

HYPERSONIC FLOW THEORY



Dr. S.R. Srikrishna
2021

Session -01:
Course
Introduction

ANARITA
ANALYTICAL AND NUMERICAL RESEARCH INSTITUTE

COURSE OBJECTIVE & SYLLABUS

Course Objective: At the end of the course the students are expected to demonstrate their understanding of the concepts of hydrostatics, hydrodynamics and fluid mechanics. They are expected to be able to apply the concepts of hydrostatics, hydrodynamics and fluid mechanics to solve problems in areas such as fluid statics, fluid dynamics, fluid mechanics and other practical situations. To understand and apply the concepts of fluid mechanics in various areas like design.

Syllabus:

UNIT I	UNIT II	UNIT III
Fluid statics and hydrostatics	Fluid dynamics and hydrodynamics	Fluid mechanics and fluid statics
Fluid statics and hydrostatics	Fluid dynamics and hydrodynamics	Fluid mechanics and fluid statics

COURSE PLAN: **OVERVIEW OF THE COURSE**

- The course plan will follow the sequence:
 - Introduction to hypersonic flight. Complete phenomena related to hypersonic aerodynamics. Flight path
 - Analysis of hypersonic flow using inviscid theoretical models
 - Study of the application of the Method of Characteristics (MOC)
 - Analysis of viscous phenomena

TEXT BOOK & REFERENCES



Text Book

- John D. Anderson, "Hypersonic and High Temperature Gas Dynamics," McGraw-Hill, 2003.

References

- Matthew A. Haines and Michael A. Gallis, "Hypersonic Flow Theory," 2nd edition, Academic Press.
- Fundamentals of Hypersonic Aerodynamics, Fletcher, 2008.

SYLLABUS



1992

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HYPersonic FLOW THEORY

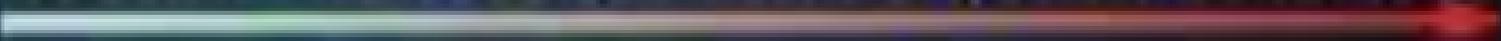


Download-Use
Hypersonic Flight

Dr. A. B. Kulkarni, Department of Aerospace
Engineering

SAARITA
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SUPERSONIC ($M > 1$) TO HYPERSONIC ($M > 5$)



- Not a step-change!
 - Quite common in supersonic
 - The only detail is "which rules are dominant/being dominant for physics of the problem?"
- More a continuously broadening gradient between **supersonic & hypersonic regimes**
- Nothing dramatically happens at $M=5$ – The changes are gradual and increasingly become prominent at higher Mach numbers
- **A hypersonic vehicle can be an airplane, missile, or spacecraft.**

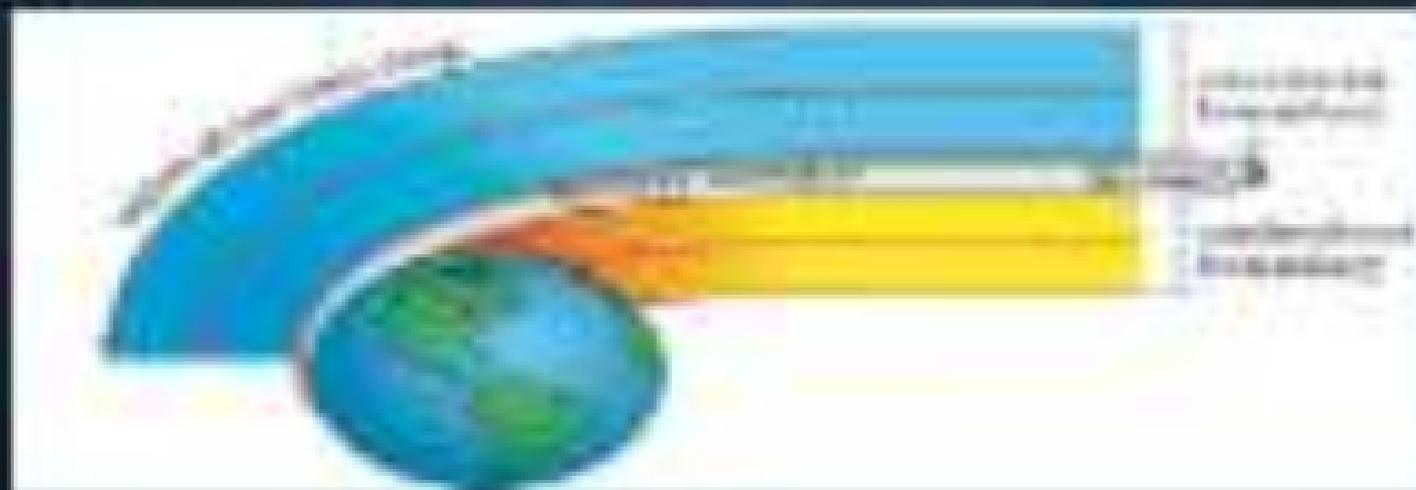
ATMOSPHERIC RE-ENTRY

- H - SET - SIX

- Shape formation of the nose
- Supersonic temperature ?



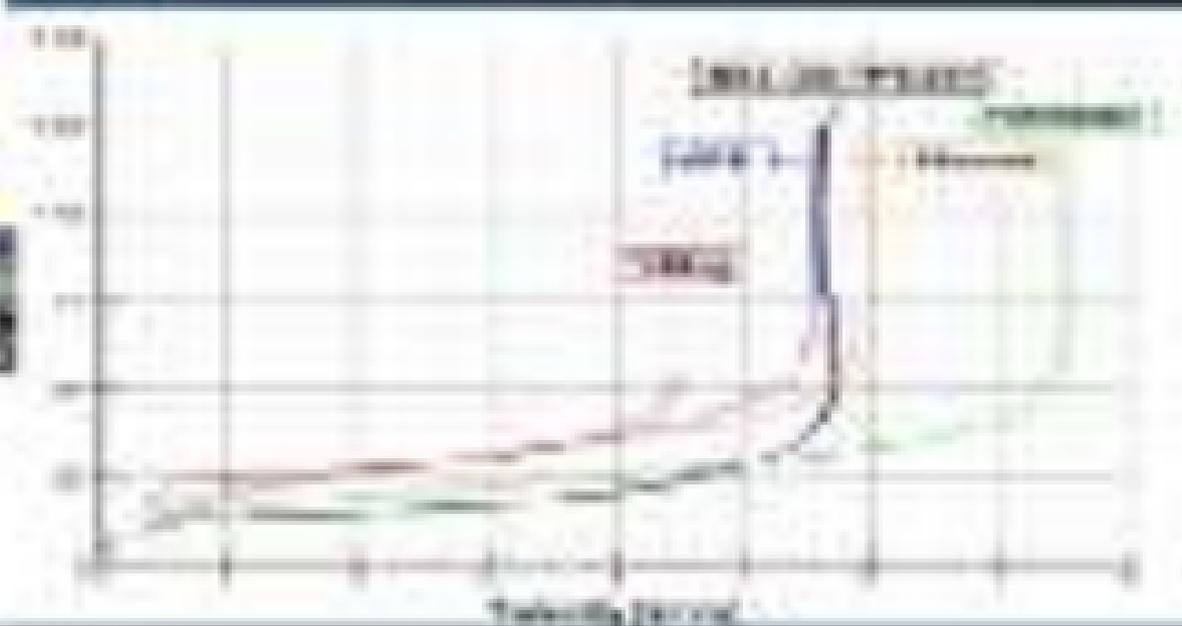
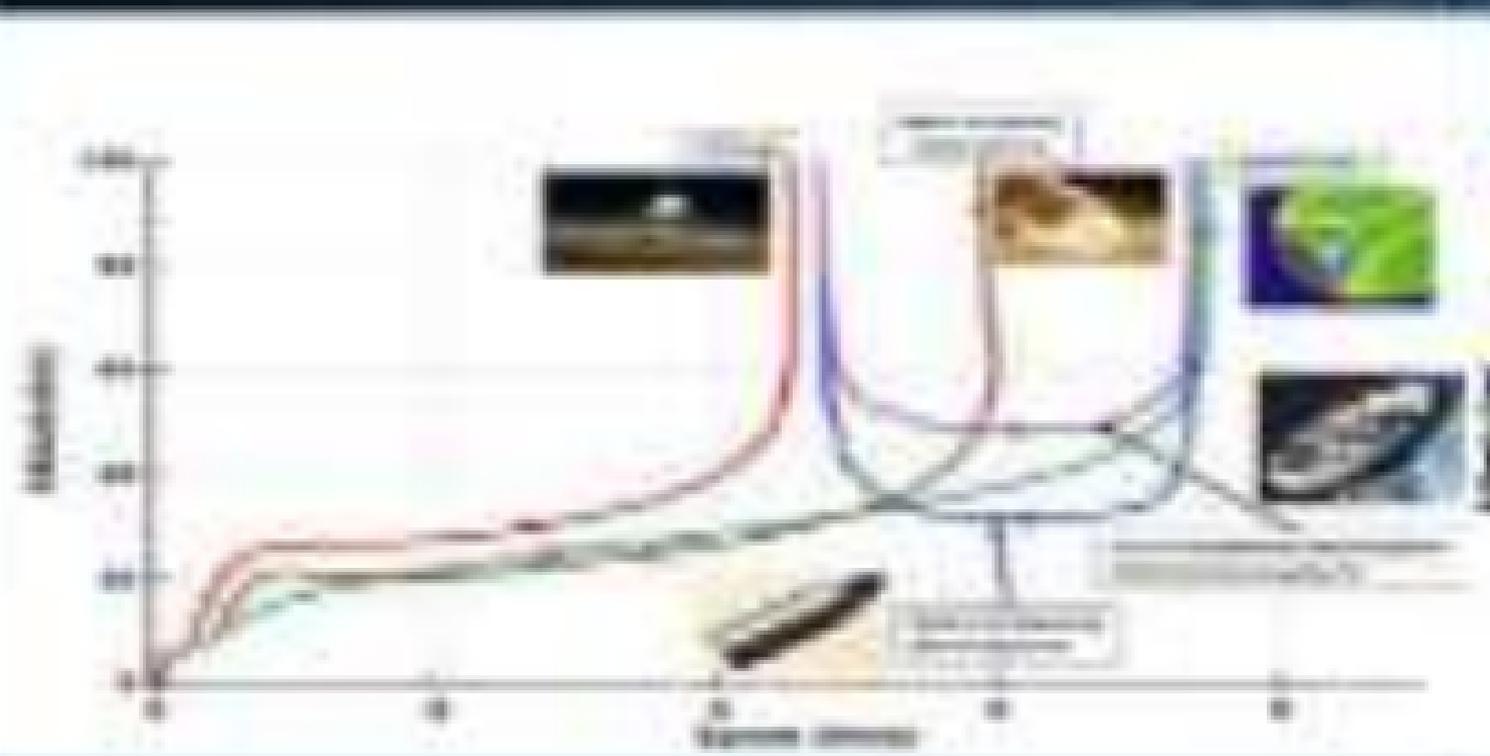
THE RE-ENTRY DILEMMA: MISS THE ATMOSPHERE OR GET BURN'T IN IT !



THE RE-ENTRY PROFILE (FOR SPACE SHUTTLE)

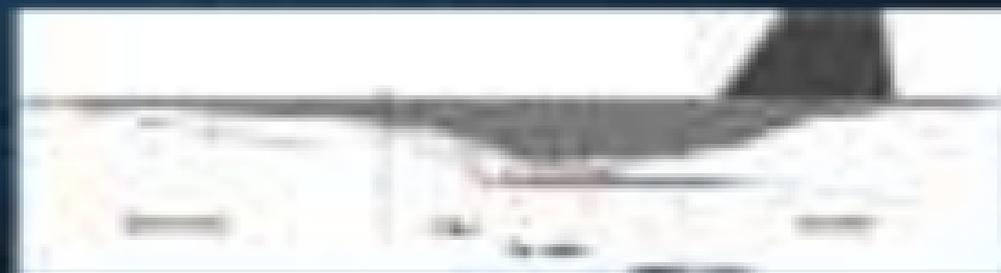


HYPERSONIC FLIGHT TRAJECTORY



HYPERSONIC AIRBREATHING "CONCEPT"

- Flow acceleration from hypersonic shock waves
- Compression of oblique shock waves
- Scramjet as core (SFA)



CONDENSED FORM

- The overall value of β is **highly** lower a **smallly**
- Some **regions** have **higher** **values** of β
- **Value** **generally** **increases** **from** **the** **center** **to** **the** **outer** **regions** **of** **the** **structure**
- These **values** **are** **directly** **related** **to** **the** **size** **of** **the** **structure**
- **Finally** **these** **values** **are** **related** **to** **the** **size** **of** **the** **structure**



WHAT DOES THIS MISSION ACCOMPLISH OVER THE CONVENTIONAL ROCKET LAUNCHES ?

- No **expensive** parts – like a rocket, for which each stage is thrown away after the first use
 - Lower price (100 million, **300 million**)
- **Flexibility** in mission profile and payload (for **commercial**)
- **REUSABILITY**

SINGLE/DOUBLE STAGE TO ORBIT CONCEPT (SSTO/DTSTO)

- Low cost solution to space
- Low risk, low operational and maintenance
- Fully reusable launch vehicle
- DTSTO - 2 stage launch vehicle
 - 1. First stage lift off and accelerate the vehicle during ascent
 - 2. Later the second stage is injected and takes it to orbit

WHAT MAKES IT TOUGH ?!

- History: Conceptualization
- Global efforts
- Present status
- **Main issues converging on to the problems associated with hypersonic aerodynamics.**
- Complexity & Diversity of the issues

DIDN'T WORK !



ALL INFORMATION CONTAINED HEREIN IS UNCLASSIFIED

HYPERSONIC FLOW THEORY



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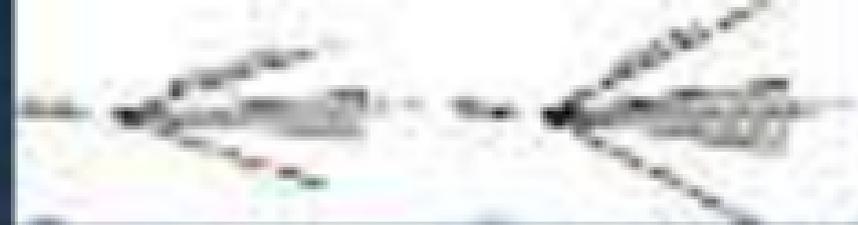
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CHARACTERISTICS OF HYPERSONIC FLOWS

- Thin shock layers
- Diamond layer
- High temperatures effects
 - Dissociation
 - Ionization



THIN SHOCK LAYER

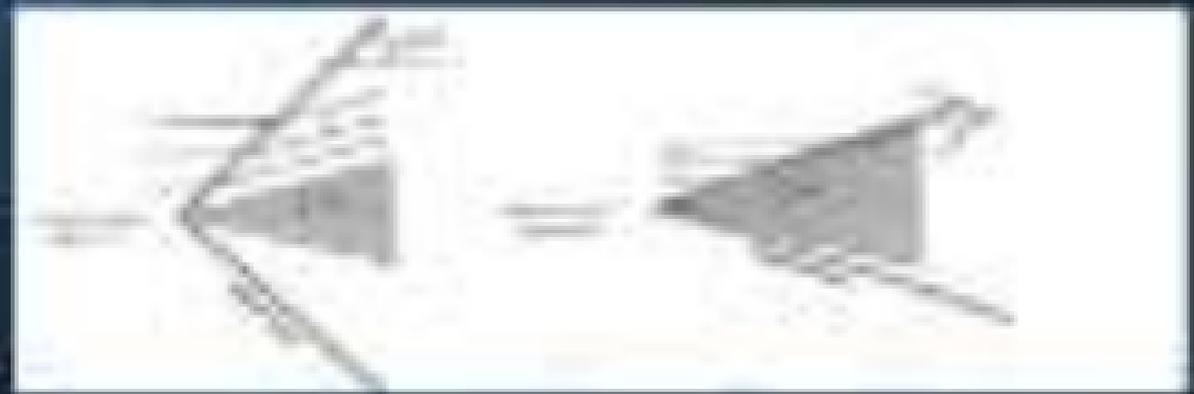


- With increase in Mach number, the wave angle θ keeps decreasing.
- In hypersonic Mach number range, the angle θ is so low that the physical distance between the body and the shock wave is very small.
- Hypersonic flows can thus be treated by very thin shock layers.



THIN SHOCK LAYER

- Thin Shock Layer
- Shock layer is much thinner
- Fully viscous shock layer



VISCOUS INTERACTION

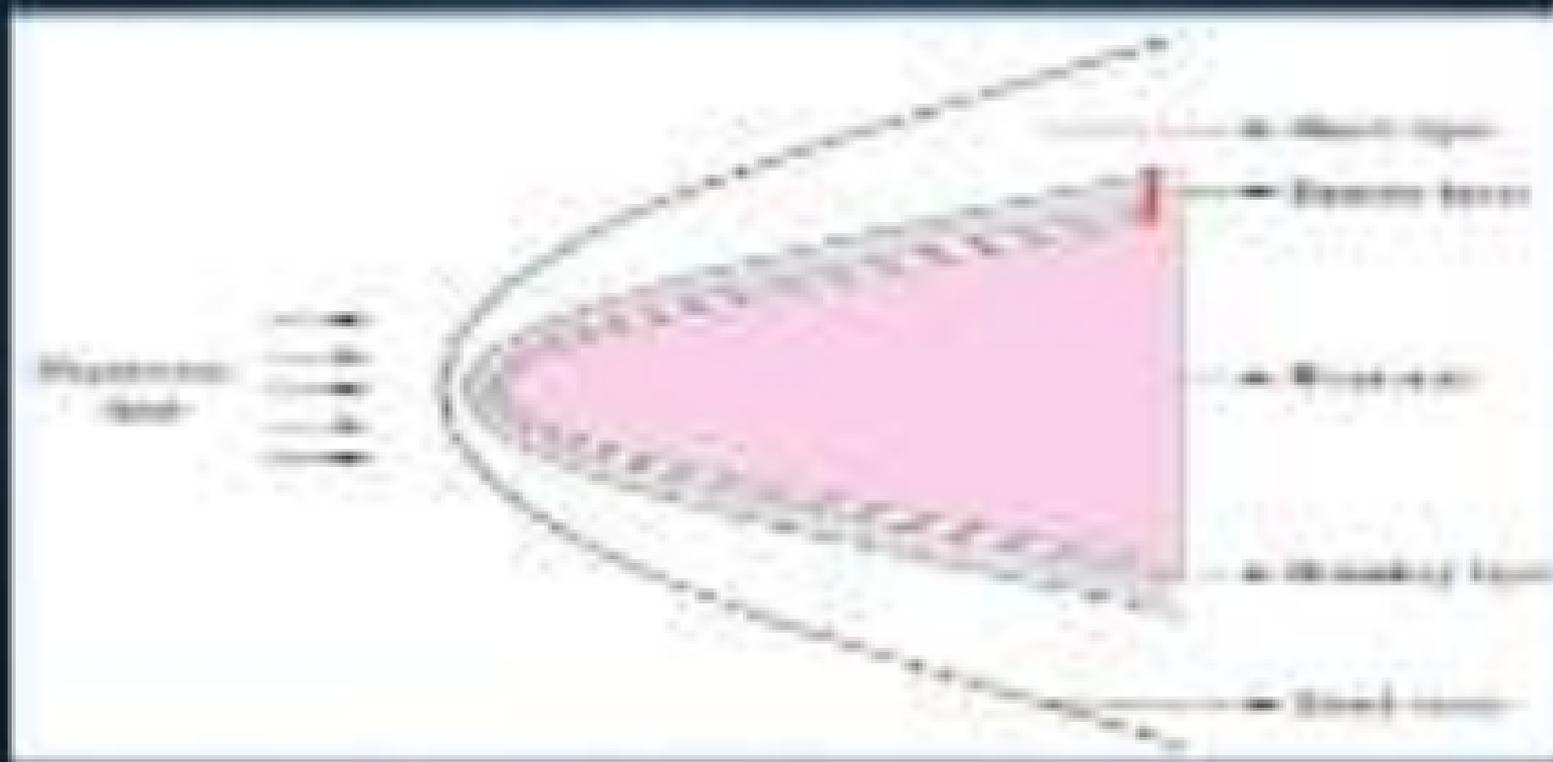
- Temperature increases the turbulent layer and the more turbulent flow
- The flow at high speeds (including flow inside veins) close to the body heats over the body surface causes the turbulent layer to increase further
- The turbulent layer is shown in a cross-section of a vessel (see the diagram in the next slide of the next lecture)
- The width of these layers is the boundary layer thickness δ in the square of the fluid velocity:

- Thus, as blood velocity increases, the boundary layer does not move rapidly as might be very high flow
 - Thus, it means that the vessel diameter is less the body
 - Vessel diameter can have a great influence on the actual pressure distribution and also on the body
 - Impact the flow velocity and thereby mass transport of the body

ENTROPY LAYER

- Flow within these boundary layers, the velocity starts increasing rapidly as we go
- Entropy is constant across a shock, and the entropy is much higher than it is on the shock strongly increases.
- Since flow over the nose passes through a nearly constant shock, it will experience a much smaller change in entropy compared to flow passing through the curved shock over the upper fuselage area, the boundary layer.
- Thus, along entropy gradients will flow the leading edge gasdynamics or "entropy layer" that flows downstream along the body surface.
- The thickness of the entropy layer is small relative





HIGH TEMPERATURE EFFECTS



- If hypersonic vehicles are still development of kinetic energy to thermal energy
 - Needs to high temperatures that cause chemical changes to occur in the fluid through which they fly
- **The most notable changes in properties as temperature increases are:**

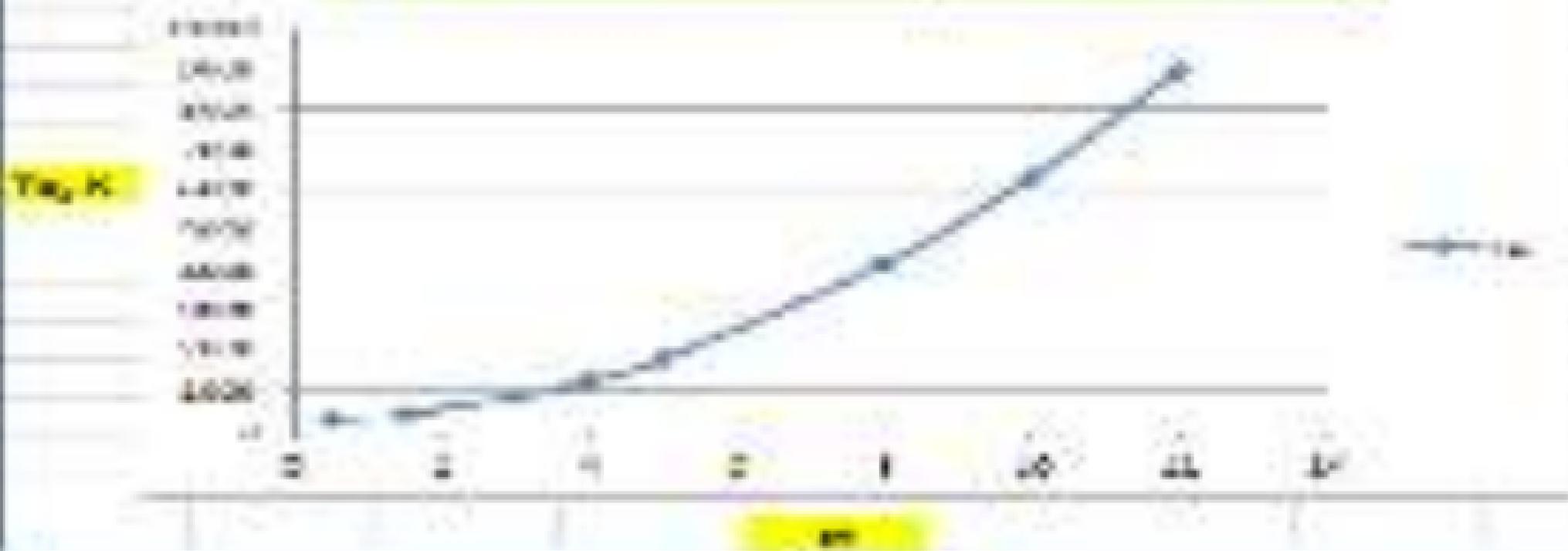


- Perfect gas assumptions do not hold at high temperatures
- **Chemically reacting boundary layer**
- **The aerodynamic flow field becomes strongly nonsteady**

