) Measures the ratio of heat transferred into a fluid by convection to the thermal capacity of fluid
- Similarity analysis of flow over flat plate leads to CH = [illegible]

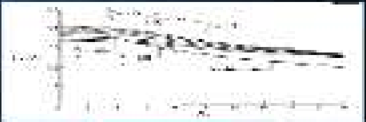
REYNOLDS ANALOGY

- The relation between momentum transfer and heat transfer is:

[illegible]

[illegible]

REYNOLDS ANALOGY



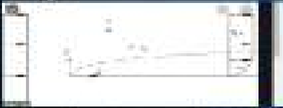
SUMMARY OF THE BOUNDARY LAYER ANALYSIS

SUMMARY OF THE BOUNDARY LAYER ANALYSIS – CTD.

- [illegible]
- [illegible]
  - Shear stress can be used to calculate skin friction which leads to the drag force
  - Aerodynamic forces can then be calculated
- [illegible]

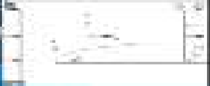
NUMERICAL PROBLEM

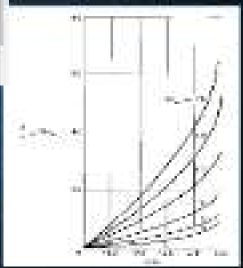
- [illegible]



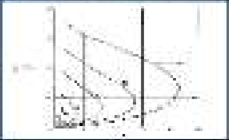
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]

MORE RESULTS





TEMPERATURE PROFILE IN HYPERSONIC BL



RECOVERY FACTOR

- If the deceleration at the wall is without heat loss, wall temp Tw will be equal to T0
- Since there is heat loss, the temperature of an adiabatic wall will be lower than T0. This is quantified by a recovery factor:

$$r = \frac{T_{aw} - T_e}{T_0 - T_e} = \frac{T_{aw} - T_e}{T_{0e} - T_e}$$

- Typically, recovery factor can be estimated as $r = \sqrt{Pr}$

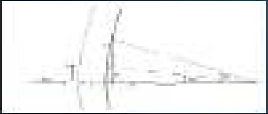
NUMERICAL PROBLEM

- Consider a flat plate exposed to hypersonic velocity stream of air at [illegible] m/s under ambient conditions of pressure = [illegible] N/m² & temperature = [illegible] K. Length of the plate is [illegible]. Calculate shear stress at a location [illegible] m from the leading edge. Assume the wall temperature to be maintained constant at [illegible] K. Calculate the heat flux at the location.

SOLUTION

[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]

STAGNATION POINT BL OF BLUNT BODY SURFACE



SPECIFIC SIMPLIFICATIONS

- [illegible] the similarity analysis with specific simplifications that apply to the stagnation point [illegible]
  - In this region, velocity is low [illegible]
  - [illegible] stagnation
  - Hence: 
  - Also, due to the low velocity in the region, incompressible flow relations [illegible] can be applied.

THE TRANSFORMED GOVERNING EQUATIONS

- The above approach leads to the non-singular governing equations at the singularity (region):

[illegible]

STAGNATION POINT HEAT TRANSFER

- A solution for the above equations as applied to a cylindrical leading edge is obtained as:

$$q_w = 0.57\,Pr^{-0.6}(\rho_e \mu_e)^{1/2}\sqrt{\frac{du_e}{dx}}\,(h_{aw} - h_w)$$

- There are the velocity gradient terms in the above solution. Using Euler's equation for edge of the BL and approximating surface pressure to the Newtonian theory, it can be shown that:

- R: The radius of curvature

$$\frac{du_e}{dx} = \frac{1}{R}\sqrt{\frac{2(p_e - p_\infty)}{\rho_e}}$$

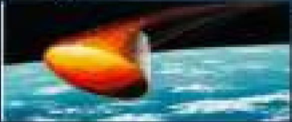
- Hence, the wall heat transfer to the stagnation region:

$$q_w \propto \frac{1}{\sqrt{R}}$$

- Stagnation-point heating varies inversely with the square root of the nose radius. Hence, *to reduce the heating, increase the nose radius*