High Temperature Effects on Air

Temperature (K)	Chemical Change
300	All chemical reactions
2000	Large increase in C_p of molecules
3000	4 major species in 2% dissociation 10% dissociation at 4000K
4000	Over 20% dissociation at 4000K

T_0 for a Perfect Gas (when $T = 300\text{ K}$)



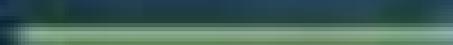


Steps of Molecular IRM Study

HIGH TEMPERATURE REACTIONS

• Dissociation

CO_2



NO_2

Measuring at $\sim 2000\text{K}$

NO



NO

Measuring at $\sim 1500\text{K}$

• Ionization (above 4000K) Formation of charged species

$\text{O} \rightarrow \text{O}^+$ Releasing electron

$\text{N} \rightarrow \text{N}^+$ Releasing electron



HYPERSONIC FLOW THEORY

Session 01-05

*Asymptotic Limit
Approximations*

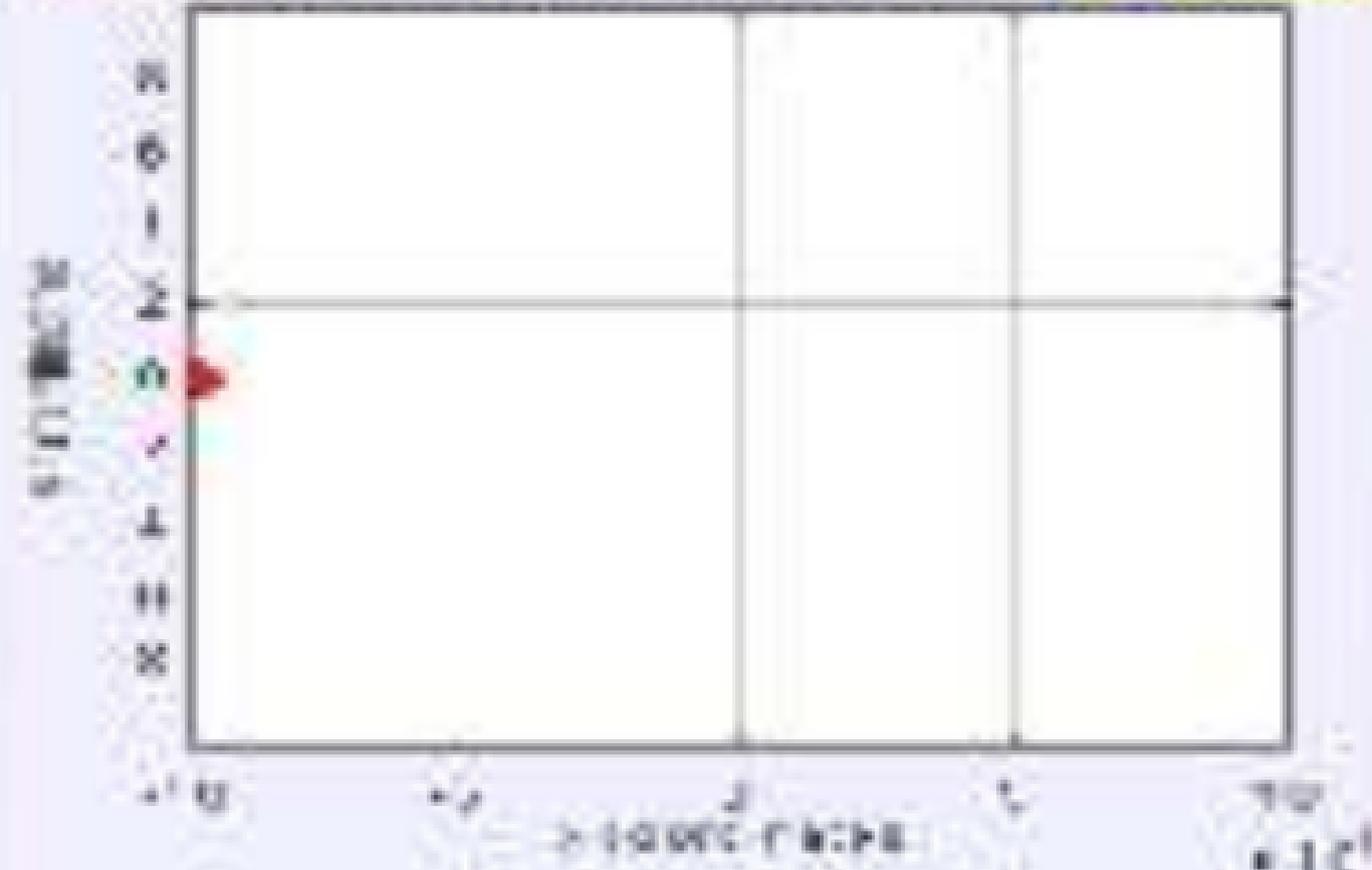


Dr. G. G. Kikvidze

2024

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INVISCID HYPERSONIC FLOW THEORY



SHOCK-EXPANSION THEORY



THE APPROACH

- Oblique shock and expansion waves
- Basic definitions & Governing Equations
- Property Relations de $M \rightarrow \infty$
- Applications

REVIEW: OBLIQUE SHOCK RELATIONS

- Review of the formulation
- Comparison with normal shock relations



REVIEW: OBLIQUE SHOCK RELATIONS – CTD.

Governing equations

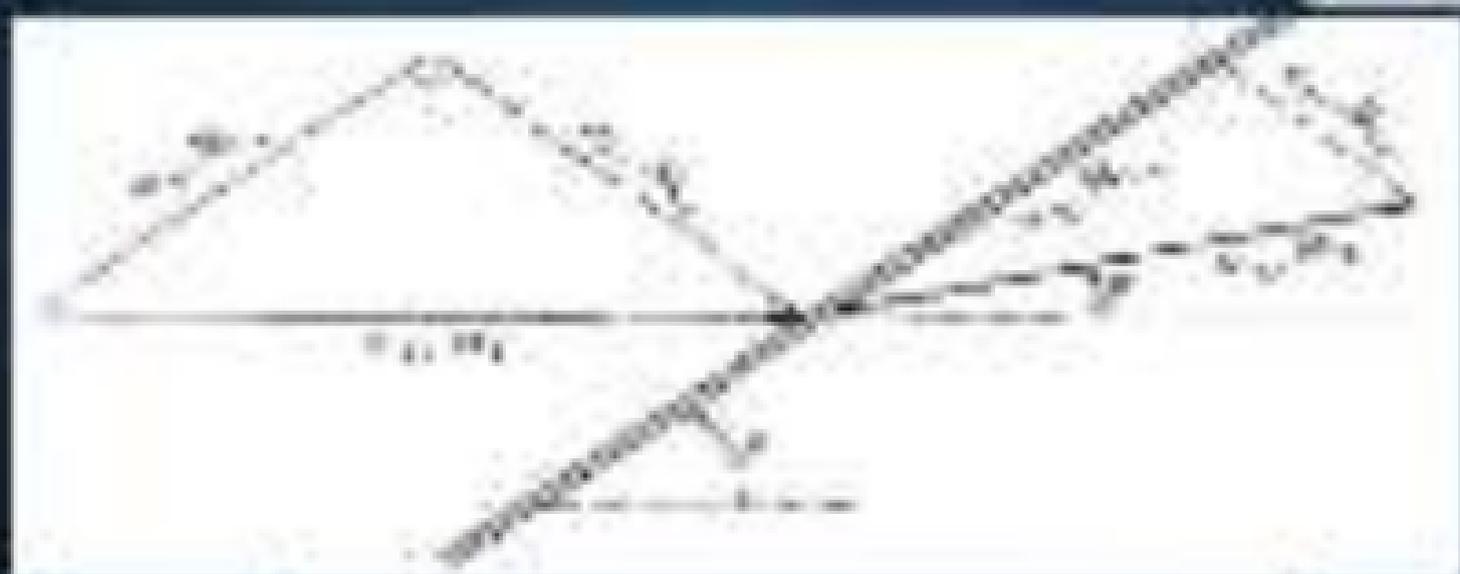
- Continuity equation
 - Mass flow rate normal and tangential components of velocity
 - Tangential component across undisturbed flow is conserved
- Momentum equation
 - Normal to normal and tangential components of velocity
- One-dimensional energy equation



- Assumptions:** Steady, inviscid, adiabatic flow, one-dimensional except across the shock, Perfect gas

$$M_1 \sin \beta = M_2 \sin \beta'$$

OBLIQUE SHOCK FORMULATION



$$M_{n1} = M_1 \sin \beta$$

REVIEW: OBLIQUE SHOCK RELATIONS – CTD.

- Mach Number, based on normal component of the velocity, downstream of the shock:

$$M_{n2}^2 = \frac{M_1^2 + (\frac{\gamma}{\gamma-1})}{(\frac{\gamma}{\gamma-1})M_1^2 + 1}$$

REVIEW: OBLIQUE SHOCK RELATIONS - CTD.

- Static pressure ratio
- Density ratio
- Static temperature ratio

$$\frac{P_2}{P_1} = 1 + \frac{\gamma}{\gamma + 1} (M_{n1}^2 - 1)$$

$$\frac{\rho_2}{\rho_1} = \frac{(\gamma + 1)M_{n1}^2}{(\gamma - 1)M_{n1}^2 + 2}$$

$$\frac{T_2}{T_1} = \frac{P_2 \rho_1}{P_1 \rho_2}$$

RELATION BETWEEN DEFLECTION ANGLE & SHOCK ANGLE

$$\tan \theta = \frac{2 \cot \beta (M_1^2 \sin^2 \beta - 1)}{M_1^2 (k + \cos 2\beta) + 2}$$

θ - β - M Relation

$$\tan \theta = \frac{2 \sin \beta (M^2 \sin^2 \beta - 1)}{M^2 (1 + \cos^2 \beta - \mu^2 - 1)}$$

Key Observations:

- The maximum deflection for small M is $\theta = \beta$.
- The Weak shock solution $\beta = 0$.
- The Strong shock solution.
- Maximum deflection angle for attached shock solutions.

The maximum deflection angle for attached shock solutions is $\theta = \beta$ for small M .
The maximum deflection angle for attached shock solutions is $\theta = \beta$ for small M .
The maximum deflection angle for attached shock solutions is $\theta = \beta$ for small M .



APPROXIMATIONS FOR HYPERSONIC LIMIT

• Static pressure ratio across oblique shock

$$\frac{p_2}{p_1} = 1 + \frac{\gamma}{\gamma + 1} \frac{2\gamma + 1}{\gamma} \sin^2 \theta \quad N \gg 1$$

• Area A_2/A_1

$$\frac{A_2}{A_1} = \frac{1}{\sin \theta} \sqrt{\frac{2\gamma + 1}{\gamma + 1}} \quad N \gg 1$$

• Density ratio across oblique shock:

• Area A_2/A_1

$$\frac{\rho_2}{\rho_1} = \frac{2 + \gamma \sin^2 \theta}{\gamma + 1}$$

$$\frac{\rho_2}{\rho_1} = \frac{1 + \frac{1}{2}(\gamma + 1) \sin^2 \theta}{1 + \frac{1}{2}(\gamma + 1) \sin^2 \theta} \quad N \gg 1$$

THE O-D-M RELATION

- Exact relation for asymptotic flow:

$$\text{Re}(\theta) = \frac{2}{\pi} \arctan \left[\frac{\pi \Gamma(\gamma) \Gamma(\frac{\gamma+1}{2})}{\Gamma(\frac{\gamma}{2})} \frac{\beta}{\alpha} \right]$$

(12.4)

- For asymptotic flow, as $\theta \rightarrow 0$, $\beta \rightarrow 0$, β allow $\theta \rightarrow 0$

- Assume

$$\beta = \frac{2}{\pi} \left[\frac{\pi \beta^{\frac{\gamma}{2}}}{\alpha \Gamma(\frac{\gamma}{2})} + \theta \right]$$

- Neglecting θ and θ^2 for high values of β :



- θ_0

$$\theta_0 = \frac{2}{\pi} \left[\frac{\pi \beta^{\frac{\gamma}{2}}}{\alpha \Gamma(\frac{\gamma}{2})} + \theta \right]$$

$$\left| \begin{array}{l} \beta = \gamma + 1 \\ \theta = \frac{2}{\pi} \end{array} \right.$$

EXPANSION WAVE

- The magnetic field decreases
- Temperature and density increase



Prandtl-
Meyer
Expansion

REVIEW: PM EXPANSION

- Steady, laminar, incompressible
- Temperature is a function of axial position, increases with flow



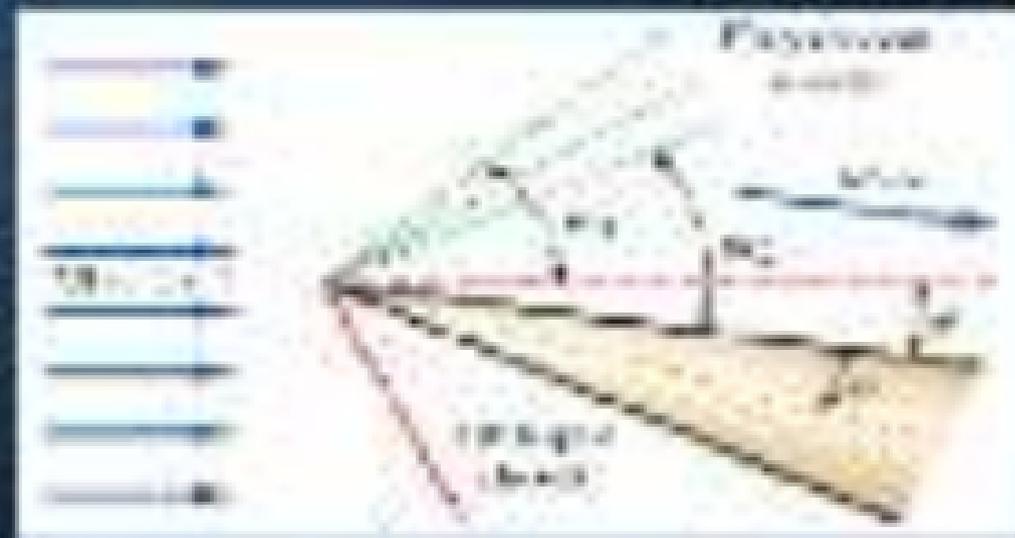
$$\rho u \frac{dV}{dx} = \rho u \frac{d}{dx} \left[\frac{\pi r^2}{2} \right]$$

$$\left[\frac{d}{dx} \left(\frac{\pi r^2}{2} \right) \right] = \frac{d}{dx} \left(\frac{\pi r^2}{2} \right) = \frac{d}{dx} \left(\frac{\pi r^2}{2} \right)$$

$$\rho u \frac{dV}{dx} = \rho u \frac{dV}{dx}$$

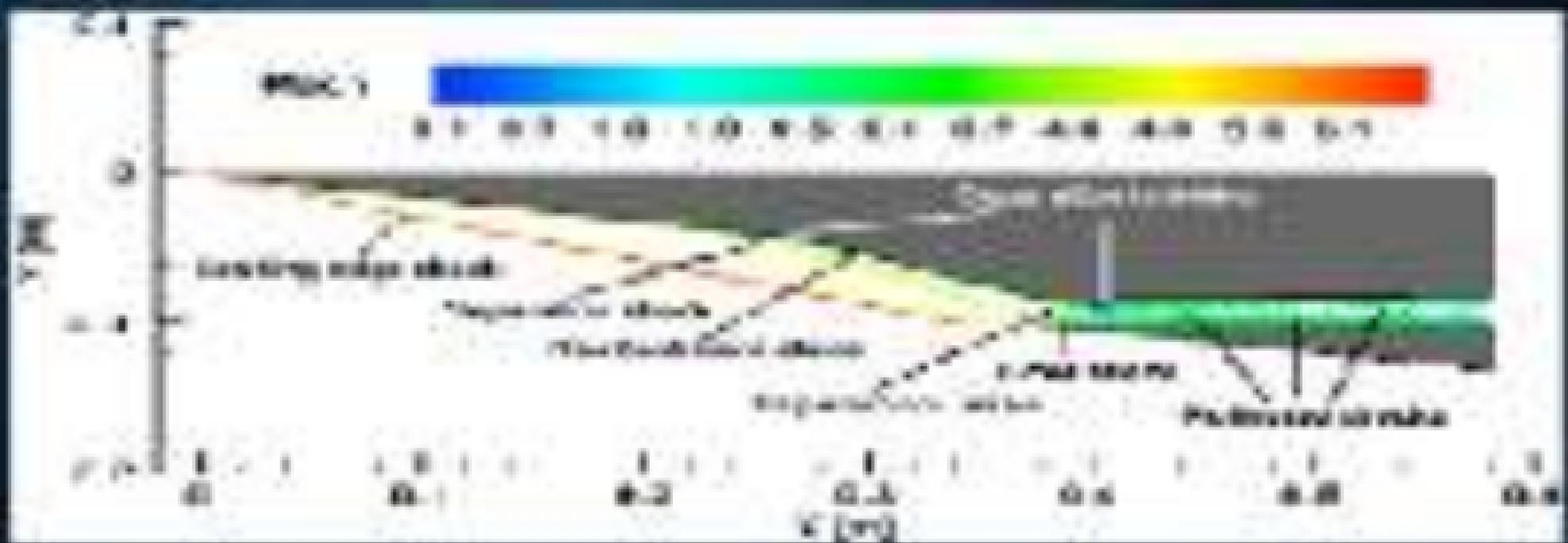
P-M EXPANSION..

- Define P-M Expansion
- **Major components**
- Define process steps
- **Expansion process in various process**
- Expansion in injection molding process





APPLICATIONS



HYPERSONIC LIMIT

As $M \rightarrow \infty$, $\sqrt{\gamma M^2 - 1} \approx \gamma M$

$$u(M) \approx \sqrt{\frac{\gamma+1}{\gamma-1}} \tan^{-1} \left(\sqrt{\frac{\gamma-1}{\gamma+1}} M \right) \rightarrow \tan^{-1}(\gamma M)$$

$$\tan^{-1} x \approx \frac{\pi}{2} - \tan^{-1} \left(\frac{1}{x} \right)$$

$$\tan^{-1} \left(\frac{1}{x} \right) \approx \frac{1}{x} - \frac{1}{3x^3} + \frac{1}{5x^5} - \frac{1}{7x^7} + \dots$$



P-M FUNCTION

$$\ln \frac{1+z}{1-z} = \sqrt{\frac{z+1}{z-1}} \left(\frac{z}{2} - \sqrt{\frac{z-1}{z+1}} \frac{z^3}{4} + \dots \right) = \left(\frac{z}{2} - \frac{1}{2z} + \dots \right)$$

MACH NUMBER - DEFLECTION ANGLE RELATION

$$\theta = \nu(M_2) - \nu(M_1)$$

$$\theta = \frac{1}{M_2} - \left(\frac{\gamma+1}{\gamma-1}\right) \frac{1}{3M_2} = \frac{1}{M_1} - \left(\frac{\gamma+1}{\gamma-1}\right) \frac{1}{3M_1}$$

$$\theta = \frac{2}{\gamma-1} \left(\frac{1}{M_1} - \frac{1}{M_2} \right)$$

STATIC PRESSURE RATIO

- How do you measure static pressure in a duct?

$$\frac{P_2}{P_1} = \left[\frac{1 + (\gamma - 1)/2 M_1^2}{1 + (\gamma - 1)/2 M_2^2} \right]^{(\gamma - 1)/\gamma}$$

- The Mach 1 (Stagnation) ratio

$$\frac{P_2}{P_1} = \left(\frac{A_2^*}{A_1^*} \right)^{(\gamma - 1)/\gamma}$$

PRESSURE COEFFICIENT

• Eq. 8.18 is convenient for the calculation of pressure

$$C_p = \frac{p - p_\infty}{\frac{\rho V_\infty^2}{2}}$$

• The pressure coefficient C_p is defined as:

$$C_p = \frac{p - p_\infty}{\frac{\rho V_\infty^2}{2}}$$

• The definition:

• This is not suitable for compressible flows

• In hypersonic aerodynamics we replace it in terms of **Mach number**:

$$C_p = \frac{p - p_\infty}{\frac{\rho V_\infty^2}{2}} = \frac{p - p_\infty}{\frac{\rho V_\infty^2}{2}} = \frac{p - p_\infty}{\frac{\rho V_\infty^2}{2}} = \frac{p - p_\infty}{\frac{\rho V_\infty^2}{2}}$$



$$C_p = \frac{p - p_\infty}{\frac{\rho V_\infty^2}{2}}$$



$$C_p = \frac{p - p_\infty}{\frac{\rho V_\infty^2}{2}} = \frac{2}{\gamma M_\infty^2} (C_p - 1)$$

COMING NEXT

- Specific theoretical formulations for pressure distribution over hypersonic bodies
- Calculation of aerodynamic forces
- Evaluation & applicability of the models

HYPERSONIC FLOW THEORY



Session 06- Expansion
Waves: Hypersonic
Approximation

Dr. J. D. Anderson
2005

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EXPANSION WAVE

- The magnetic field decreases
- The pressure in front of the wave increases



Prandtl-
Meyer
Expansion

REVIEW: PM EXPANSION

- Steady, laminar, incompressible
- Temperature is a function of axial position, increases with flow



$$\rho u \frac{dV}{dr} = \rho u \frac{d}{dr} \left(\frac{1}{2} \pi r^2 \right)$$

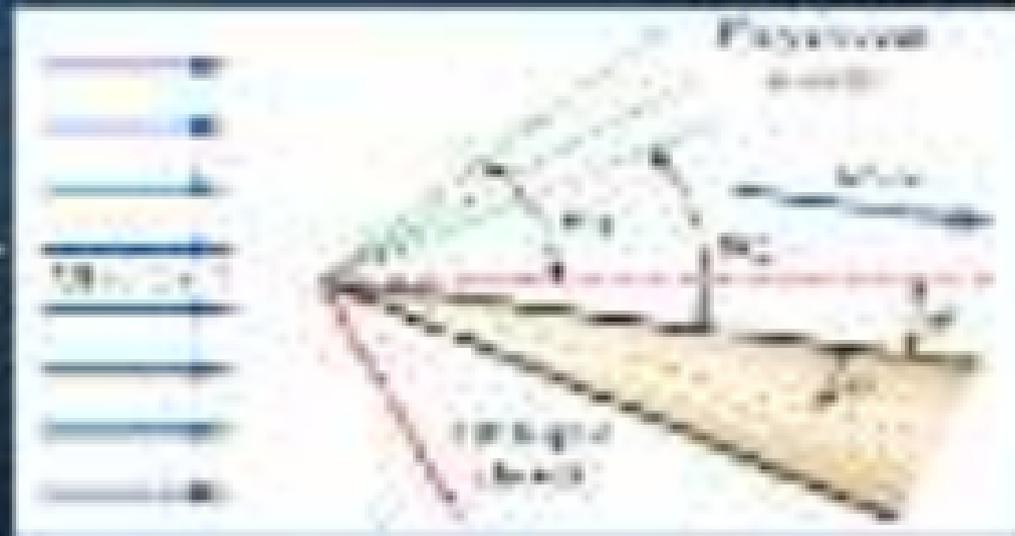
$$\left[\rho u \frac{d}{dr} \left(\frac{1}{2} \pi r^2 \right) \right] = \rho \frac{d}{dr} \left(\frac{1}{2} \pi r^2 u \right) = \rho u \frac{d}{dr} \left(\frac{1}{2} \pi r^2 \right)$$

$$\rho u \frac{d}{dr} \left(\frac{1}{2} \pi r^2 \right) = \rho u \frac{d}{dr} \left(\frac{1}{2} \pi r^2 \right)$$

P-M EXPANSION..

• Layers P-M Expansion:

- **Highly compressible**
- **Lowest pressure depth**
- **Longitudinal pressure transmits pressure**
- **Longitudinal compression or stretch**



HYPERSONIC LIMIT

As $M \rightarrow \infty$, $\sqrt{\gamma M^2 - 1} \approx \sqrt{\gamma} M$

$$\omega(M) = \sqrt{\frac{\gamma+1}{\gamma-1}} \tan^{-1} \sqrt{\frac{\gamma-1}{\gamma+1}} M \rightarrow \tan^{-1}(\sqrt{\gamma}) M$$

$$\tan^{-1} \sqrt{\gamma} = \frac{\pi}{2} - \tan^{-1} \left(\frac{1}{\sqrt{\gamma}} \right)$$

$$\tan^{-1} \left(\frac{1}{\sqrt{\gamma}} \right) = \frac{1}{\sqrt{\gamma}} - \frac{1}{3(\sqrt{\gamma})^3} + \frac{1}{5(\sqrt{\gamma})^5} - \frac{1}{7(\sqrt{\gamma})^7} + \dots$$



P-M FUNCTION

$$\ln \frac{1+z}{1-z} = \sqrt{\frac{z+1}{z-1}} \left(\frac{z}{2} - \sqrt{\frac{z-1}{z+1}} \frac{z^3}{4} + \dots \right) = \left(\frac{z}{2} - \frac{z^3}{24} + \dots \right)$$

MACH NUMBER - DEFLECTION ANGLE RELATION

$$\theta = \nu(M_2) - \nu(M_1)$$

$$\theta = \frac{1}{M_2} - \left(\frac{\gamma+1}{\gamma-1}\right) \frac{1}{3M_2} = \frac{1}{M_1} + \left(\frac{\gamma+1}{\gamma-1}\right) \frac{1}{3M_1}$$

$$\theta = \frac{2}{\gamma-1} \left(\frac{1}{M_1} - \frac{1}{M_2} \right)$$

STATIC PRESSURE RATIO

- How do you calculate the static pressure ratio?

$$\frac{P_2}{P_1} = \left[\frac{1 + (\gamma - 1)/2 M_1^2}{1 + (\gamma - 1)/2 M_2^2} \right]^{(\gamma - 1)/\gamma}$$

- The Mach 1 (Stagnation) ratio

$$\frac{P_2}{P_1} = \left(\frac{A_2^*}{A_1^*} \right)^{(\gamma - 1)/\gamma}$$

PRESSURE COEFFICIENT

• Cp is a convenient factor for correlating pressure

$$C_p = \frac{p - p_\infty}{\frac{\rho U_\infty^2}{2}}$$

• The dynamic pressure, q, is defined as:

$$q = \frac{\rho U_\infty^2}{2}$$

• The dynamic pressure

• This is not available for compressible flows

• In hypersonic aerodynamics we replace q in terms of **stagnation enthalpy**:

$$h = \frac{U^2}{2} + \int_p^p \frac{1}{\rho} dp = \frac{U^2}{2} + \frac{U^2}{\gamma - 1} = \frac{\gamma + 1}{2} \frac{U^2}{\gamma}$$



$$q = \frac{\rho U^2}{2}$$



$$C_p = \frac{p - p_\infty}{\frac{\rho U_\infty^2}{2}} = \frac{2}{\gamma M_\infty^2} (C_p - 1)$$

COMING NEXT

- Specific theoretical formulations for pressure distribution over hypersonic bodies
- Calculation of aerodynamic forces
- Evaluation & applicability of the models

HYPERSONIC FLOW THEORY



Session: 07-08
Newtonian Theory for
Hypersonic Flows

Dr. A. R. Krishnamoorti

2009

AMARITA

CALCULATING SURFACE PRESSURE DISTRIBUTION

- An integral part of aerodynamic design
 - Several methods for calculating the pressure distribution
 - Analytical, empirical, experimental, hybrid
 - Each method has its own pros and cons
- It usually starts with an analytical approach followed by iterations for fine-tuned flow
- Trade-off is a must
- Further developments improve the utility of the formulation

NEWTON'S THEORY

- Flow impinging is either normal or at an angle of attack
- Newton: Force normal to surface must equal flow momentum in the normal direction and flow tangentially along

NEWTON'S THEORY

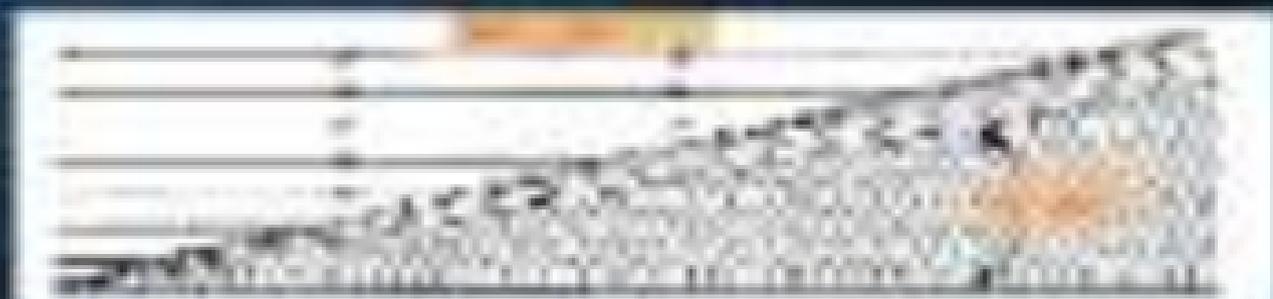
Flow Impinging

Flow impinging on a surface at an angle of attack α is decomposed into normal and tangential components. The normal component is responsible for the pressure force, and the tangential component is responsible for the shear force.



RELEVANCE OF THE MODEL IN HYPERSONIC FLOW

- Connected $M_1 \rightarrow M_2$ approximations = 3 shock discontinuities
- Eulerian shock solutions given
+ $p = 12.2n$
- **Shock were nearly parallel!**
(at this view)



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BASIC FORMULATION



- Mass force per area

$$\rho_{\infty} V_{\infty}^2 \sin^2 \theta$$

- Drag coefficient C_D is defined as $\frac{D}{\rho_{\infty} V_{\infty}^2 A}$, where D is the drag force, ρ_{∞} is the free stream density, V_{∞} is the free stream velocity and A is the reference area.

$$\rho_{\infty} V_{\infty}^2 \sin^2 \theta \sin \theta = \rho_{\infty} V_{\infty}^2 \sin^3 \theta$$

$$\rho_{\infty} V_{\infty}^2 A \sin^2 \theta$$

- There is also a choice of reference area

- Hence the drag coefficient

$$C_D = \rho_{\infty} V_{\infty}^2 A \sin^3 \theta$$

$$\frac{C_D}{A} = \rho_{\infty} V_{\infty}^2 \sin^3 \theta$$

PRESSURE COEFFICIENT

• Steps 2nd and 3rd are identical to the flow velocity derivation

$$p - p_{\infty} = \rho_{\infty} V_{\infty}^2 C_p$$

$$\frac{p - p_{\infty}}{\rho_{\infty} V_{\infty}^2} = C_p$$

$$C_p = \frac{p - p_{\infty}}{\rho_{\infty} V_{\infty}^2}$$

Can be used to
analyze the
aerodynamic
efficiency

PRESSURE DISTRIBUTION AS PER N'S THEORY

- Case line investigated showed **no flow over distribution**
- Shows the flow does not reach around the line because (as per Bernoulli theory) the water is exposed to the flow (see Open). It is at free surface static pressure



WHY NEWTON'S THEORY FOR HYPERSONIC FLOWS ?

- No upstream stream-line turning in $M \gg 1$ flow
 - Flow follows the surface in the sense approximated by Newton
- Mach number independence of C_D in hypersonic flow
 - In accordance with more accurate models
- Flow tangential flow over the surface due to low sweep angle
 - "lagging and being parallel" (slaved) to the surface – as Newton conceptualized

AERODYNAMIC FORCES AS PER N'S THEORY

(This is a simplified version of the theory)

→ Kinematic frame conditions: $\alpha = \beta = \gamma = 0$

→ This and length are used to define C_L & C_D

→ Assume we have flow velocity V
 This would allow us to measure lift
 by pressure distribution over the
 airfoil and we can find C_L & C_D



→ C_L and C_D are →

$$C_L = \frac{L}{\frac{1}{2} \rho V^2 S}$$

$$C_D = \frac{D}{\frac{1}{2} \rho V^2 S}$$



$$C_L = C_L(\alpha)$$

$$C_D = C_D(\alpha)$$



$$C_D = C_{D0} + C_{Di}$$

MODIFIED N'S THEORY



- **Levi's modification:**

- "An attempt to reconcile the results with classical theory (see II - **Modeling of the displacement path**)"

- "The water has a fixed body area with deformed shape"

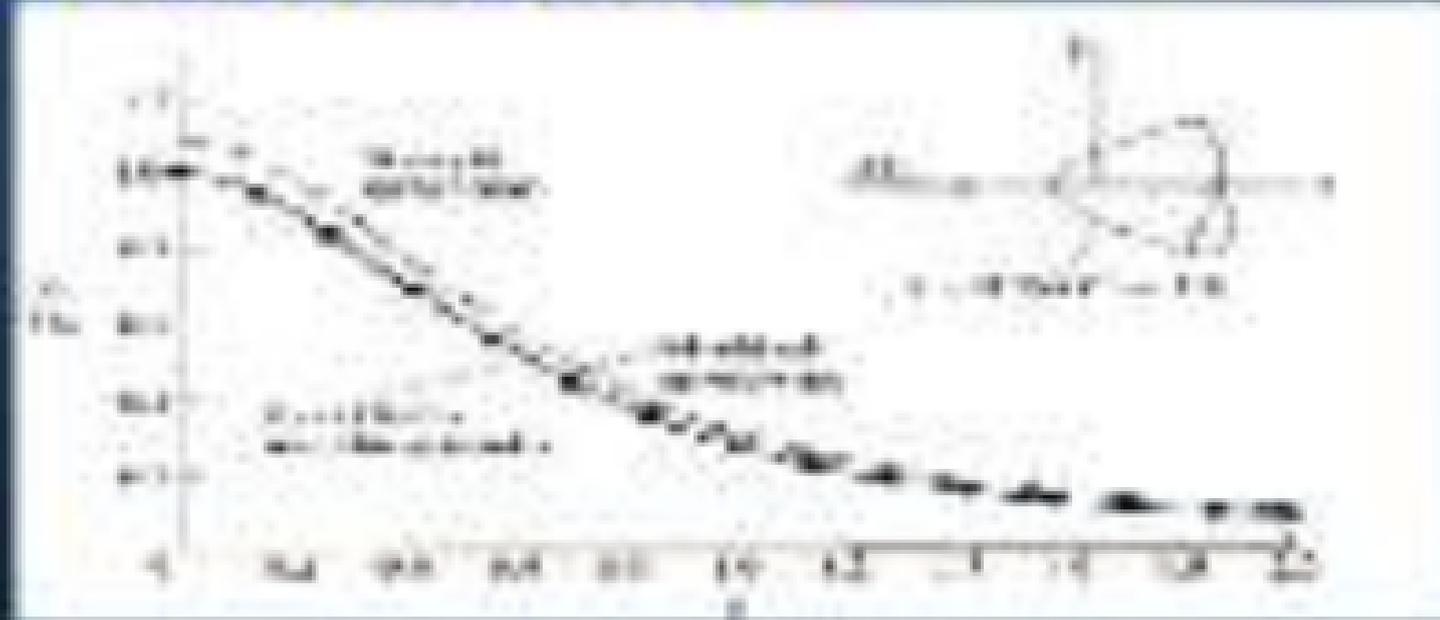
- σ_{ij} corresponds to the water pressure, but not the normal stress

$$\sigma_{ij} = C_{ijkl} \epsilon_{kl}$$

$$\epsilon_{ij} = \frac{1}{2} (\partial_i u_j + \partial_j u_i)$$



THE EFFECT OF LEE'S MODIFIED FORMULATION



SURFACE PRESSURE DISTRIBUTION



NOTE ON NEWTONIAN APPROACH - WHY HYPERSONIC ?

At hypersonic conditions, the flow over blunt bodies exhibits a shock wave that is curved and the gas behind the shock is highly compressed and heated.

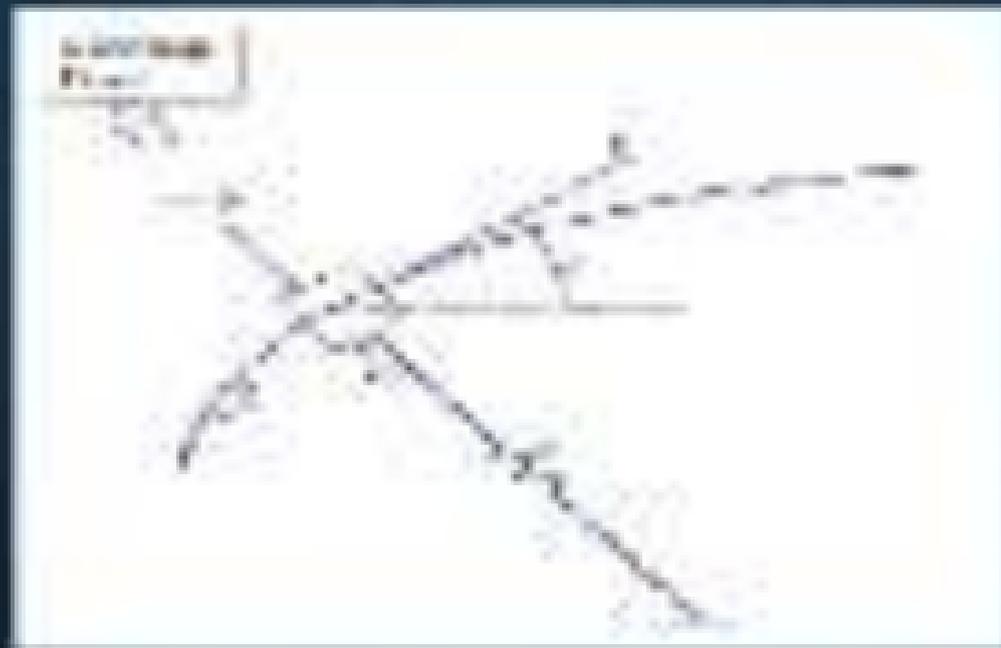
Therefore, the Newtonian approach is not applicable at hypersonic speeds. The flow is highly compressible and the shock wave is curved.



MODIFICATION TO INCLUDE CURVATURE EFFECT OF BLUNT BODIES

- **Standard Boundary Layer**
- Introduce additional terms to reflect the effect of curvature, because that's what the flow sees.
- The velocity profile is predicted in the presence of the curvature very significantly from some models.





HYPERSONIC FLOW THEORY



Session 09 Shock
Expansion Method

Dr. A. M. Srinivasan
2022

AMARITA

MODIFICATION TO INCLUDE CURVATURE EFFECT OF BLUNT BODIES

- **Standard Boundary Layer**
- Introduce additional terms to reflect the effect of curvature, because that's what the θ curvature is.
- The velocity field is predicted in the presence of the curvature very significantly from some models.



SHOCK EXPANSION "TURNING" METHOD

- An approach for calculating aerodynamic forces on non-dimensional supersonic/hypersonic bodies using methods similar to those used by swept wings
 - Flat plate at an angle to shock
 - Direct analysis
- Using the velocity shock relations and the shock deflection theorem
 - static pressure on all surfaces of the diamond body and the plate and the lift and drag forces can be calculated.
- Applicable to sharp curved bodies with an attached shock wave





WAVE DRAG



- D' also due to turbulent subsonic flow past a body gives **WAVE DRAG** (impinging wings affect)
- **Thin airfoil approximation**
 - Drag limited due to wave drag due to the deflection of pressure lines
- Drag goes to 0 as

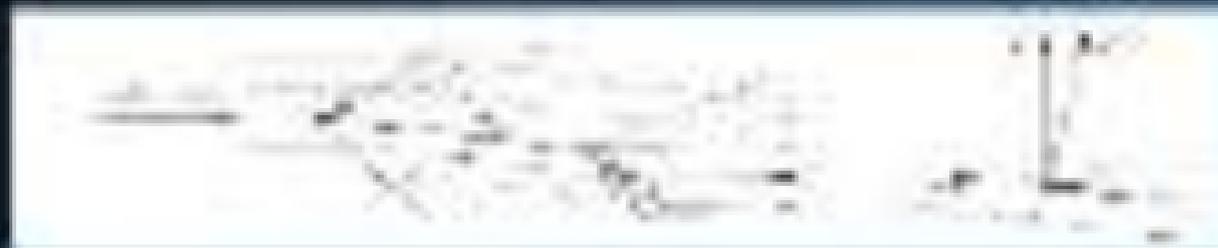
$$D = \frac{1}{2} \rho V^2 c_d \quad [c_d = 2(\alpha - \alpha_0)^2] = 2(\alpha - \alpha_0)^2 \frac{\rho}{2}$$

$$D = (\alpha - \alpha_0)^2 \rho V^2$$

WAVE DRAG
DUE TO
DEFLECTION OF
PRESSURE LINES

DRAG
Goes to 0
as $\alpha \rightarrow \alpha_0$

WAVE DRAG – ON FLAT PLATE



Wave Drag on a Flat Plate

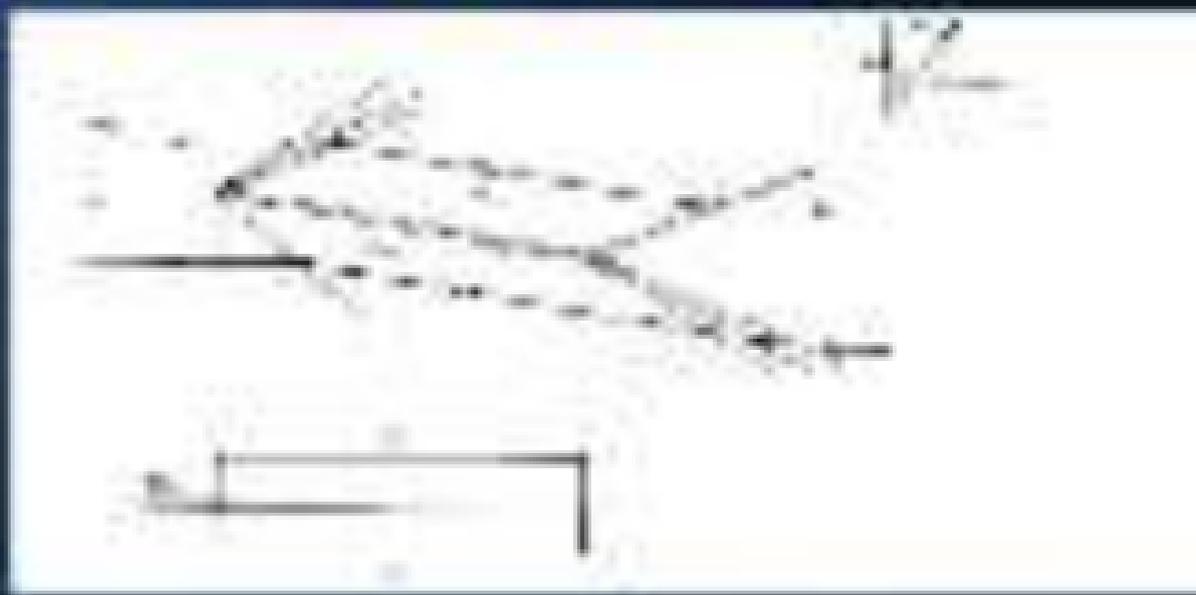
$$\Delta F \approx \Delta D \approx \rho U_\infty^2 \alpha^3$$

$$\Delta F^* \approx \Delta D^* \approx \rho U_\infty^2 c \alpha^3$$

$$\Delta F \approx \Delta D \approx \rho U_\infty^2 c \alpha^3$$

SHOCK EXPANSION CALCULATIONS

- Calculate PE using PM regression method
- Calculate PM using O'Hagan shock expansion
- **Tab A Drop from the previous distribution**



ACCURACY OF SHOCK-EXPANSION MODEL



Accuracy from a model is 100% at the beginning of 1980 period.