BLUNT BODIES IN HYPERSONIC FLOWS

- This shows why hypersonic applications typically have blunt nose/blunt leading edges



APPENDIX

NON-DIMENSIONALIZED NS EQUATIONS, STEADY, 2D

[illegible]

CONTINUITY EQUATION

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \mathbf{v}) = 0$$

MOMENTUM EQUATIONS

ENERGY EQUATIONS

[illegible]

HYPERSONIC FLOW THEORY

Session 34: Hypersonic TURBULENT flow-1. Transition

Dr. A.R. Srikrishnan

THE CONCEPT OF TRANSITIONAL REYNOLDS NUMBER

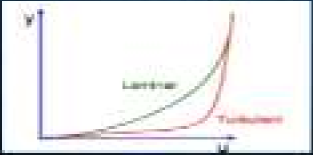
[illegible]

- Essential to incorporate the impact of skin friction & heat transfer into the analysis
- Reliance on empirical data in the absence of theoretical means to predict transition accurately

THE TRANSPORT PROCESSES

- In a laminar boundary layer any exchange of mass or momentum takes place only between adjacent layers on a microscopic scale [which is not visible to the eye]
- Turbulent boundary layer is characterized by mixing across several layers of it.
  - The mixing is on a macroscopic scale
  - Packets of fluid may be seen moving across
  - Thus there is an exchange of mass, momentum and energy on a much bigger scale compared to a laminar boundary layer

TYPICAL VELOCITY PROFILES



TRANSITION TO TURBULENCE

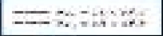
[illegible]

[illegible]

EFFECT OF MACH NUMBER

- In hypersonic range, increase in Mach number delays the transition to turbulence
- Relatively large region of laminar flow that can occur over a body at hypersonic speeds

EFFECT OF ANGLE OF ATTACK



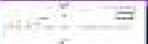
EFFECT OF BLUNTNESS

- Impact of the entropy layer—the thermodynamic properties such as T [illegible] through the [illegible] in the layer
- Small nose radius: by adding a blunt nose to the cone, transition has been delayed to a distance farther downstream of the nose tip.
  - The transition Reynolds number is increased by such bluntness
- In contrast, for large bluntness transition can occur prematurely

EFFECT OF BLUNTNESS

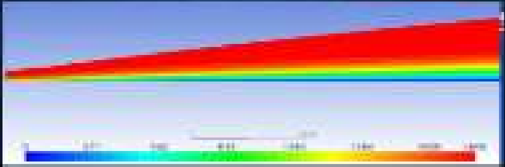
For low values of bluntness, the transition is delayed (occurs at higher Reynolds No)

The trend is found to be the opposite for higher values of bluntness (larger nose radius)



EFFECT OF WALL TEMPERATURE

- [illegible]
- [illegible]



HYPERSONIC FLOW THEORY

Session 36 37: Hypersonic TURBULENT flow-2: Effects of Turbulence

Dr. A.R. Srikrishnan

TURBULENCE VISCOSITY

- [illegible]
- [illegible] equation can be written as:

$$[illegible]$$

- Eddy viscosity

TURBULENT PRADNTL NUMBER & TURBULENT THERMAL CONDUCTIVITY

- Turbulent Pr Defined analogously to Prandtl number in laminar flow
- Laminar thermal conductivity:

$$k = \frac{\mu c_p}{Pr}$$

- Turbulent Thermal Conductivity

$$k_t = \frac{\mu_t c_p}{Pr_t}$$

CRITICAL INFLUENCE ON WALL FRICTION & HEAT TRANSFER

[illegible]

→

[illegible]

CP WITH MACH NUMBER

EFFECT ON HEAT TRANSFER

HYPERSONIC FLOW THEORY

Session 38 39: Viscous Interaction in Hypersonic Flow

Dr. S.R. Srikrishnan

AMRITA

IMPACT OF VISCOSITY ON FLOW/FLUID PROPERTIES

- As hypersonic flow decelerates around the body (after shock) part of kinetic energy gets dissipated as thermal energy, increasing the temperature significantly
- As we know: Density drops with temperature
  - In BL, a drop in density implies the need for increased thickness to admit the mass flow
- Viscosity of gases increases with temperature
- Self-similar solution for laminar BL leads to:

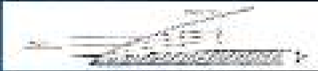
$$\delta \approx \frac{x}{\sqrt{Re}}$$

- Assuming a linear dependence of viscosity on temperature: [illegible]
- Hence: [illegible]
- Assuming complete recovery of temperature within BL ($r = 1$):

[illegible]

- This leads to: [illegible]

- Hypersonic boundary layers can be orders of magnitude thicker than low-speed boundary layers at the same Reynolds number

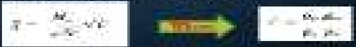


- The thick hypersonic boundary layer displaces the outer inviscid flow
  - Hence the inviscid streamlines are displaced changing the nature of the inviscid flow
  - For hypersonic viscous flow with a thick boundary layer, the inviscid streamlines are displaced upward, causing a shock wave at the leading edge
- **This is the source of the viscous interaction**

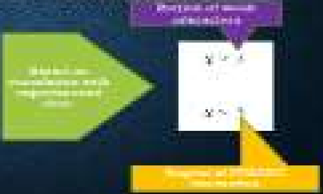
THE REGIONS OF VISCOUS INTERACTION

WEAK INTERACTION vs STRONG INTERACTION

- Rapidly growing BL interacts with the terminal shock wave.
- If the terminal shock is significantly influenced, it will feedback the impact to the viscous region, the BL. This is referred to as STRONG Interaction.
- If the shock wave is not significantly influenced, the feedback effect on the BL will not be significant either. This is referred to as WEAK Interaction.
- Both types of interaction lead to an increase in static pressure, which can be quantified as a function of the non-dimensional parameter:



THE IMPACT OF VISCOUS ITERACTIN ON SURFACE STATIC PRESSURE



SOME RELATIONS FOR SURFACE PRESSURE

- For an adiabatic plate, for the case of STRONG interaction: $\frac{p}{p_\infty} = 0.514\bar{\chi} + 0.759$
- For an adiabatic plate, for the case of WEAK interaction: $\frac{p}{p_\infty} = 1 + 0.31\bar{\chi} + 0.05\bar{\chi}^2$
- For a cold wall, where $T_w \ll T_0$, the strong interaction is $\frac{p}{p_\infty} = 1 + 0.15\bar{\chi}$
- For a cold wall, where $T_w \ll T_0$, the weak interaction is $\frac{p}{p_\infty} = 1 + 0.078\bar{\chi}$

HYPERSONIC FLOW THEORY

Session 10: High Temperature Effects in Hypersonic Flow

Dr. A.R. Srikrishnan

AMRITA

THE KEY ISSUE: KE TO INTERNAL ENERGY

- Static temperature Vs Stagnation temperature
- [illegible]
    - Due to shock
    - And due to viscous effects
- [illegible]

PERFECT GAS ASSUMPTION

- Note that the relation [illegible] relies on perfect gas assumption.
- [illegible]
- Apollo Reentry had a Mach number of [illegible] (ambient atmospheric temperature = [illegible] K).
  - If we use the perfect gas based relation for T_0 this corresponds to [illegible] K!
  - Actual stagnation temperature [illegible] K.
- Why the difference ?!

THE VALIDITY OF PERFECT GAS ASSUMPTION

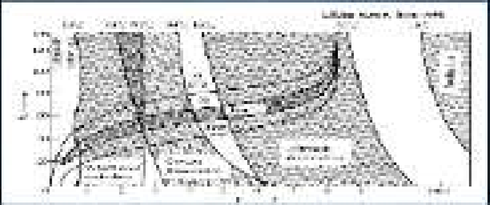
- Neglects intermolecular forces
- Applies under:
  - Conditions where pressure is not too high – less molecular collisions
  - Temperatures are sufficiently high (> 20 K)
  - When the gases are stable and no reactions
- Under [illegible] conditions the [illegible] are [illegible] and hence the perfect gas assumption is [illegible] accurate