EXAMPLE: APPLICATION OF NEWTONIAN THEORY

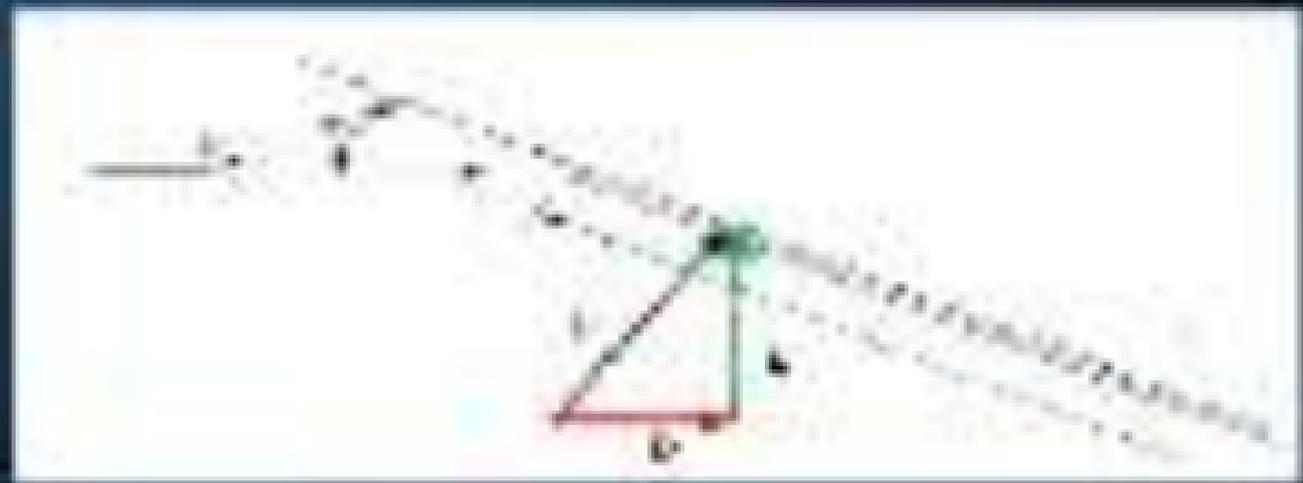
- Calculate the L/D Ratio for a flat plate in 2D flow in 25°Cs. If, with an angle of attack = 10°

- Normal Force $D = \rho_{\infty} V_{\infty}^2 \frac{c_D}{2} A \cos^2 \theta$

- Lift = $D \cos \theta$

- Drag = $D \sin \theta$

- $L/D = \cot \theta = 5.67$



EXAMPLE: SHOCK-EXPANSION METHOD

Calculate pressure distribution over flat plate angle of 7.5° with a Mach 2.5 stream with static pressure = 10 kPa.

→ Region 1:

$$M_1 = \frac{u_1}{a_1} = \frac{u_1}{\sqrt{\gamma R T_1}} = 2.5$$

$$\Delta H_0 = 0.475 \cdot (2.5)^2 = 3.031$$

$$\rightarrow P_0/P_1 = 0.475$$

→ Region 2:

$$\rightarrow P_2/P_1 = 0.7209$$

$$\rightarrow P_2/P_0 = 0.338$$



HYPERSONIC FLOW THEORY

Session 10 Shock-
Expansion Method for
HYPERSONIC FLOWS



Dr. A. M. S. Srinivasan
2009

AMARITA

REGAP

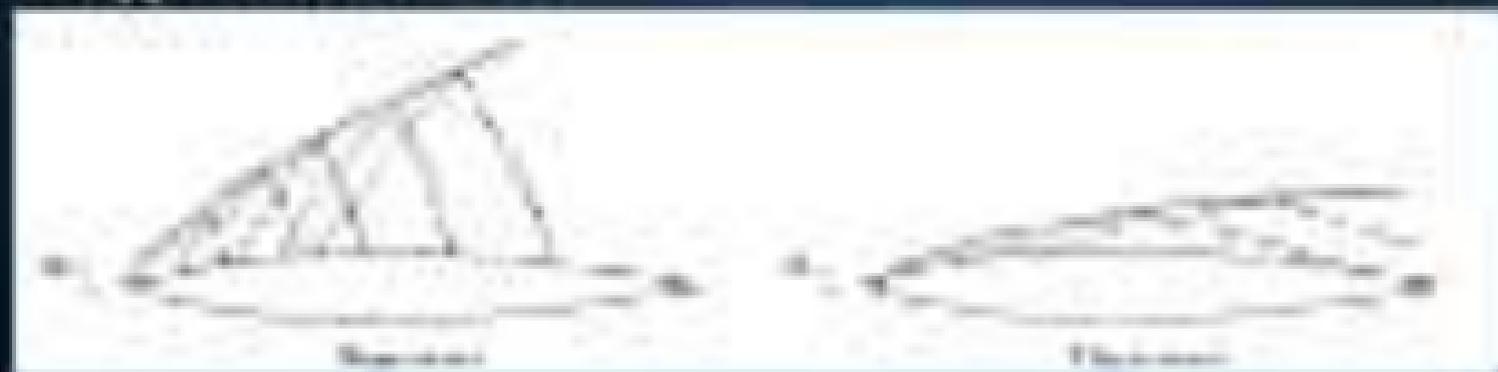
regularized generalized eigenvalue problem

- An approach for estimation of non-symmetric tensors via non-dimensional regularization. Regularized ridge method can be approximated as limited by weight loss
 - Flat plate, no weight is added
 - Directed matrix
- Using the singular value solution and the directed matrix from low
 - matrix process on all elements of the directed matrix and the plate and the full and they become one for individual
- Applicable to sharp curved bodies with an attached sheet, more



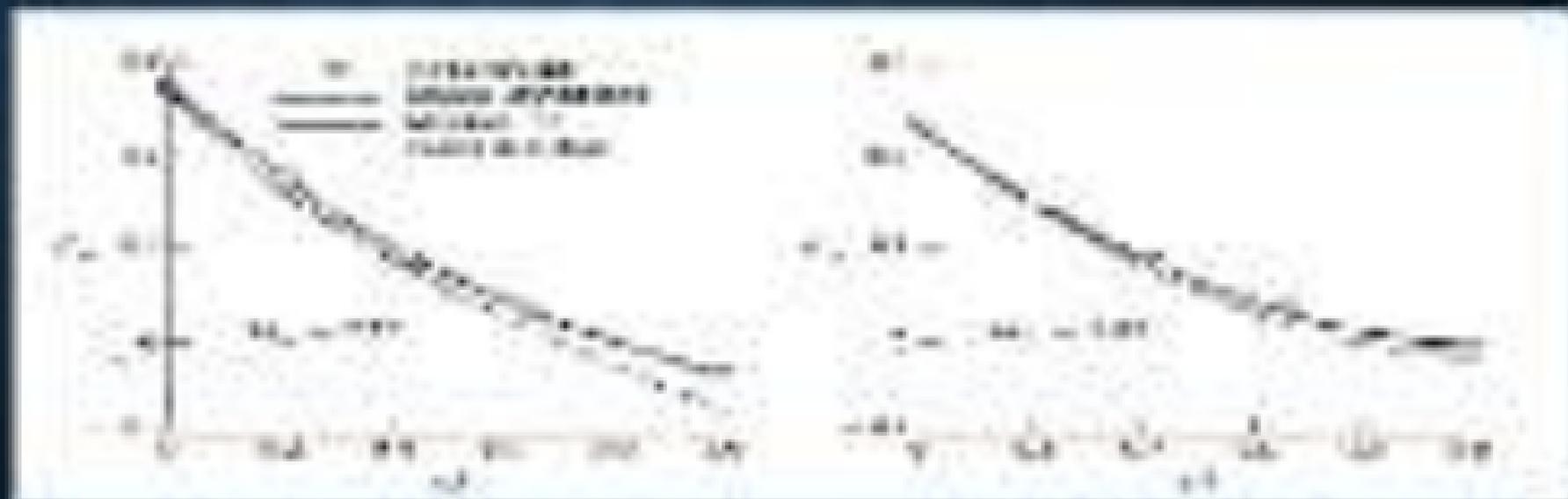
SUPERSONIC VS HYPERSONIC APPLICATION

- ↳ Relatively large nose angle results in reflected waves in supersonic flow
- ↳ **Flow over blunt bodies at hypersonic**
- ↳ Linear theory applies as M increases, but it becomes difficult to calculate. → Perturbations with distance smaller in hypersonic flow



COMPARISON

- Compare results with a numerical analysis in
Approximate Data



EXAMPLE: APPLICATION OF NEWTONIAN THEORY

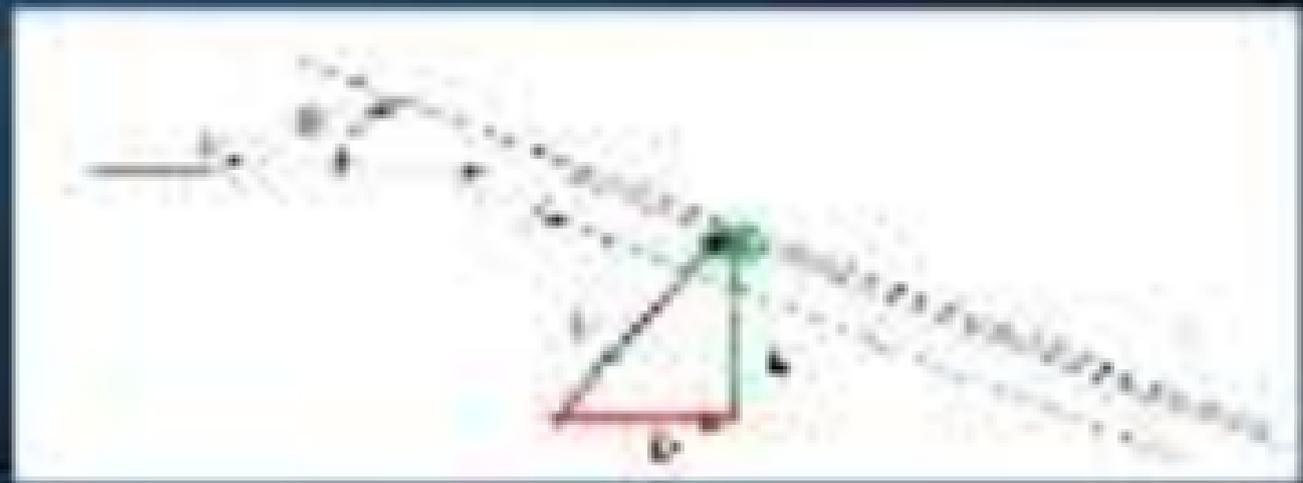
- Calculate the L/D Ratio for a flat plate in 2D flow in 25°C air with an angle of attack = 10 deg

- Reynolds Number $Re = \rho_{\infty} V_{\infty} L / \mu_{\infty}$

- $L/D = 1 / \sin \theta$

- $\sin 10 = 0.1736$

- $L/D = \cot \theta = 5.67$



EXAMPLE: SHOCK-EXPANSION METHOD

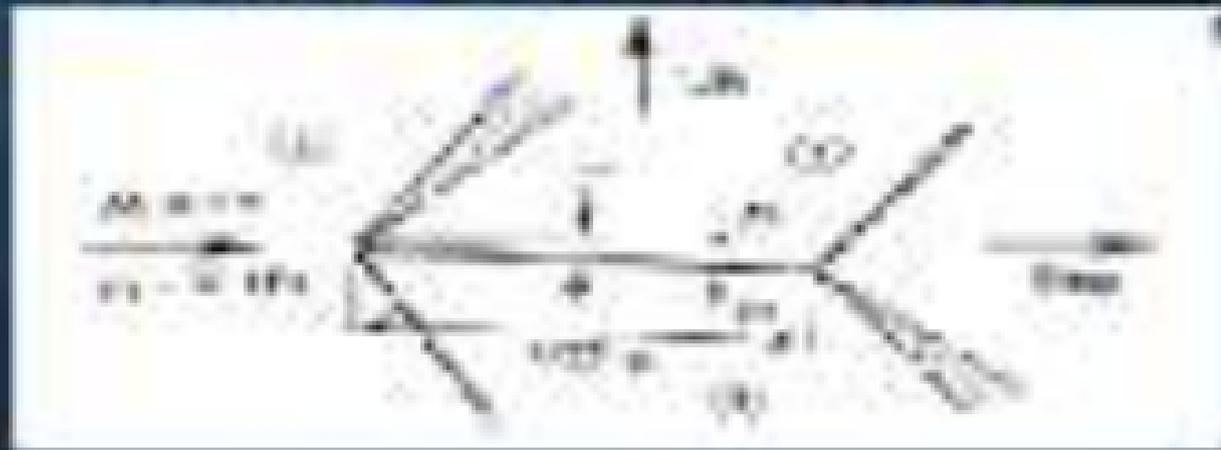
Calculate pressure distribution on a flat plate at angle of 3 deg with a Mach 1.1 stream with static pressure of 0.974

→ Region 1

- $\rho_1 = 1.225 \text{ kg/m}^3$
- $\mu_1 = 1.81 \times 10^{-4} \text{ Pa}\cdot\text{s}$
- $T_1 = 288 \text{ K}$
- $p_1 = 0.974 \text{ Pa}$

→ Region 2

- $\rho_2 = 1.225 \text{ kg/m}^3$
- $\mu_2 = 1.81 \times 10^{-4} \text{ Pa}\cdot\text{s}$



NUMERICAL PROBLEM: PRESSURE DISTRIBUTION ON A DIAMOND AF IN HYPERSONIC FLOW

- Calculate pressure distribution over the body in Mach 12 flow. Half angle $\alpha = 20^\circ$. Free stream conditions: $p_\infty = 10^{-4}$ Pa

$$D = 10^{-2} \text{ m}, L = 10^{-2} \text{ m} \Rightarrow \text{Re} = 10^6 \Rightarrow \mu = 1.8 \cdot 10^{-4} \text{ Pa}\cdot\text{s}$$



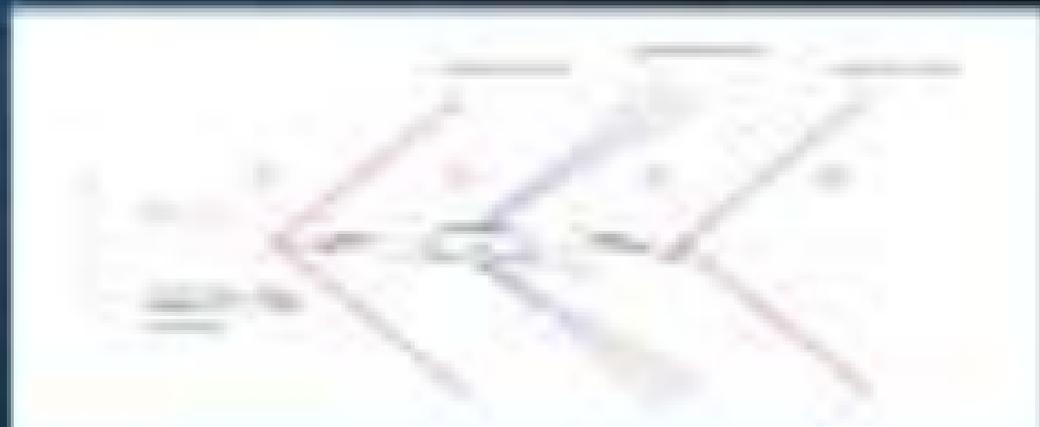
$$\frac{p_c}{p_\infty} = \frac{1}{1 + \frac{\gamma - 1}{2} M^2 \sin^2 \theta} \rightarrow p_c = p_\infty \left(1 + \frac{\gamma - 1}{2} M^2 \sin^2 \theta \right)^{-1}$$

$\text{with } \gamma = 1.4, M = 12, \theta = 20^\circ \Rightarrow p_c = 10^{-4} \cdot 10.721 \text{ Pa}$

- Minimum angle about $\theta = 10^\circ$ along

$$\theta = \frac{1}{M} \left(\frac{1}{\gamma} - \frac{1}{M^2} \right) \rightarrow \theta = 10.11^\circ$$

- Calculate pressure distribution over the body $\Rightarrow p_c$



$$\frac{p_c}{p_\infty} = \left(1 + \frac{\gamma - 1}{2} M^2 \sin^2 \theta \right)^{-1}$$

RECONP

RECONSTRUCTION AND ANALYSIS OF NETWORKS

- Tools estimate the unknown processes as a function of the known processes realization

- Insight forward application

- Despite the complex methodology

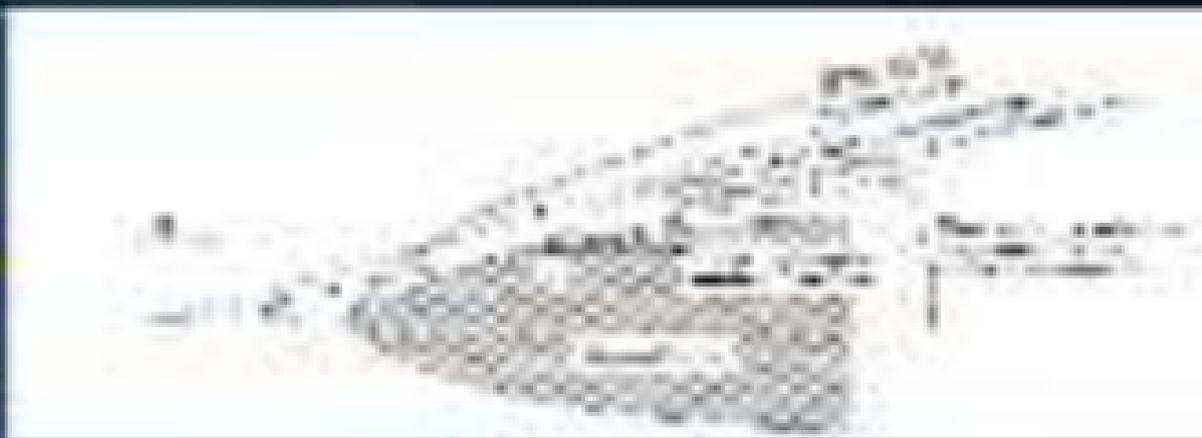
It is for large generative models
works well with experiments

- Close match for homogeneous data



THE PROCEDURE..

- Draw tangents at the ends of the wedge and find out the center of gravity by the suspension method with two different velocity centers.
- Using this angle and dimensions Mark's number calculate the probable velocity at each angle.
- In this above angle calculate the maximum angle made by the projectile. Mark's number does not change through out the movement of projectile at any point on the CD orbit.
 - Draw any point of the trajectory at any CD point and draw tangent to the CD surface at 2 m point. This is the velocity through out the movement of the projectile.
 - From the above velocity make and use the Mark's number and use above angle to calculate the velocity at that point.
- Follow this procedure at 40 points on the wedge surface and calculate the pressure on it.



THE IMPORTANCE OF SURFACE INCLINATION APPROACH

- Several mathematical methods for aerodynamic analysis have proven sufficiently accurate for use at such a critical stage
- Limitations of experimental facilities:
 - Access related to flow quality
 - Test time
 - Instrumentation cost

EXPERIMENTAL CHALLENGES

- Low-magnification microscopy has little potential for super-resolution, but includes high-throughput screens
- High-magnification microscopy has the potential for super-resolution, but includes:
 - Limited FOV
 - Poor SNR (limited resolution) and challenging

Low & High Magn. Micro

Table 1

Summary table of used FOV, magnification and resolution for differentiating. Basic Microscopy, Super-Resolution and High-magnification FOV are shown with a resolution of 200.

FOV	Mag	Res	SNR	SNR	SNR	SNR
Basic Micro	100x	200nm	0.01	0.01	0.01	0.01
High-Mag	1000x	20nm	0.1	0.1	0.1	0.1

HYPERSONIC FLOW THEORY



Dr. A. R. Subramanian

Session 13- Hypersonic
Small Perturbation
Theory

ANARITA
ANALYTICAL RESEARCH INSTITUTE

SMALL PERTURBATIONS – THE GENERAL APPROACH

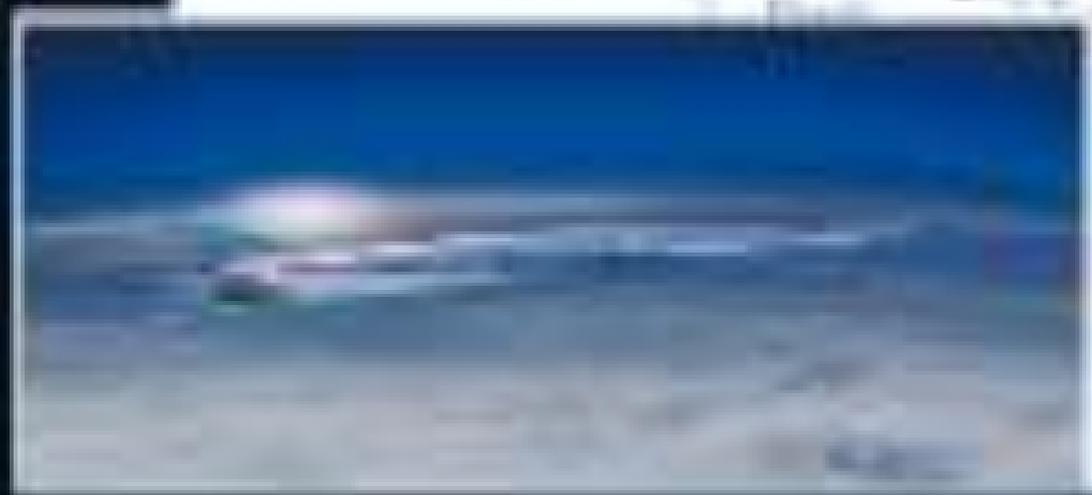
- Linearized analysis of the perturbations of a known flow pattern
 - Like, equilibrium flow, the general flow always propagates as a result of a disturbance
 - The perturbation
- Flow always satisfies the MFR $\rho(\mathbf{V} \cdot \mathbf{n}) = \rho_0(\mathbf{V} \cdot \mathbf{n}) + \rho_1(\mathbf{V} \cdot \mathbf{n})$
- The velocity field of the unperturbed flow pattern is $\mathbf{v} = \mathbf{V}_0 + \mathbf{v}_1 + \mathbf{v}_2 + \dots$
- The perturbation may be expressed as a perturbation $\mathbf{v}_1 = \mathbf{v}_1(x, y, z, t)$
- Modify the governing equations using the perturbed variables (simplify by neglecting small quantities)
- **Small** perturbation. As in **linearized** analysis, analysis of small region of flow.

STATIC PRESSURE RATIO ACROSS SHOCKS AT **SLENDRE** AND **ROUNDED** BODIES

Mach	Slender Bodies						Rounded Bodies					
	Re	AR	β	α	β	α	Re	AR	β	α	β	α
1.5	100	1.00	0.00	0.00	0.00	0.00	100	1.00	0.00	0.00	0.00	0.00
1.5	100	1.00	0.00	0.00	0.00	0.00	100	1.00	0.00	0.00	0.00	0.00
1.5	100	1.00	0.00	0.00	0.00	0.00	100	1.00	0.00	0.00	0.00	0.00
1.5	100	1.00	0.00	0.00	0.00	0.00	100	1.00	0.00	0.00	0.00	0.00
1.5	100	1.00	0.00	0.00	0.00	0.00	100	1.00	0.00	0.00	0.00	0.00

$$P = P_0 \left(\frac{\rho_0}{\rho} \right)^{\frac{\gamma}{\gamma-1}}$$

SLIMMER DESIGNS: CHARACTERIZE HYPERSONONIC AIR-BREATHING CONCEPTS



SLENDERNESS & PERTURBATION

- **Transition: Change in velocity due to the incompressibility**

$$\begin{aligned} \rho &= \rho_0 + \rho' \\ \eta &= \eta' \end{aligned}$$

$$\frac{d}{dt} \rho_0 + \rho_0 \frac{d}{dt} \ln \eta' = \rho_0 \frac{d}{dt} \ln \eta'$$

Velocity is constant
in the axial direction
due to incompressibility

$$\frac{d}{dt} \ln \eta' = \frac{d \ln \eta'}{dt} = \frac{1}{\eta'} \frac{d \eta'}{dt}$$



- If $\rho = \rho_0$ the increase of absolute value is due

$$\frac{d}{dt} \ln \eta' = \frac{1}{\eta'} \frac{d \eta'}{dt} = \frac{d \ln \eta'}{dt}$$

If $\rho = \rho_0$ and $\eta = \eta'$ then $\frac{d \rho}{dt} = 0$

There will be no change in density with respect to time and distance if $\rho = \rho_0$ and $\eta = \eta'$ in any given velocity

GOVERNING EQUATIONS IN PERTURBATION FORMS

$$\begin{aligned}
 & \text{Set } V_0 = 1, \quad \rho_0 = \rho_0, \quad \mu_0 = \mu_0 \\
 & \rho_0 \frac{dV_0}{dt} = \rho_0 \frac{d}{dt} \left(\frac{V_0}{\rho_0} \right) = \rho_0 \left(\frac{1}{\rho_0} \frac{dV_0}{dt} - \frac{V_0}{\rho_0^2} \frac{d\rho_0}{dt} \right) \\
 & \rho_0 \frac{dV_0}{dt} = \rho_0 \left(\frac{1}{\rho_0} \frac{dV_0}{dt} - \frac{V_0}{\rho_0^2} \frac{d\rho_0}{dt} \right) \\
 & \rho_0 \frac{dV_0}{dt} = \rho_0 \left(\frac{1}{\rho_0} \frac{dV_0}{dt} - \frac{V_0}{\rho_0^2} \frac{d\rho_0}{dt} \right) \\
 & \rho_0 \frac{dV_0}{dt} = \rho_0 \left(\frac{1}{\rho_0} \frac{dV_0}{dt} - \frac{V_0}{\rho_0^2} \frac{d\rho_0}{dt} \right)
 \end{aligned}$$

Steady component
 and transient as
 perturbation
 to the
 steady-state
 value

NON-DIMENSIONALIZATION OF PERTURBATION EQUATIONS



- We are focusing on **2D/3D** models
- Masses $\rho \rightarrow \rho^*$
- $\rightarrow \rho^* \rightarrow \rho$
- If we can non-dimensionalize the governing equations such that the key terms are of order unity, then terms involving products of $O(\epsilon)$ can be neglected
- An example of such a non-dimensionalization:

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \mathbf{u}) = 0$$

$$\rho \frac{D\mathbf{u}}{Dt} = -\nabla p + \nabla \cdot \mathbf{T}$$



ORDERS OF MAGNITUDE - 2

→ Similarly,

$$\frac{P_2}{P_1} = \frac{\gamma + 1}{\gamma - 1}$$

then

$$P = \rho \alpha^2 / \rho_0$$

is $O(\alpha^2)$

ALSO

$$\frac{u_2}{V_{\infty}} \rightarrow 1 - \frac{\gamma \sin^2 \theta}{\gamma + 1}$$



$$\frac{\Delta u}{V_{\infty}} = \frac{u_2 - u_1}{V_{\infty}} \rightarrow \frac{\gamma \sin^2 \theta}{\gamma + 1} \rightarrow O(\alpha^2)$$



$$\Delta u = \alpha / V_{\infty} \rightarrow O(\alpha)$$

→ Standardly

$$\frac{\Delta u}{u} = \frac{\Delta v}{v} \rightarrow \frac{\Delta u}{u} = \frac{\Delta v}{v} \rightarrow \frac{\Delta u}{u} = \frac{\Delta v}{v} \rightarrow \frac{\Delta u}{u} = \frac{\Delta v}{v}$$



$$\frac{\Delta u}{u} = \frac{\Delta v}{v} \rightarrow \frac{\Delta u}{u} = \frac{\Delta v}{v} \rightarrow \frac{\Delta u}{u} = \frac{\Delta v}{v}$$

HYPERSONIC PERTURBATION EQUATIONS

$$\frac{\partial}{\partial x} \left(\frac{\rho}{\rho_0} \right) + \frac{\partial}{\partial y} \left(\frac{\rho v}{U_0} \right) + \frac{\partial}{\partial z} \left(\frac{\rho w}{U_0} \right) = 0$$

$$\frac{\partial}{\partial x} \left(\frac{\rho u}{U_0} \right) + \frac{\partial}{\partial y} \left(\frac{\rho v}{U_0} \right) + \frac{\partial}{\partial z} \left(\frac{\rho w}{U_0} \right) = 0$$

$$\frac{\partial}{\partial x} \left(\frac{\rho u^2}{U_0^2} \right) + \frac{\partial}{\partial y} \left(\frac{\rho u v}{U_0^2} \right) + \frac{\partial}{\partial z} \left(\frac{\rho u w}{U_0^2} \right) = 0$$

$$\frac{\partial}{\partial x} \left(\frac{\rho u v}{U_0^2} \right) + \frac{\partial}{\partial y} \left(\frac{\rho v^2}{U_0^2} \right) + \frac{\partial}{\partial z} \left(\frac{\rho v w}{U_0^2} \right) = 0$$

$$\frac{\partial}{\partial x} \left(\frac{\rho u w}{U_0^2} \right) + \frac{\partial}{\partial y} \left(\frac{\rho v w}{U_0^2} \right) + \frac{\partial}{\partial z} \left(\frac{\rho w^2}{U_0^2} \right) = 0$$

$$\frac{\partial}{\partial x} \left(\frac{\rho u^2}{U_0^2} \right) + \frac{\partial}{\partial y} \left(\frac{\rho u v}{U_0^2} \right) + \frac{\partial}{\partial z} \left(\frac{\rho u w}{U_0^2} \right) = 0$$

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$$\frac{\partial}{\partial x} \left(\frac{\rho u^2}{U_0^2} \right) + \frac{\partial}{\partial y} \left(\frac{\rho u v}{U_0^2} \right) + \frac{\partial}{\partial z} \left(\frac{\rho u w}{U_0^2} \right) = 0$$

$$\frac{\partial}{\partial x} \left(\frac{\rho u v}{U_0^2} \right) + \frac{\partial}{\partial y} \left(\frac{\rho v^2}{U_0^2} \right) + \frac{\partial}{\partial z} \left(\frac{\rho v w}{U_0^2} \right) = 0$$

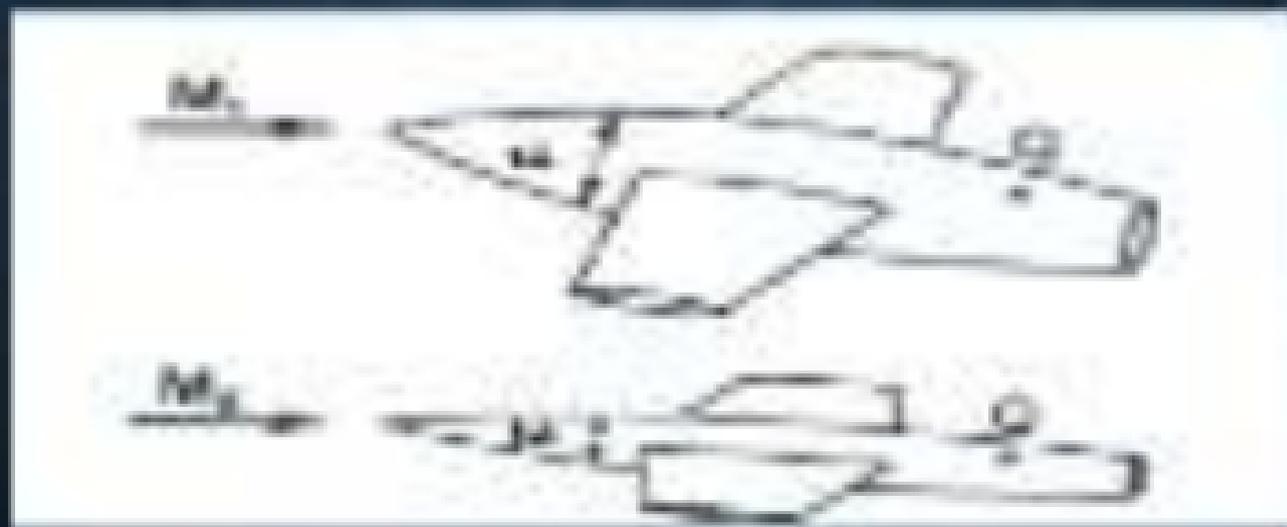
$$\frac{\partial}{\partial x} \left(\frac{\rho u w}{U_0^2} \right) + \frac{\partial}{\partial y} \left(\frac{\rho v w}{U_0^2} \right) + \frac{\partial}{\partial z} \left(\frac{\rho w^2}{U_0^2} \right) = 0$$

(Assuming velocity components are already perturbed)
 Continuity: $\rho_0 \frac{\partial u}{\partial x} + \rho_0 \frac{\partial v}{\partial y} + \rho_0 \frac{\partial w}{\partial z} = 0$
 Momentum: $\rho_0 U_0 \frac{\partial u}{\partial x} + \rho_0 U_0 \frac{\partial v}{\partial y} + \rho_0 U_0 \frac{\partial w}{\partial z} = 0$

THE HYPERSONIC SMALL-DISTURBANCE EQUATIONS

- A set of coupled, nonlinear partial differential equations
 - No general analytical solutions
- Reflects the inherently non-linear nature of hypersonic flow physics
- Leads to hypersonic similarity analysis

SIMILARITY ANALYSIS



SIMILARITY APPROACH

Experimental requirements

- Facilities not identical
- situations different
- which there is some
- common phenomena
- and hence that are
- NOT identical

Complex physical phenomena can be represented in a simplified form with all the parameters that govern the phenomenon as part of the domain

Similarity approach reduces the number of parameters that describe the phenomena

THE EQUATIONS

$$\begin{array}{l} \frac{d}{dt} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} -\lambda x \\ -\lambda y \\ -\lambda z \end{pmatrix} \\ \frac{d}{dt} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} -\lambda x \\ -\lambda y \\ -\lambda z \end{pmatrix} \\ \frac{d}{dt} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} -\lambda x \\ -\lambda y \\ -\lambda z \end{pmatrix} \end{array}$$

Exponential Growth/Decay
The rate of change of a quantity is proportional to the quantity itself.

Approximation to Exponential Growth/Decay
The amount of a quantity increases/decreases over time.

Exponential Growth/Decay
The amount of a quantity increases/decreases over time.

THE BOUNDARY CONDITIONS

• Tangential flow along the surface implies:

→ **no normal component of velocity at the boundary**

$$v_n = \mathbf{v} \cdot \mathbf{n} = 0 \quad \text{on } \partial\Omega$$

$$(\mathbf{v} \cdot \mathbf{n}, \mathbf{v} \cdot \mathbf{t}) = 0 \quad \text{on } \partial\Omega$$

• Using the new dimensionalized approach:

$$\left(\frac{1}{\rho} \nabla \cdot \sigma\right) \cdot \mathbf{n} = \mathbf{v} \cdot \mathbf{t} \quad \text{on } \partial\Omega$$

$$\mathbf{v} = \mathbf{v}_1 \mathbf{e}_1 + \mathbf{v}_2 \mathbf{e}_2$$

$$v_1 = v_x, \quad v_2 = v_y$$

$$\mathbf{t} = \sqrt{1 + \alpha^2} \mathbf{e}_1 = \sqrt{1 + \alpha^2} \mathbf{e}_x, \quad \mathbf{n} = \mathbf{e}_2 = \mathbf{e}_y$$

$$\sigma_{ij} = -p \delta_{ij} + \mu \nabla_i v_j + \mu \nabla_j v_i$$

• As $\beta \rightarrow 0$ & $\theta \rightarrow 0$ (slender body with low angle of attack)

• $\sin \beta \approx \sin \theta \approx \theta$

• If geometry of the body at surface: $dy/dx = \theta = \tau$

WITH SLENDER BODY ASSUMPTIONS...

$$\frac{d}{dt} \int_{\text{CV}} \rho \mathbf{v} dV = \int_{\text{CS}} \rho \mathbf{v} \cdot \mathbf{n} dA$$

$$\frac{d}{dt} \int_{\text{CV}} \rho \mathbf{v} dV = \int_{\text{CS}} \rho \mathbf{v} \cdot \mathbf{n} dA$$



$$\frac{d}{dt} \int_{\text{CV}} \rho \mathbf{v} dV = \int_{\text{CS}} \rho \mathbf{v} \cdot \mathbf{n} dA$$

$$\frac{d}{dt} \int_{\text{CV}} \rho \mathbf{v} dV = \int_{\text{CS}} \rho \mathbf{v} \cdot \mathbf{n} dA$$

$$\frac{d}{dt} \int_{\text{CV}} \rho \mathbf{v} dV = \int_{\text{CS}} \rho \mathbf{v} \cdot \mathbf{n} dA$$

$$\frac{d}{dt} \int_{\text{CV}} \rho \mathbf{v} dV = \int_{\text{CS}} \rho \mathbf{v} \cdot \mathbf{n} dA$$

M_τ
 Referred to as the
 Hypersonic Similarity
 Parameter

THE SOLUTIONS ARE EXPECTED AS:

$$\begin{aligned} \bar{T} &= \bar{T}(S, Q, N, \mu, T) \\ \bar{P} &= \bar{P}(P, Q, N, \mu, T) \\ \bar{U} &= \bar{U}(S, Q, N, \mu, T) \end{aligned}$$

IMPLICATIONS OF *HYPERSONIC SIMILARITY*

11/11/2024
12:00
44

$$\lambda = M^{-1/2}$$



- Subflows for two different flows over two different bodies with similar geometry but different scale in one dimension but *affinity related bodies* (bodies that have essentially the same mathematical shape, but that differ by a scale factor on one direction, such as different radius or diameter) will lead to the same values of non-dimensional parameters components γ^* and u^* if they have the same value of $\lambda = M^{-1/2}$

$$\lambda = M^{-1/2}$$



SIMILARITY RELATIONS FOR **SMALL** **ANGLES OF ATTACK**

- The small values of angle of attack of the laminar airfoil are considered as

$$\beta = \beta(\alpha, C_u, C_{u_0}, \gamma, M_{\infty}, \frac{r_0}{c})$$
$$\beta = \beta(\gamma, C_u, C_{u_0}, \gamma, M_{\infty}, \frac{r_0}{c})$$

PRESSURE COEFFICIENT

$$C_p = \frac{p - p_\infty}{\frac{\rho V_\infty^2}{2}}$$

$$= \frac{p - p_\infty}{\rho V_\infty^2} \cdot \frac{2}{2}$$

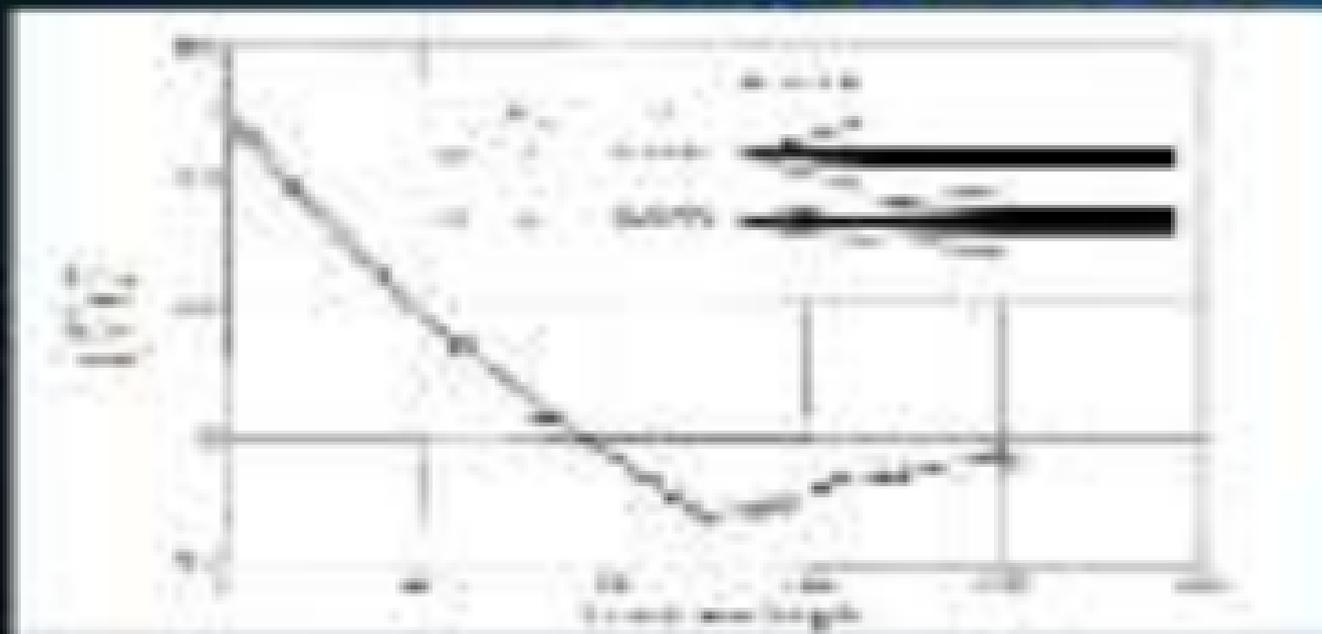


A diagram showing a circular object in a flow field. The flow velocity is V_∞ . The pressure coefficient is C_p . The pressure is p . The dynamic pressure is $\frac{\rho V_\infty^2}{2}$.

$$C_p = \frac{2(p - p_\infty)}{\rho V_\infty^2} = 2 \left(\frac{p}{\rho V_\infty^2} - \frac{p_\infty}{\rho V_\infty^2} \right)$$

$$\left| \frac{p}{\rho V_\infty^2} = \frac{1}{2} (C_p + 1) \right|$$

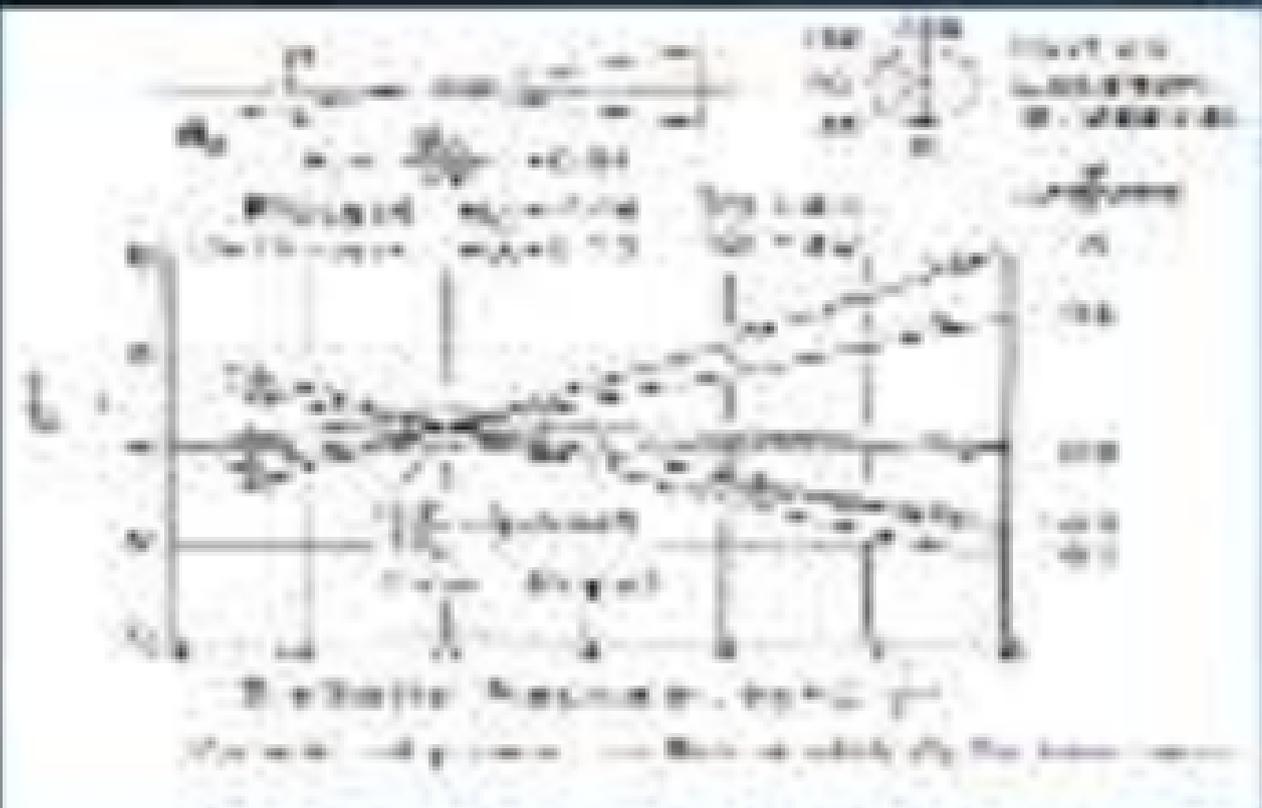
VALIDATION OF HYPERSONIC SIMILARITY – USING EXACT METHODS



Validation of Hypersonic Similarity
Mach 10

EXPERIMENTAL VALIDATION

- If you want to maximize low probability and the value of the target parameter for the low probability events, you should use a low probability point to estimate the value of the resulting parameter only.
- **Single experiment of low probability events is not recommended. The purpose of an experiment is to measure the value of a parameter, not to measure the value of a parameter. The results of the experiment are not the results of a parameter.**



Agreement with the
 approximation of similarity to
 observed data in that the
 conditions
 for the present study, provide
 an insight into the
 possibility
 of the present study

Figure 1

CORNERS WITH VARIOUS ANGLES

LEARNING OBJECTIVES

CLASSIC ANGLES



$$K = \frac{P}{N \cdot L}$$

$P = P_1 + P_2 + P_3 + P_4$
 $P = P_1 + P_2 + P_3 + P_4$
 $P = P_1 + P_2 + P_3 + P_4$

$$\sum_{i=1}^n \frac{1}{(1+r)^t} = \frac{1}{r} \left(1 - \frac{1}{(1+r)^n} \right)$$



| Year |
|------|------|------|------|------|------|------|------|------|------|
| 1 | 1990 | 1991 | 1992 | 1993 | 1994 | 1995 | 1996 | 1997 | 1998 |
| 2 | 1991 | 1992 | 1993 | 1994 | 1995 | 1996 | 1997 | 1998 | 1999 |
| 3 | 1992 | 1993 | 1994 | 1995 | 1996 | 1997 | 1998 | 1999 | 2000 |
| 4 | 1993 | 1994 | 1995 | 1996 | 1997 | 1998 | 1999 | 2000 | 2001 |
| 5 | 1994 | 1995 | 1996 | 1997 | 1998 | 1999 | 2000 | 2001 | 2002 |

HYPERSONIC FLOW THEORY



Sessions 18-:
Hypersonic Equivalence

Dr. A. M. S. Srinivasan
2005

AMARITA

SIMILARITY TO EQUIVALENCE

- Hyperbolic similarity: For slender bodies at small angles of attack

Flows over affinely related bodies with the same values of γ , M_∞ , and α will show the same values of C_D/C_L at the corresponding normalized locations

- Based on small perturbation analysis
- **Hyperbolic Equivalence:** Extension of approximate analysis to blunt-nosed slender bodies

THE COMPARISON OF INVISCID GOVERNING EQUATIONS

INCOMPRESSIBLE FLOW

$$\begin{aligned} \rho_0 \left(\frac{D\mathbf{u}}{Dt} + \nabla\phi \right) &= \rho_0 \left(\frac{\partial \mathbf{u}}{\partial t} + \mathbf{u} \cdot \nabla \mathbf{u} \right) + \rho_0 \nabla\phi = \rho_0 \mathbf{f} \\ \nabla \cdot \mathbf{u} &= 0 \end{aligned}$$

$$\begin{aligned} \rho_0 \left(\frac{\partial \mathbf{u}}{\partial t} + \mathbf{u} \cdot \nabla \mathbf{u} \right) + \rho_0 \nabla\phi &= \rho_0 \mathbf{f} \\ \nabla \cdot \mathbf{u} &= 0 \end{aligned}$$

COMPRESSIBLE FLOW

$$\begin{aligned} \rho \left(\frac{D\mathbf{u}}{Dt} + \nabla\phi \right) &= \rho \left(\frac{\partial \mathbf{u}}{\partial t} + \mathbf{u} \cdot \nabla \mathbf{u} \right) + \rho \nabla\phi = \rho \mathbf{f} \\ \frac{D\rho}{Dt} + \rho \nabla \cdot \mathbf{u} &= 0 \end{aligned}$$

CYLINDRICAL (BLAST) WAVES

- Differs from spherical waves in that it has a $1/r$ decay
- Spherical waves are $1/r^2$ and cylindrical waves are $1/r$
- **Area** $\propto r^2$ **Area** $\propto r$ **Area** $\propto r$



HYPERSONIC EQUIVALENCE

The principle

Prandtl-Glauert
singularity

The hypersonic equivalence principle is essentially the statement that the **steady hypersonic flow over a slender body is equivalent to an unsteady hypersonic flow in subsonic conditions**.