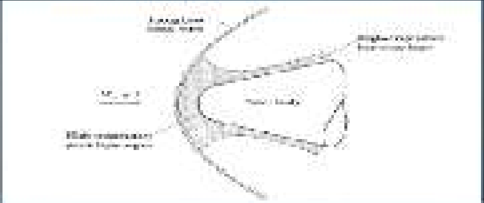
THE NEED TO INCORPORATE HIGH TEMPERATURE EFFECTS

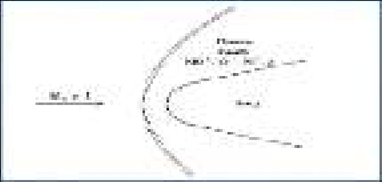
- Note the difference in stagnation temperatures calculated in the Apollo case cited above
- In order to have any meaningful estimate, the ensuing chemical changes need to be accounted for.
- What are they ?
  - Reacting flows
  - The thermodynamic properties vary significantly
  - Transport properties (μ, k) vary significantly with temperature in this range
  - Cp and gamma are NOT constant

THE "CHEMICAL" EFFECTS IN RE-ENTRY

VELOCITY-ALTITUDE MAP: FLIGHT REGIONS ASSOCIATED WITH VARIOUS "CHEMICAL" EFFECTS IN AIR







REACTING FLOWS

- As stated previously, at high temperature air becomes chemically reacting
  - [illegible]
- [illegible]
  - Chemical Equilibrium: [illegible]
- Non equilibrium
  - For situations where the gas is not in equilibrium
    - [illegible]

REACTION TIME

- [illegible]
  [illegible]
- [illegible]
  - [illegible]
- [illegible]
  - [illegible]
  - [illegible]
  - [illegible]

LOW DENSITY FLOWS/RARIFIED FLOWS

- For very high speeds at low density which might correspond to a high atmospheric altitude, the mean free path, which is the average distance that a molecule or atom travels between collisions with its neighbours, can be large than a characteristic length of the vehicle.
- The air density can become low enough that only a few molecules impact the surface per unit time, and after these molecules reflect from the surface they do not interact with the incoming molecules. This is the regime of free molecular flow.
- The similarity parameter that governs these different regimes is the Knudsen number, defined as $Kn = \lambda/L$ where L is a characteristic dimension of the body [illegible]
  - [illegible]
  - [illegible]

- The effects of free molecular flow begin around a value of Kn > 0.2 and extend on to the limit of Kn (approaching infinity)
  - Hence, the transitional regime is nominally contained within $0.03 < Kn < 1.0$
- A hypersonic vehicle entering the atmosphere from space will encounter the full range of these low-density effects, down to an altitude below which the full continuum assumptions are valid

HYPERSONIC FLOW THEORY

Sessions 42-43: Shock Interactions

Dr. A.R. Srikrishnan

SHOCK-SHOCK INTERSECTION

- The patterns produced when two shocks intersect or interfere are classified into what are now commonly acknowledged as six types
  - Variants may exist in some circumstances

TYPE I INTERSECTION

- Two shocks of opposite families intersect at a p

- Shock C1: Pressure jumps from p1 to p3
- Shock C2: Pressure jumps from p1 to p2
- Incompatible downstream conditions requires the formation of 2 more shocks C3 & C4
  - So that the flow adopts a common direction subsequently
  - Region 2 & 3 have same static pressure but different velocities & stagnation pressures

- [illegible] C1 & C2 [illegible] [illegible] (CE) [illegible] C1 & C2
    - The [illegible]
    - [illegible]

TYPE VI

- Type VI interference occurs when two oblique shocks from the same family (C1 and C2) meet in a region where they [illegible]. Both weak oblique shocks from the same family.
  - The flow pattern is simpler than in the previous cases.
- For two shocks, (C1) and (C2) meet at higher point [illegible] shock (C3) is formed [illegible]

[illegible]

HYPERSONIC FLOW THEORY

COURSE CONCLUSION

Dr. R.R. Srikrishnan

AMRITA

- [illegible]
- [illegible]
- [illegible] heat transfer- was examined [illegible] with the application of the [illegible] modes of analysis.
- The complications that arise out of turbulence and high temperature effects are discussed.
- [illegible] the theoretical conclusions

Thank you !!