THE ANALOGY

- This analogy establishes the relationship between an unsteady flow and a steady motion with one more spatial dimension.
- Based on this analogy, Prandtl successfully found a semi-empirical way to estimate the pressure distribution for blunt-nosed bodies at hypersonic speeds.
- This analogy theoretically reduces the general shock waves and with appropriate scale
 - Only holds for slender configurations where the small perturbation hypothesis can be used.

BLAST WAVE ANALOGY

• Analogy between

- Steady flow past a streamlined body of revolution or non-axisymmetrical shape

AND

- An instantaneous energy release in the form of a concentrated explosion at the point fully stagnated by the flow of velocity.



expansion of shock wave due to energy release is called blast wave

50 ?

Study of three dimensional steady hypersonic flow over a slender object is equivalent to study two dimensional unsteady flow over the same

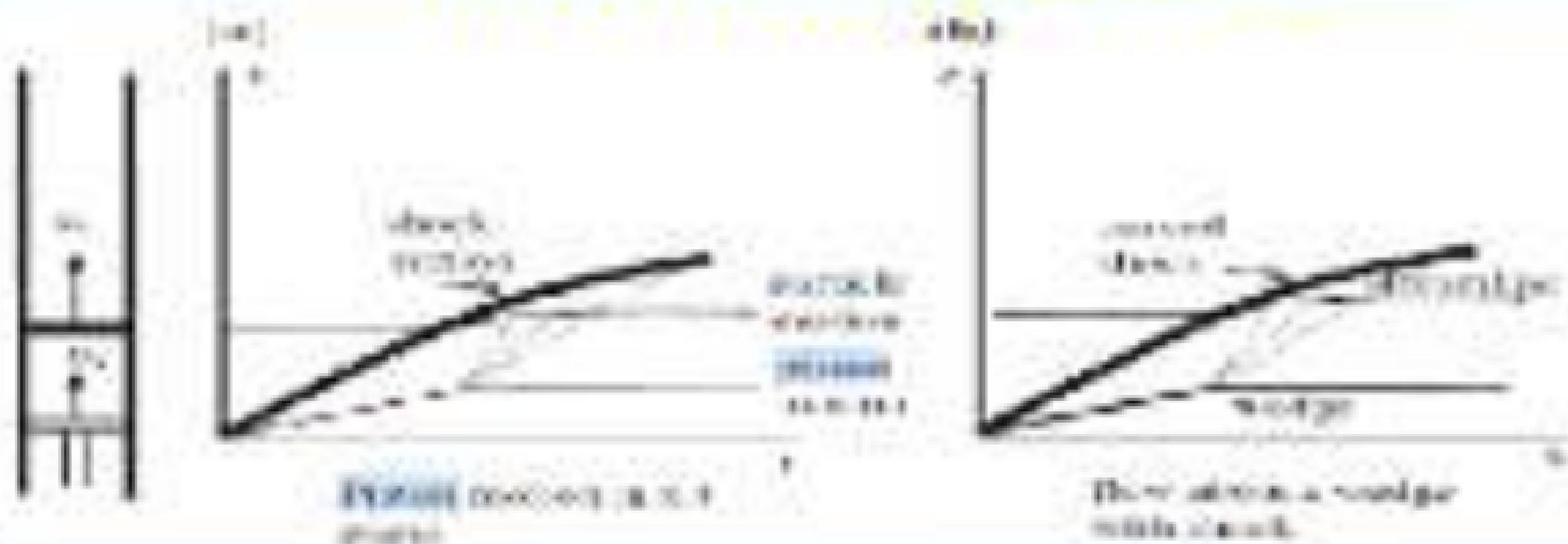
the steady hypersonic flow over a slender object is equivalent to study two dimensional unsteady flow over the same slender object

The asymptotic asymptotic parameters already hyperbolic close over a distance δ in asymptotic for all asymptotic show the case δ asymptotic dimension

$$\frac{1}{\delta} \rightarrow \frac{1}{\delta} \rightarrow \frac{1}{\delta}$$

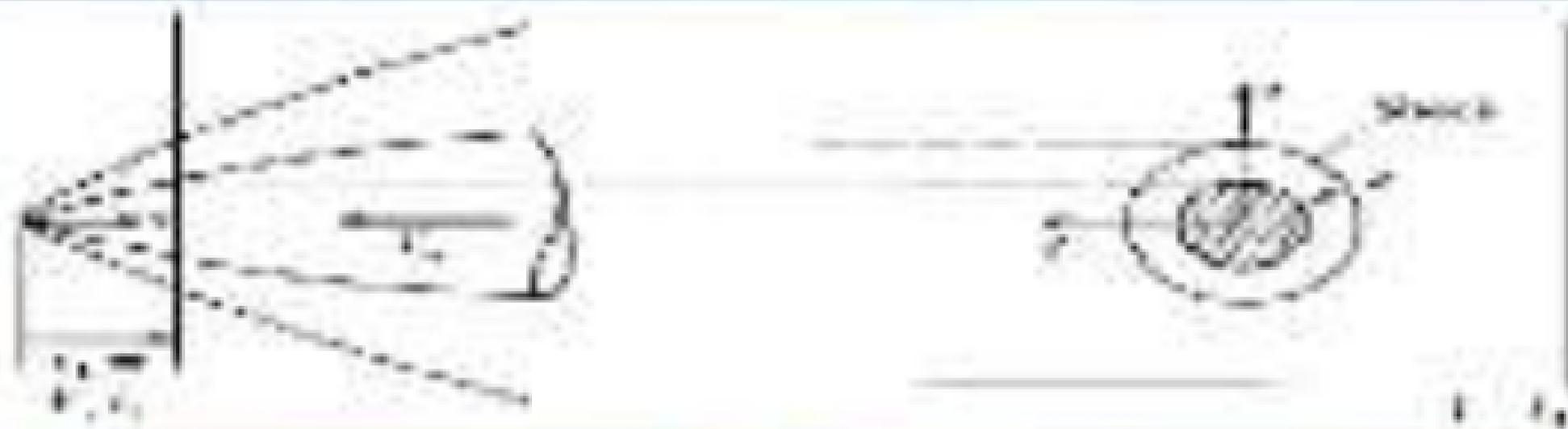
|| δ ||

PISTON-EQUIVALENCE

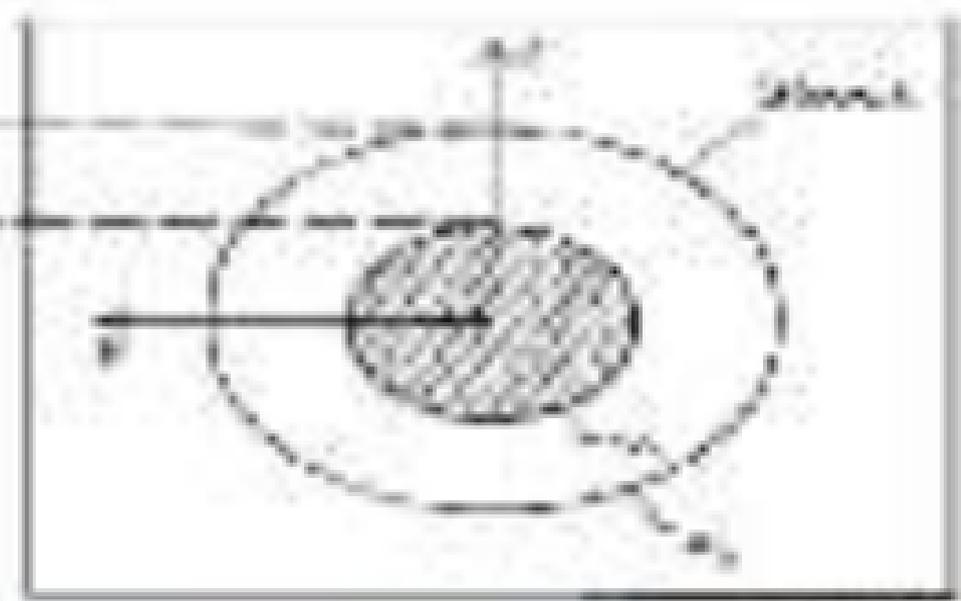
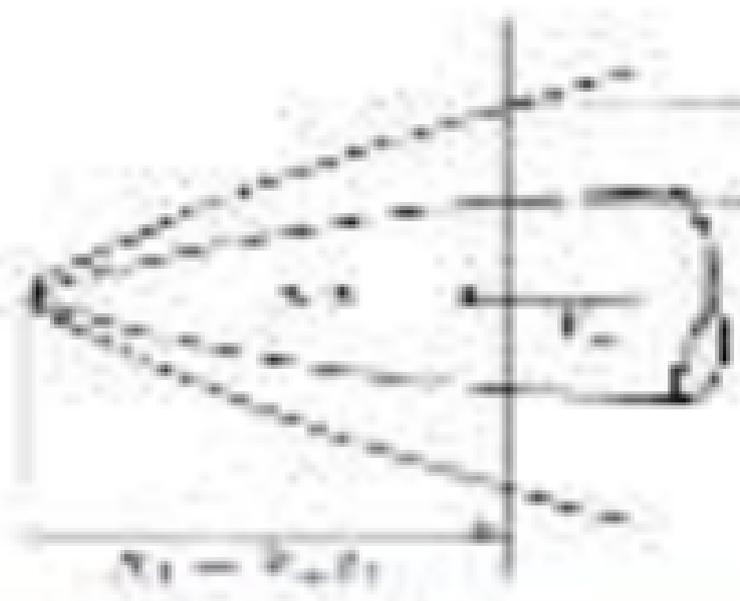


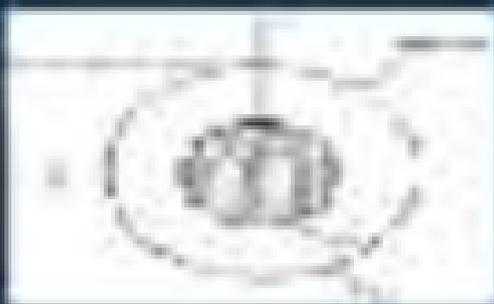
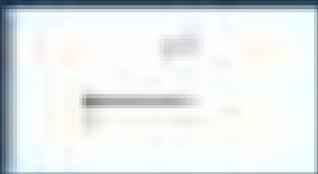
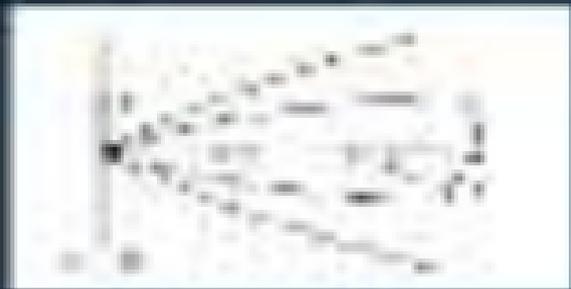


1 - Part



The diagram shows a lens system. On the left, a biconvex lens is shown in cross-section, mounted on a vertical axis. It has a central hole. On the right, a circular lens element is shown, also mounted on a vertical axis. It has a central hole and is surrounded by a circular frame. A horizontal line connects the two lenses, and a vertical line is on the far right. The diagram is labeled '1 - Part' at the top.





Cell Cycle
Prokaryotes: One
stage or 2ndly
cyclic
eukaryotes
many
cyclic
prokaryotes
many
cyclic

FROM CLASSIC THEORY OF BLAST WAVES – CYLINDRICAL BLAST WAVE

Pressure and velocity
behind the front of
propagating LB
pressure front
decrease as

$$P \propto \frac{1}{R^2}$$

$$u \propto \frac{1}{R}$$

$$P = \frac{E}{R^2}$$



$$d_t = \frac{1}{2} \left(\frac{1}{R} - \frac{1}{R_0} \right)$$

DRAC VS ENERGY RELEASE

- **E** Energy released per unit length $E = \rho_{drac} \cdot \dot{Q}_{drac}$
The **drac** is much denser than air, so the energy is **high**
The **drac** length measured in **sub-inches**
- **Q** $\dot{Q} = \rho_{drac} \cdot v_{drac} \cdot A_{drac} \cdot \dot{Q}_{drac}$
The **drac** velocity is **high** (1000 ft/sec)
The **drac** area is **small** (0.1 in²)



The shock wave generated by this energy release is a cylindrical blast wave, propagating outward in the radial direction.

RELATING THE PHENOMENA

$$E - E_0 = \frac{1}{2} \mu_0 V_0 \epsilon_0 \omega^2$$



$$\frac{d^2 \rho}{dt^2} + \omega^2 \rho = 0$$


Wave Pulse

$$\frac{d^2 \rho}{dt^2} = -0.0661 N_0 \sqrt{E_0}$$

$$\frac{d}{dt} \left(\frac{1}{2} m v^2 \right) = \frac{d}{dt} (m v \cdot v) \quad (1)$$

$$m \frac{d}{dt} (v \cdot v)$$



$$= m \frac{d}{dt} (v \cdot v)$$

CYLINDRICAL (BLAST) WAVES

- $\Delta p/p_0 = \frac{1}{2} \frac{v}{c}$ (small $\Delta p/p_0$)
 - $\Delta p/p_0 = \frac{1}{2} \frac{v}{c}$ (large $\Delta p/p_0$)
 - $\Delta p/p_0 = \frac{1}{2} \frac{v}{c}$ (large $\Delta p/p_0$)
- wavefront \rightarrow $\rho_0 c v$



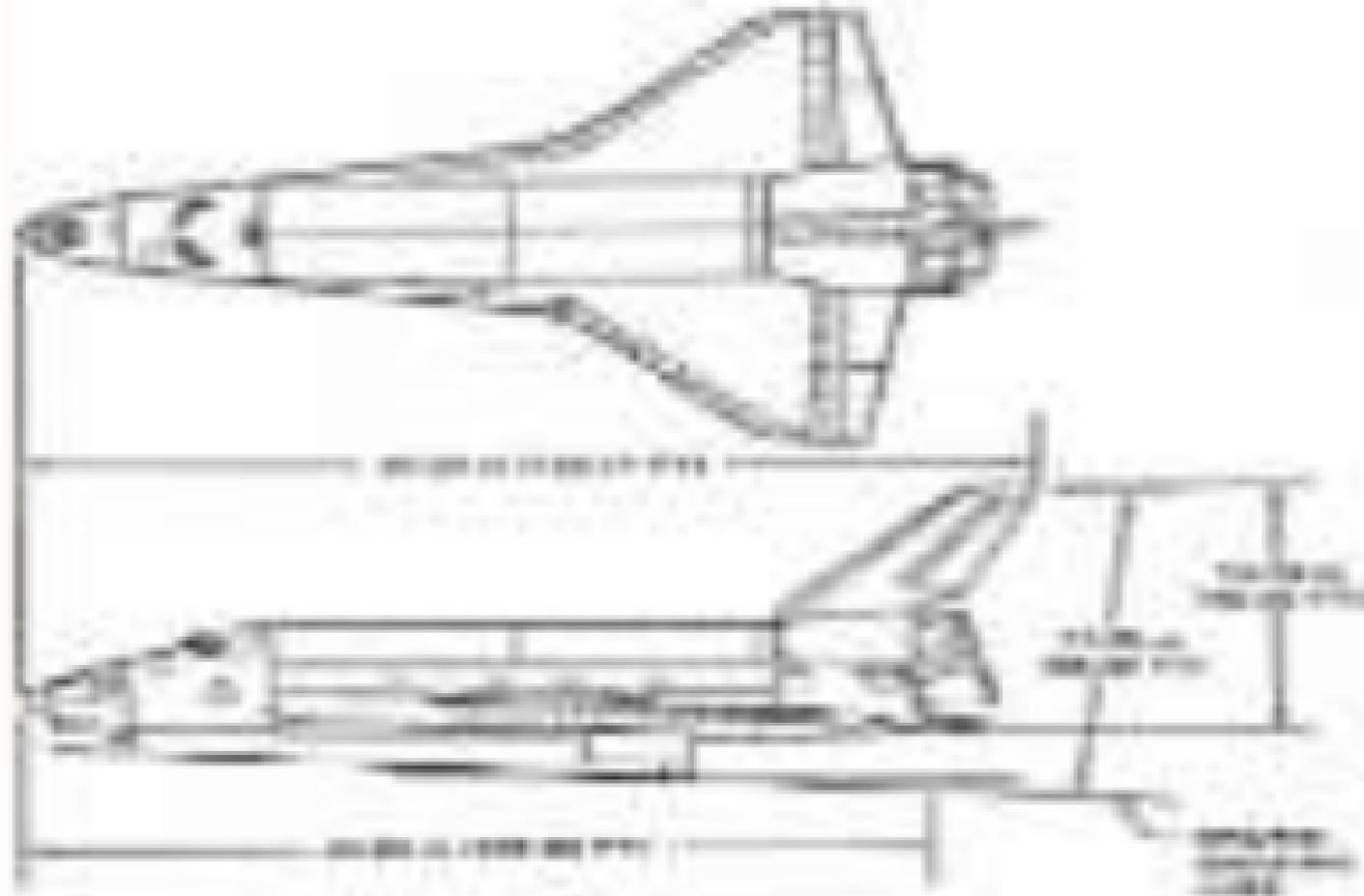
PRACTICAL CONSIDERATIONS

- **Simulation of the Approach:**

- A computer is the standard technique, solutions of transient and steady-state flows at a point or line in space.
- A discretized body does not add energy to the flow, rather it just is the energy problem, primarily at a point or along a line.

- **Utility**

- Physical measuring instead the sensing devices, the steady flow over a hydrodynamic slender body and the unsteady flow at one free space downstream.



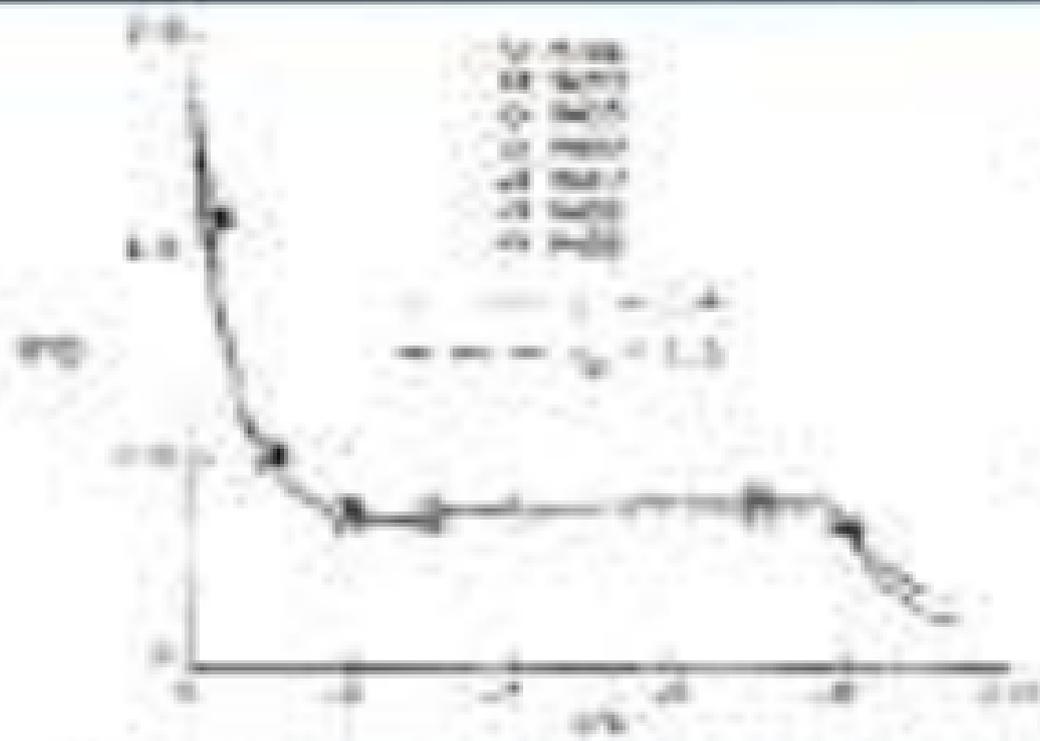


Fig. 10. Viscosity-concentration relationship for polyacrylamide (M_w = 20,000, 0.1% NaCl, 25°C).



Fig. 11. Viscosity-concentration relationship for polyacrylamide (M_w = 20,000, 0.1% NaCl, 25°C).



Fig. 10. Decay of relative humidity in a 1000 ft³ room. $R_{0.1} = 0.1$, $\alpha = 0.001$, and $\beta = 0.001$.



Fig. 11. Decay of relative humidity in a 1000 ft³ room.

HYPERSONIC FLOW THEORY



Session 27-28: Viscous
Flow: Introduction

Dr. A. M. Kulkarni

2011

AMARITA

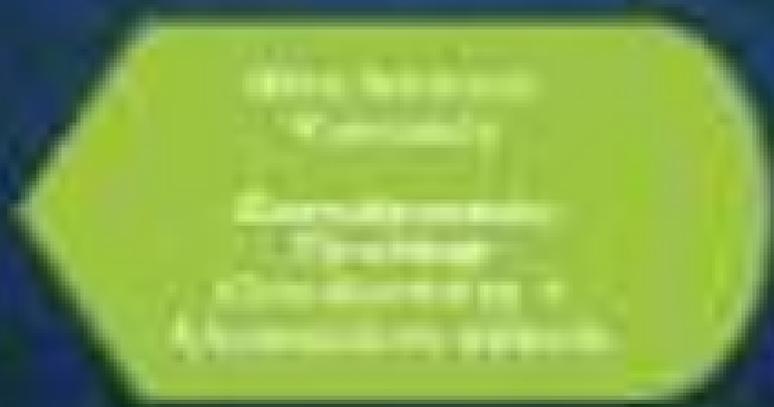
CAPABILITIES OF THE INVISCID APPROACH

- Shock angle
- Pressure distribution
- Wave drag



WHY VISCOUS ??

- Friction Drag
- Heat Transfer

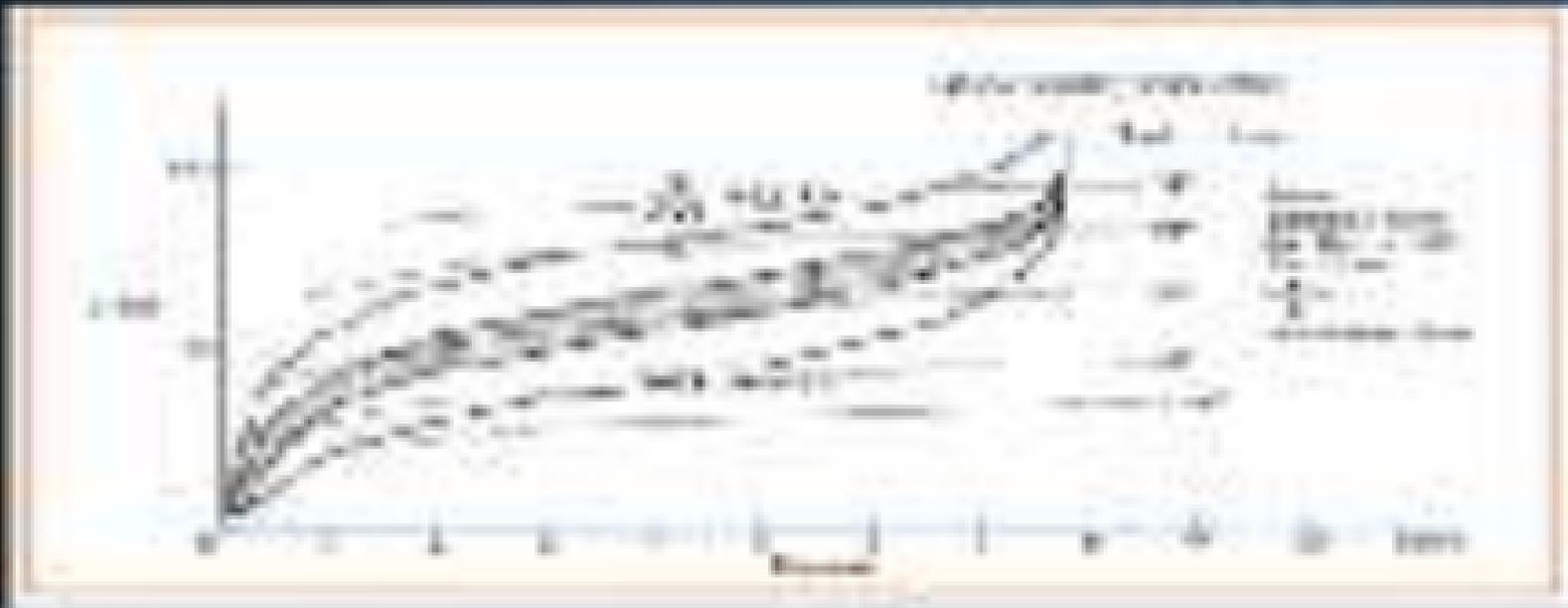


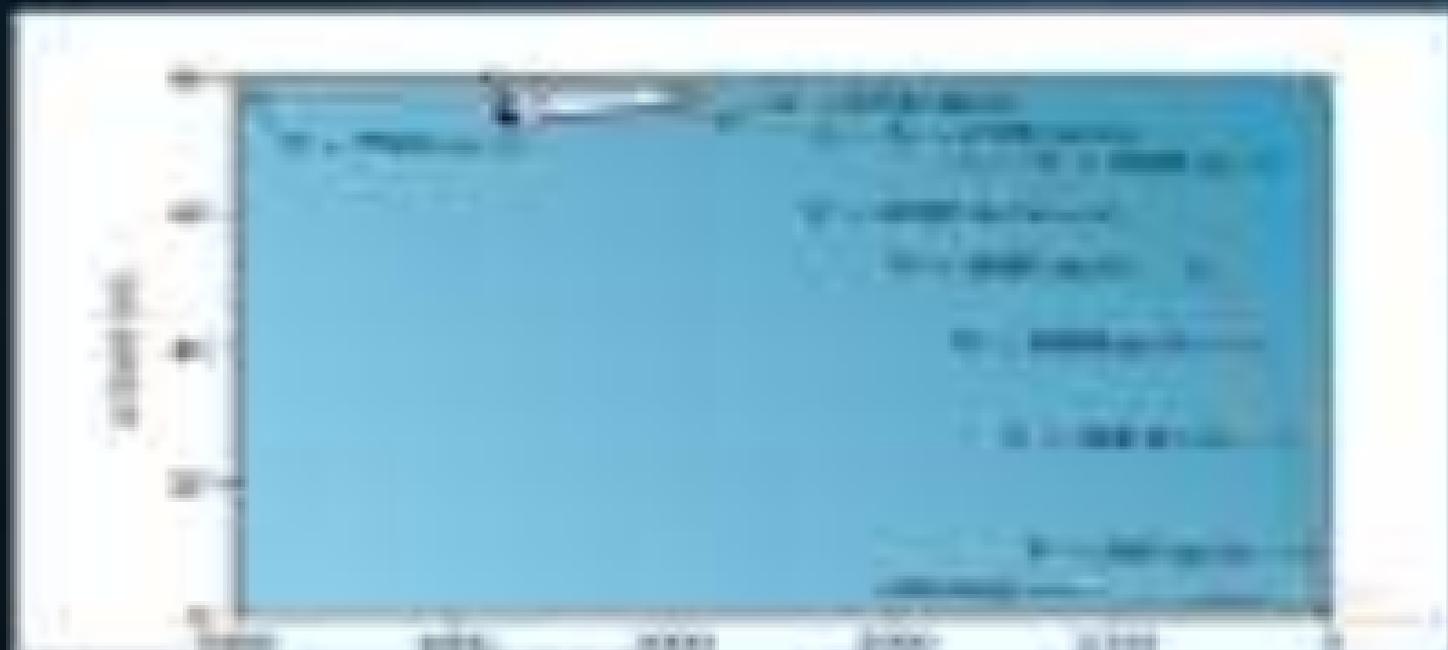
All these phenomena typically occur simultaneously in fluid applications.

VELOCITY-ALTITUDE MAP FOR RE-ENTRY TRAJECTORIES



As hypersonic velocity is attained, the aerodynamic heating becomes so intense that the vehicle must be protected by a thermal protection system.





The highest percentage increase in the population aged 65 and over is seen in Japan, which is projected to rise from approximately 10% in 1950 to 40% in 2050. Other countries with high projected increases include France, Germany, Italy, Spain, Sweden, United Kingdom, United States, Canada, Mexico, Argentina, Colombia, Peru, Chile, Poland, Czechia, Slovakia, Hungary, Russia, Ukraine, Belarus, Latvia, Lithuania, Estonia, Finland, Norway, Denmark, Iceland, Switzerland, Austria, Netherlands, Belgium, Portugal, Greece, and Turkey.

HOW DO WE INCLUDE VISCOUS EFFECTS IN ANALYSIS ?

- **Consideration of the full Navier-Stokes equations**
 - Commercial application of CFD equations
- **Incorporating boundary layer theory**
 - "... the most important and critical achievement in fluid dynamics since the beginning of the 20th century ..."
- **Derivation of boundary layer equations**

NAVIER STOKES EQUATIONS



- The approach by derivation of:
 - Continuity
 - Momentum &
 - Energy Equations
- State the fundamental differential equations used in fluid mechanics
- CFD: All generally requires 3D solution
- Mathematical review of the equations

CONTINUITY EQUATION

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \mathbf{v}) = 0$$

It shows the conservation of mass in a fluid flow. The equation states that the mass entering a control volume must equal the mass leaving the volume.

Conservation of mass

MOMENTUM EQUATIONS

1. **1D collisions**

2. **2D collisions**

3. **Impulse**

4. **Conservation of momentum**



$$m_1 u_1 + m_2 u_2 = m_1 v_1 + m_2 v_2$$

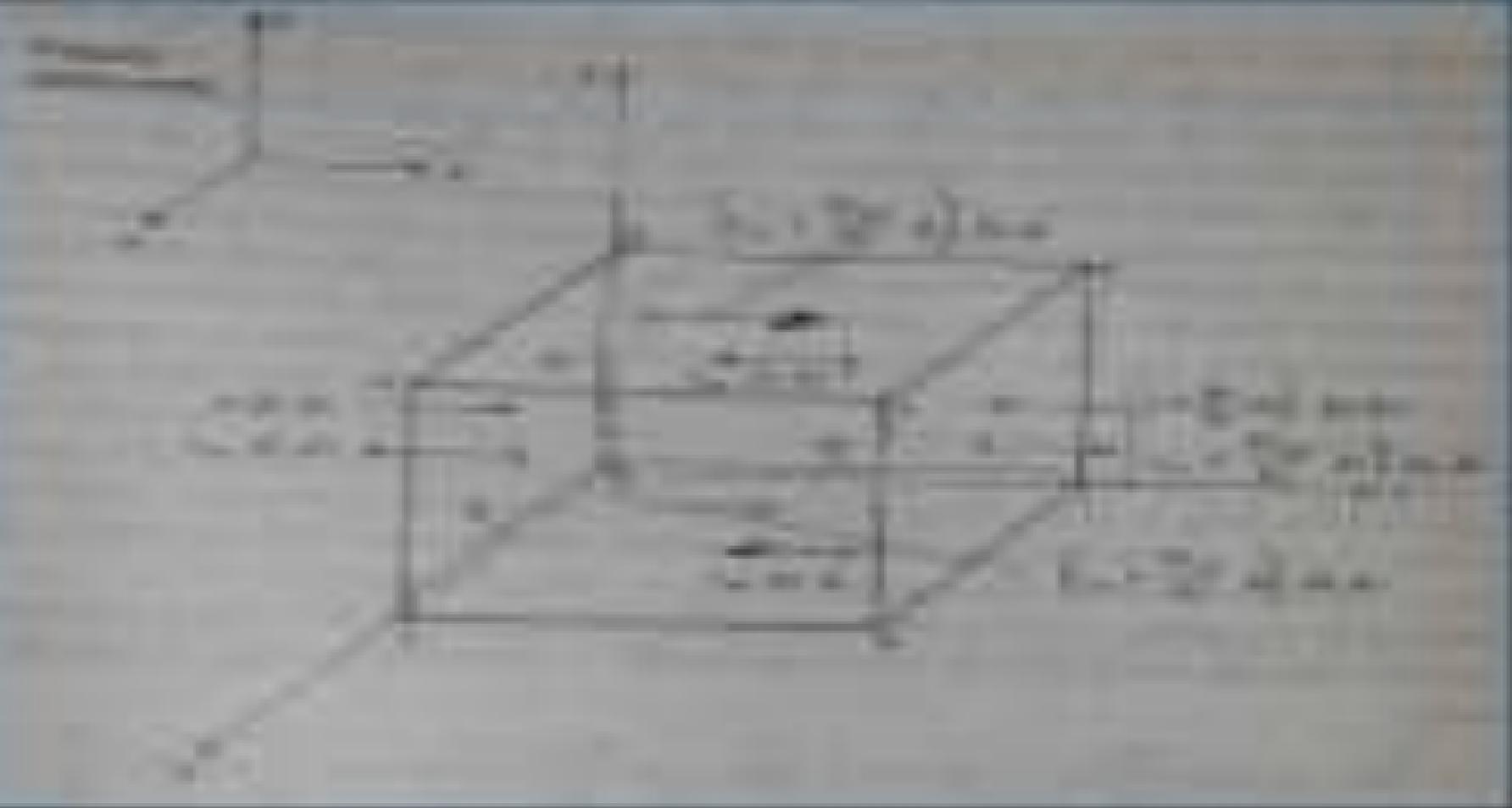
$$m_1 u_1 + m_2 u_2 = m_1 v_1 + m_2 v_2$$

$$m_1 u_1 + m_2 u_2 = m_1 v_1 + m_2 v_2$$

$$m_1 u_1 + m_2 u_2 = m_1 v_1 + m_2 v_2$$

$$m_1 u_1 + m_2 u_2 = m_1 v_1 + m_2 v_2$$

$$m_1 u_1 + m_2 u_2 = m_1 v_1 + m_2 v_2$$



SOLUTION ?

- Approximate Secondary Layer equations
- Power (BT)

NS EQUATIONS IN NON-DIMENSIONAL FORM

- Non-dimensional form of the equations

$$\begin{aligned} \frac{\partial \bar{u}}{\partial \bar{x}} + \bar{u} \frac{\partial \bar{u}}{\partial \bar{x}} + \bar{v} \frac{\partial \bar{u}}{\partial \bar{y}} &= -\frac{\partial \bar{p}}{\partial \bar{x}} + \frac{1}{\text{Pr}} \frac{\partial^2 \bar{u}}{\partial \bar{y}^2} \\ \frac{\partial \bar{v}}{\partial \bar{x}} + \bar{u} \frac{\partial \bar{v}}{\partial \bar{x}} + \bar{v} \frac{\partial \bar{v}}{\partial \bar{y}} &= -\frac{\partial \bar{p}}{\partial \bar{y}} + \frac{1}{\text{Pr}} \frac{\partial^2 \bar{v}}{\partial \bar{y}^2} \\ \frac{\partial \bar{\theta}}{\partial \bar{x}} + \bar{u} \frac{\partial \bar{\theta}}{\partial \bar{x}} + \bar{v} \frac{\partial \bar{\theta}}{\partial \bar{y}} &= \frac{\partial^2 \bar{\theta}}{\partial \bar{y}^2} \end{aligned}$$

...FOR STEADY 2-DIMENSIONAL FLOW

$$\begin{aligned}
 & \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \\
 & \rho \left(u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} \right) = -\frac{\partial p}{\partial x} + \mu \left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} \right) \\
 & \rho \left(u \frac{\partial v}{\partial x} + v \frac{\partial v}{\partial y} \right) = -\frac{\partial p}{\partial y} + \mu \left(\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2} \right) \\
 & \rho \left(u \frac{\partial \theta}{\partial x} + v \frac{\partial \theta}{\partial y} \right) = k \left(\frac{\partial^2 \theta}{\partial x^2} + \frac{\partial^2 \theta}{\partial y^2} \right) + \dot{q} \\
 & \rho \left(u \frac{\partial \omega}{\partial x} + v \frac{\partial \omega}{\partial y} \right) = D \left(\frac{\partial^2 \omega}{\partial x^2} + \frac{\partial^2 \omega}{\partial y^2} \right)
 \end{aligned}$$

DISCUSSION OF NON-DIMENSIONAL PARAMETERS

▷ **Mass ratio**

▷ **Mass ratio**
[M] = [M]

▷ **Mass ratio**
[M] = [M]

$$\frac{M_1}{M_2} = \frac{m_1}{m_2}$$

▷ **Mass ratio**
[M] = [M]

▷ **Mass ratio**
[M] = [M]

▷ **Mass ratio**
[M] = [M]

PRANDTL NUMBER: WHAT AND WHY ?

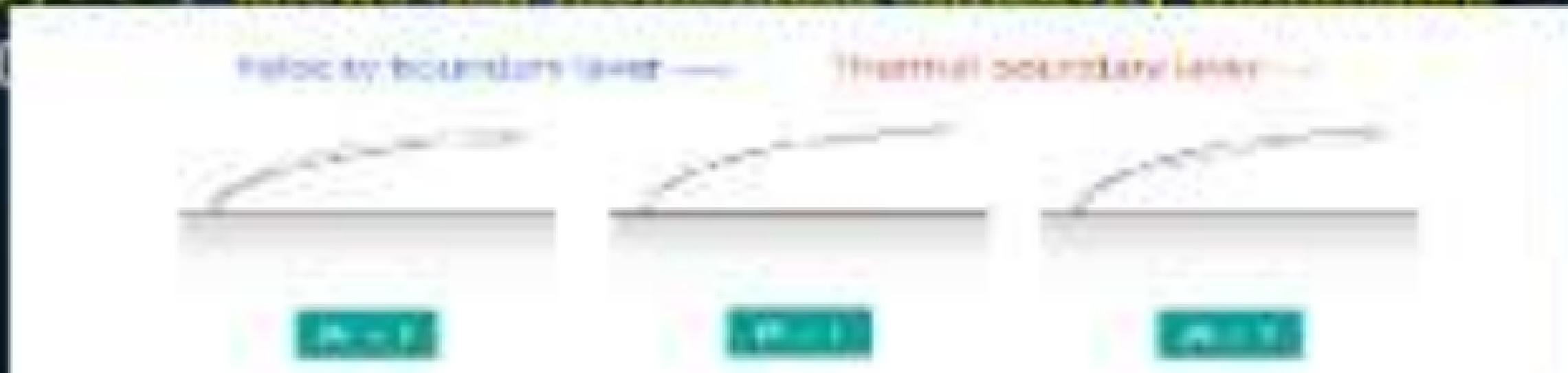
- A non-dimensional parameter generally used in heat transfer
- Used to calculate rate of heat transfer between a moving fluid and a solid body

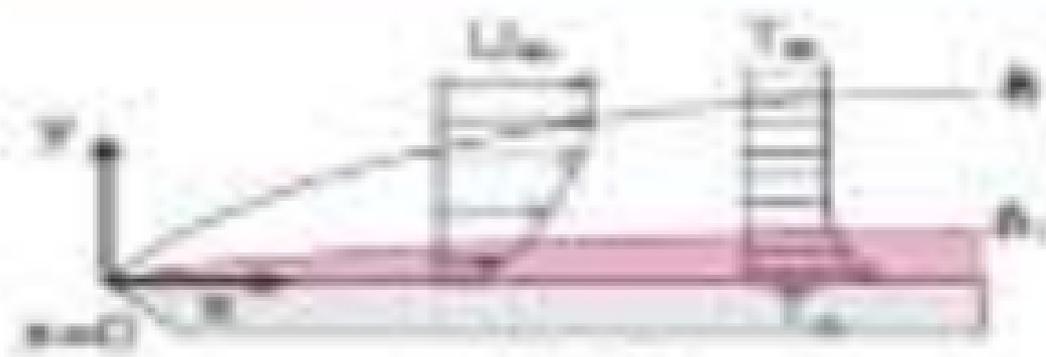
- Defined as $Pr = \frac{\rho C_p \mu}{k}$

- A property of the fluid
- $Pr > 0.7$ for all of standard substances
- **Ratio of momentum diffusivity to thermal diffusivity**

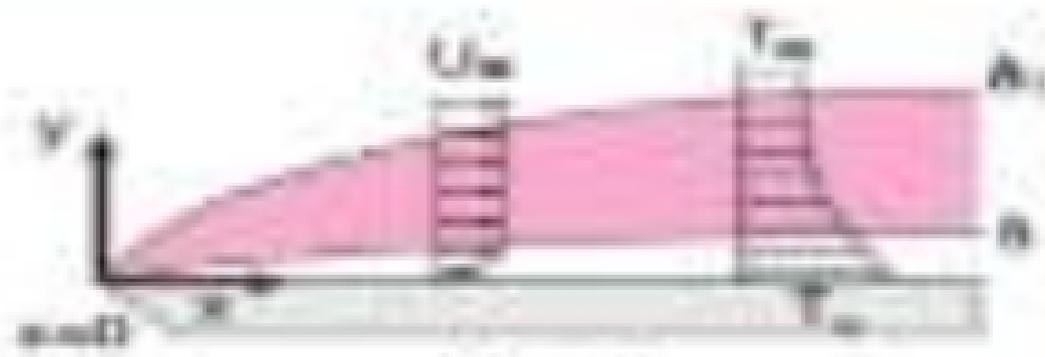
PRANDTL NUMBER: WHAT AND WHY? -CTD.

- $Pr \ll 1$ shows that thermal diffusivity dominates
- $Pr \gg 1$ shows that momentum diffusivity dominates





$P_2 > P_1$
 $B_2 > B_1$



$P_2 < P_1$
 $B_2 < B_1$

BOUNDARY CONDITIONS FOR N-S EQUATIONS (2D)

→ No slip condition at the wall (with no-shear stress condition)

$$u = v = \tau = 0$$

→ Thermal boundary conditions if wall is at constant specified temperature

$$T = T_w$$

→ Thermal boundary conditions if wall temperature varies along

$$T = T_w(x)$$

→ Thermal boundary conditions if wall heat flux is prescribed

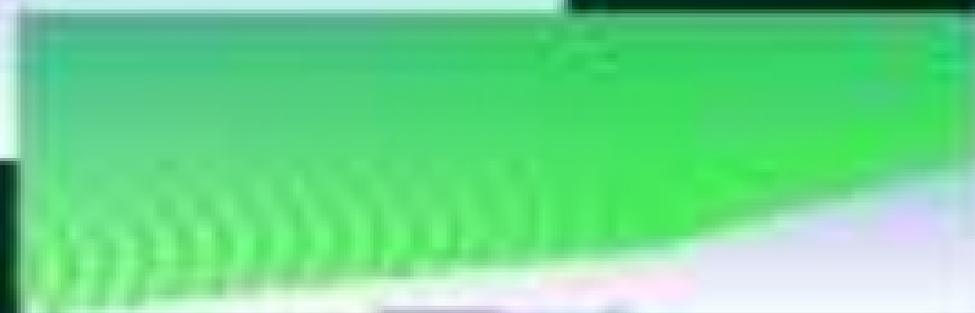
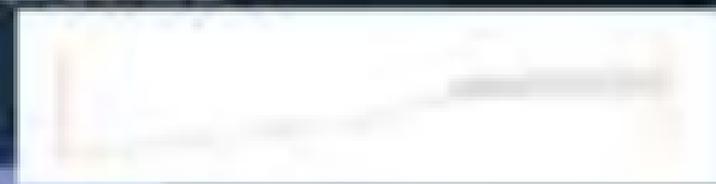
$$q_w = -k \left(\frac{\partial T}{\partial y} \right)_w$$

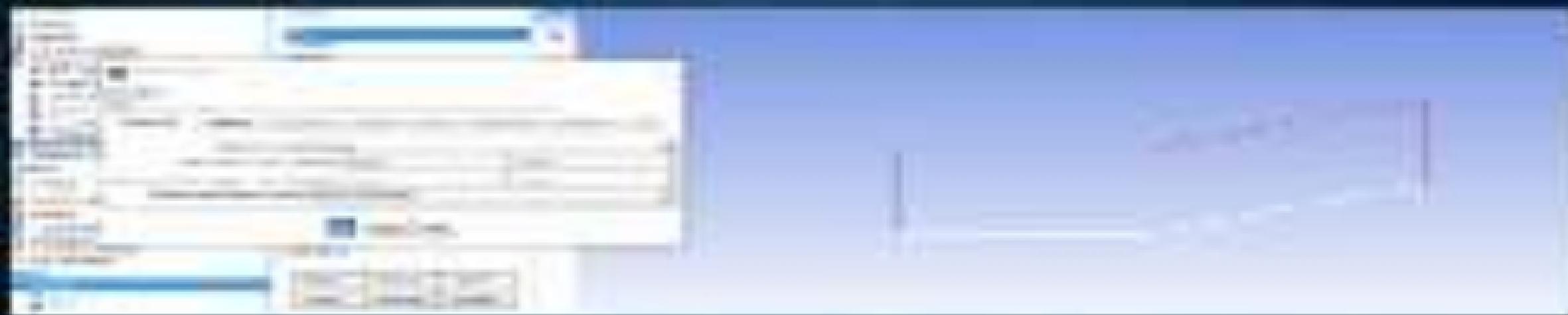
SCRAMJET INTAKE





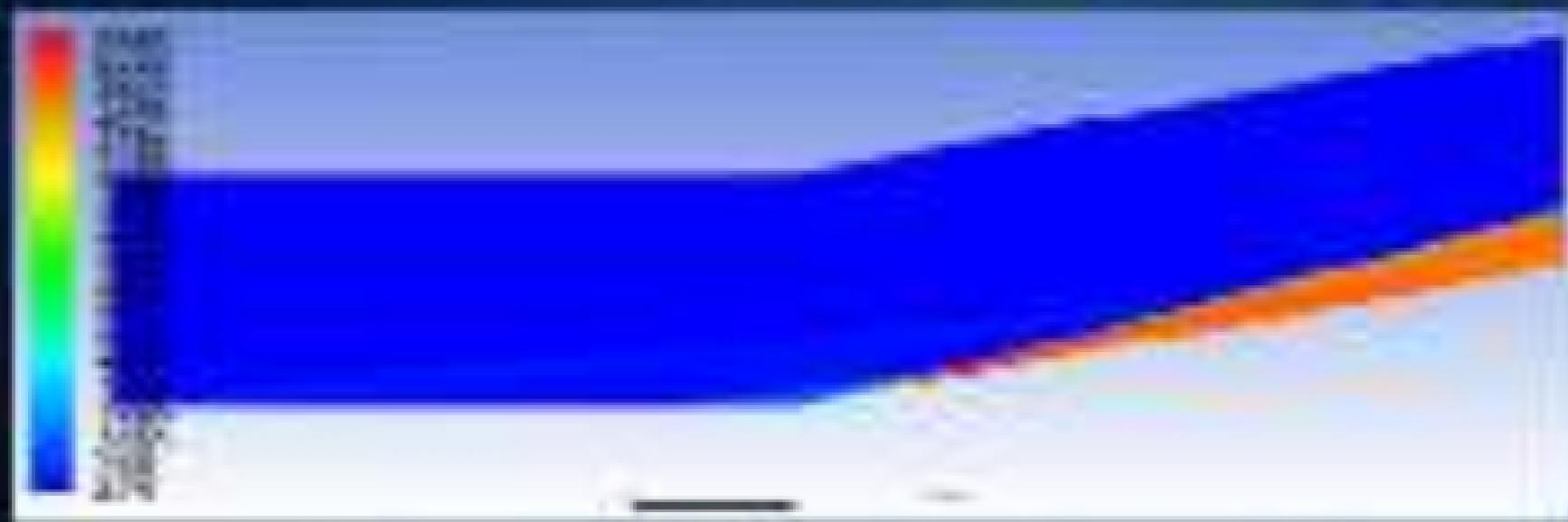
DOUBLE COMPRESSION RAMP



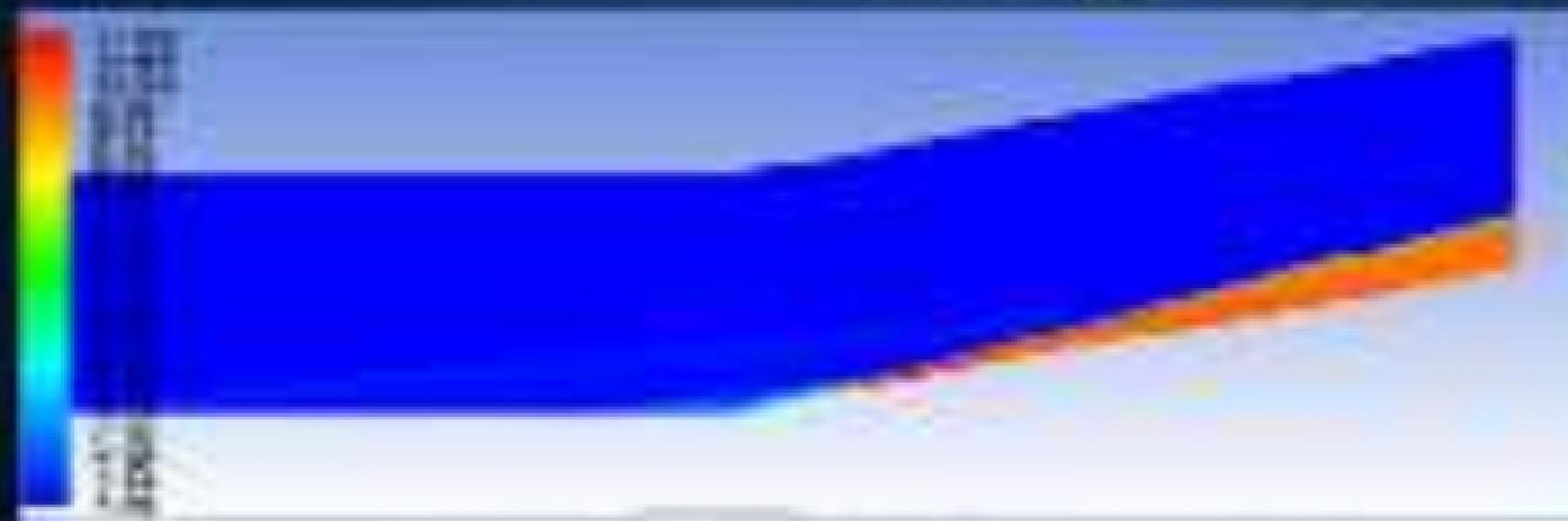


STATIC PRESSURE FIELD

(1) (2) (3) (4) (5) (6) (7) (8) (9) (10) (11) (12) (13) (14) (15) (16) (17) (18) (19) (20) (21) (22) (23) (24) (25) (26) (27) (28) (29) (30) (31) (32) (33) (34) (35) (36) (37) (38) (39) (40) (41) (42) (43) (44) (45) (46) (47) (48) (49) (50) (51) (52) (53) (54) (55) (56) (57) (58) (59) (60) (61) (62) (63) (64) (65) (66) (67) (68) (69) (70) (71) (72) (73) (74) (75) (76) (77) (78) (79) (80) (81) (82) (83) (84) (85) (86) (87) (88) (89) (90) (91) (92) (93) (94) (95) (96) (97) (98) (99) (100)



NON-DIMENSIONALIZED STATIC PRESSURE FIELD



$$\bar{p} = \frac{p}{p_{ref}}$$

SUBSTANTIAL/MATERIAL DERIVATIVE

Substantial/Material
Derivative

Substantial/Material
Derivative

There is a need
to disclose the
substance of the
substantial/material
derivative in
accordance with
the requirements

Substantial/Material
Derivative

HYPERSONIC FLOW THEORY



Session 29: Boundary Layer Equations

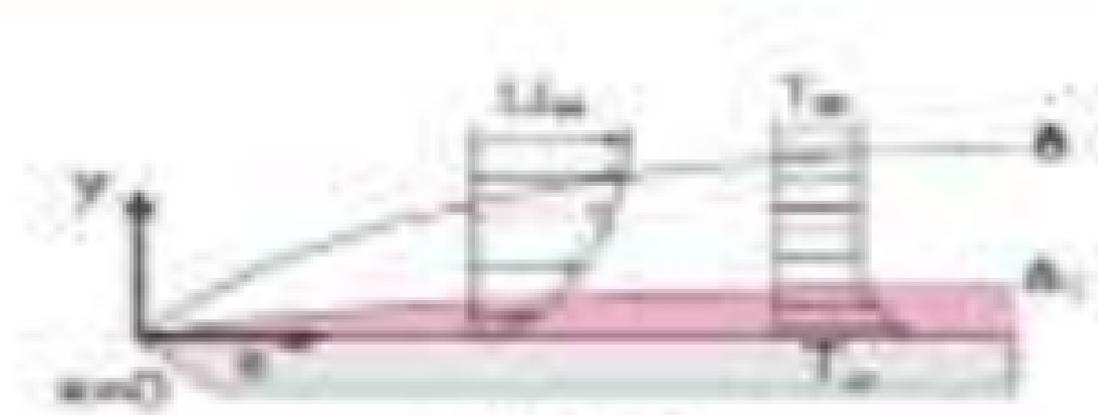
Dr. A. M. Kishor Babu
2020/21

AMARITA

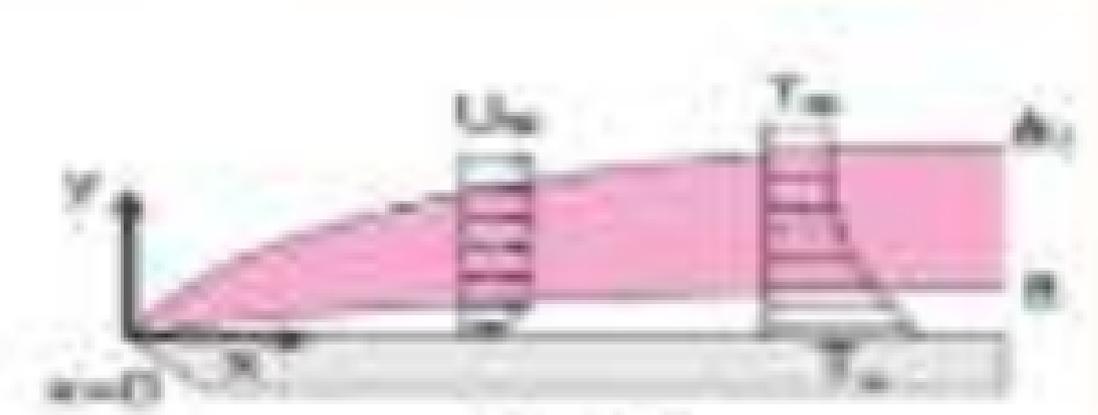
PRANDTL NUMBER: WHAT AND WHY? - CTD.

- $Pr \ll 1$ shows that thermal diffusivity dominates
- $Pr \gg 1$ shows that momentum diffusivity dominates
- [Pr is around 7.6 for water & > 400 for engine oil]





$Pr > 1$
 $b > b_1$



$Pr < 1$
 $b < b_1$

BOUNDARY CONDITIONS FOR N-S EQUATIONS (2D)

→ No slip condition at the wall (with no-shear stress condition)

$$u = v = \tau = 0$$

→ Thermal boundary conditions if wall is at constant specified temperature

$$T = T_w$$

→ Thermal boundary conditions if wall temperature varies along

$$T = T_w(x)$$

→ Thermal boundary conditions if wall heat flux is prescribed

$$q_w = -k \left(\frac{\partial T}{\partial y} \right)_w$$

$$\frac{2x^2 - 20x + 45}{x^2 - 25} = \frac{A}{x-5} + \frac{B}{x+5}$$

$$\frac{2x^2 - 20x + 45}{(x-5)(x+5)} = \frac{A(x+5)}{(x-5)(x+5)} + \frac{B(x-5)}{(x-5)(x+5)}$$

$$\frac{2x^2 - 20x + 45}{(x-5)(x+5)} = \frac{A(x+5) + B(x-5)}{(x-5)(x+5)}$$

$$\frac{2x^2 - 20x + 45}{(x-5)(x+5)} = \frac{Ax + 5A + Bx - 5B}{(x-5)(x+5)}$$

$$\frac{2x^2 - 20x + 45}{(x-5)(x+5)} = \frac{(A+B)x + (5A-5B)}{(x-5)(x+5)}$$

$$2x^2 - 20x + 45 = (A+B)x + (5A-5B)$$

$$\frac{2x^2 - 20x + 45}{x^2 - 25} = \frac{1}{x-5} + \frac{1}{x+5}$$

PRANDTL'S ORDER OF MAGNITUDE ANALYSIS

- Viscous effects are confined to a thin region near the solid surface
- This region is called boundary layer
- The thickness of boundary layer is much smaller than the width of the body



ORDERS OF MAGNITUDE

1.22. Homogeneous systems depending on λ are called **ODEs of order n** .

$$\frac{d^2x}{dt^2} + \frac{dx}{dt} = 0$$

$$\frac{d^2x}{dt^2} + \frac{dx}{dt} = 0 \quad \frac{d^2y}{dt^2} + \frac{dy}{dt} = 0 \quad \frac{d^2z}{dt^2} + \frac{dz}{dt} = 0$$

1. Constant Equations

+ λ varies from 0 (at $\omega(t)$) to ∞ (Free system - where $\omega = \infty$)

+ λ is of order of magnitude 1

+ λ varies from 0 to 1 - Hence $\mathcal{O}(t) = 1$

+ λ varies from 0 to ∞ (not to 0) - Hence $\mathcal{O}(t) = \lambda / \omega$

+ Considering unit length (no t); the order of magnitude from of continuity eqn

becomes: $\frac{d^2x(t)}{dt^2} + \frac{dx(t)}{dt} = 0$ **THIS IMPLIES THAT t IS**

$\mathcal{O}(t)$

ORDERS OF MAGNITUDE-CTD.

→ Similar analysis leads to:

$$\begin{aligned} \frac{dy}{dt} = 0 & \Rightarrow \frac{dy}{dt} = 0 & \Rightarrow \frac{dy}{dt} = 0 \\ \frac{d^2y}{dt^2} = 0 & \Rightarrow \frac{d^2y}{dt^2} = 0 & \Rightarrow \frac{d^2y}{dt^2} = 0 \end{aligned}$$

→ Also $\text{Re}(\lambda) < 0$

→ The order of magnitude analysis of y concentration equation leads to:

→ → Ratio provides no real information since that $\text{Re}(\lambda) < 0$

$$\frac{dy}{dt} = 0$$

THE BOUNDARY LAYER EQUATIONS

- Using observations based on the order of magnitude analysis, applying across the BL, the dimensional forms of steady, 2D governing equations within boundary layer become:

$$\begin{aligned} & \boxed{\rho \frac{Dv}{Dt} = \rho \frac{Dv}{Dt} = -\frac{\partial p}{\partial y} + \mu \left(\frac{\partial^2 v}{\partial y^2} \right)} \\ & \boxed{\rho \frac{Dw}{Dt} = \rho \frac{Dw}{Dt} = -\frac{\partial p}{\partial z} + \mu \left(\frac{\partial^2 w}{\partial y^2} \right)} \\ & \boxed{\rho \frac{Dw}{Dt} = \rho \frac{Dw}{Dt} = -\frac{\partial p}{\partial z} + \mu \left(\frac{\partial^2 w}{\partial y^2} \right) + \rho g_z} \end{aligned}$$

THE BOUNDARY CONDITIONS

• At the wall:

$$u = 0, \quad v = 0, \quad w = 0$$

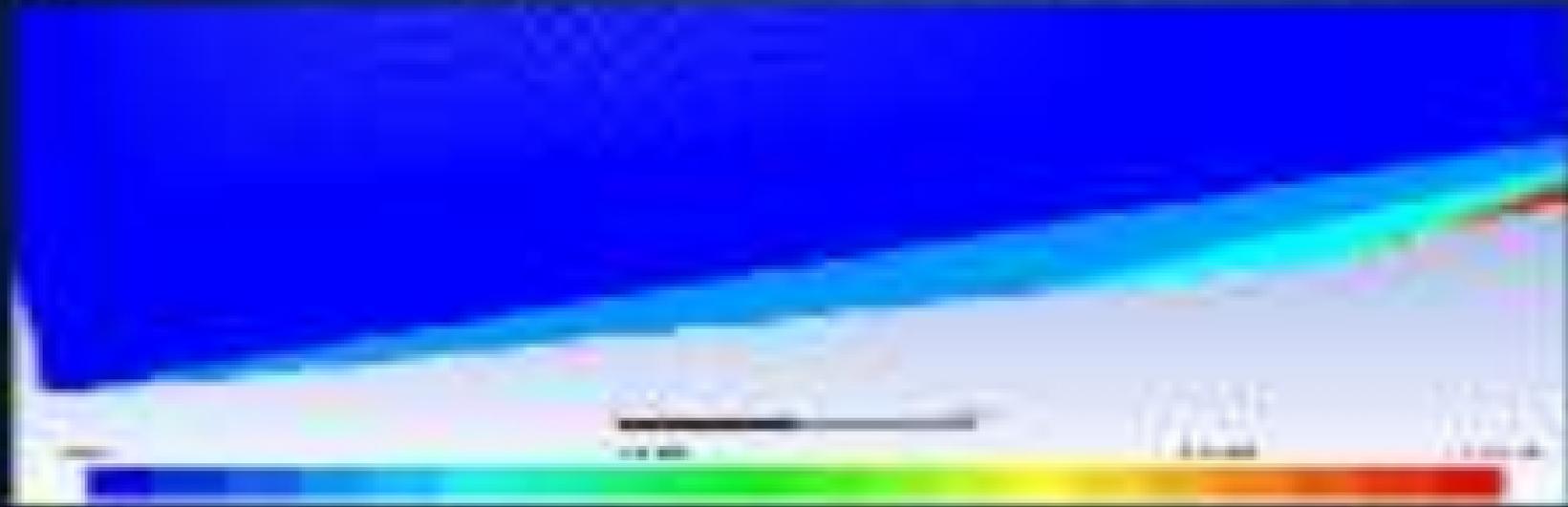
$$T = T_w$$

$$\rightarrow \left(\frac{\partial T}{\partial n} \right)_w = 0$$

• At the edge of the boundary layer:

$$T = T_\infty, \quad H = H_\infty, \quad E = T_\infty$$

Figure 10



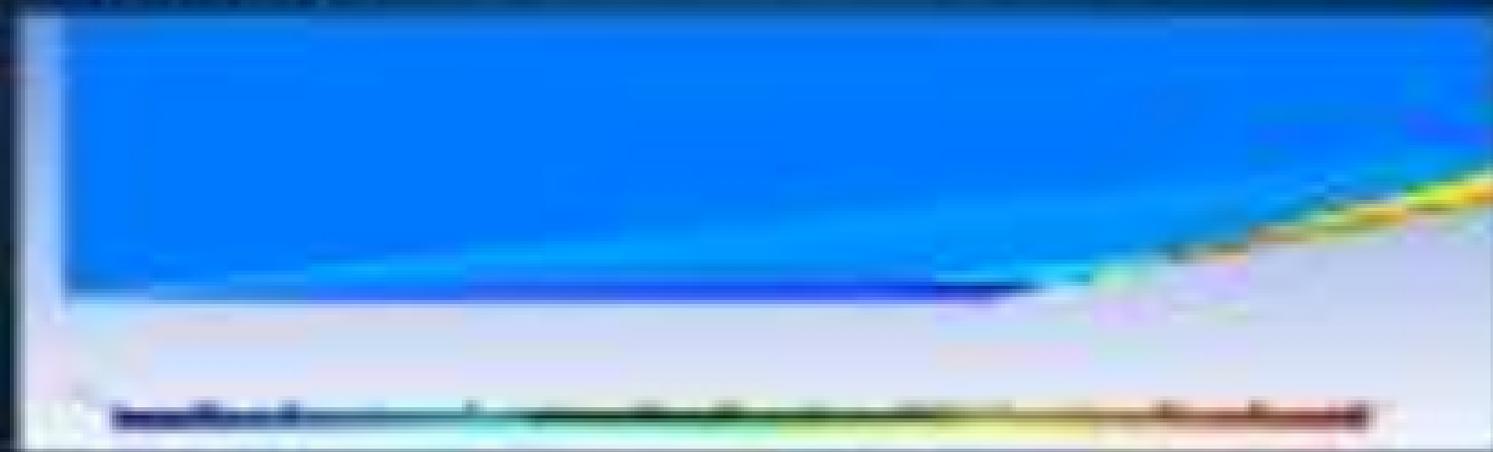
EXAMINING THE ORDERS OF MAGNITUDE...

→ 0.1 to 100



Examine your results. Group
in. Appendix

Figure 10: Numerical solution of the Burgers' equation



APPENDIX

CONTINUITY EQUATION

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \mathbf{v}) = 0$$

MOMENTUM EQUATIONS

1. **1D collisions**

2. **2D collisions**

3. **Impulse**

4. **Conservation of momentum**

5. **Conservation of energy**

$$m_1 u_1 + m_2 u_2 = m_1 v_1 + m_2 v_2$$

$$m_1 u_1 + m_2 u_2 = m_1 v_1 + m_2 v_2$$

$$m_1 u_1 + m_2 u_2 = m_1 v_1 + m_2 v_2$$

$$m_1 u_1 + m_2 u_2 = m_1 v_1 + m_2 v_2$$

$$m_1 u_1 + m_2 u_2 = m_1 v_1 + m_2 v_2$$

$$m_1 u_1 + m_2 u_2 = m_1 v_1 + m_2 v_2$$

$$m_1 u_1 + m_2 u_2 = m_1 v_1 + m_2 v_2$$

$$m_1 u_1 + m_2 u_2 = m_1 v_1 + m_2 v_2$$

$$m_1 u_1 + m_2 u_2 = m_1 v_1 + m_2 v_2$$

ENERGY EQUATIONS

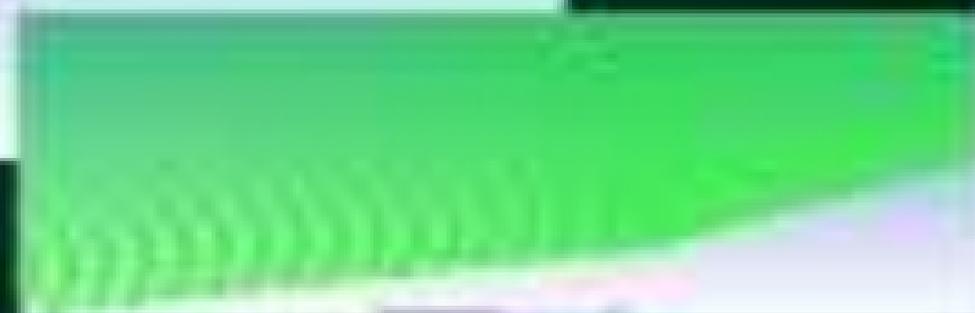
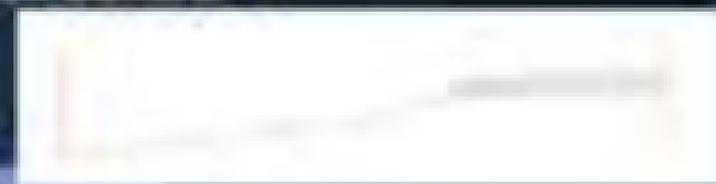
$$\begin{aligned}
 \rho \frac{D\mathcal{E}}{Dt} + \nabla \cdot (\mathcal{E}\mathbf{v}) &= \rho \left(\frac{\partial \mathcal{E}}{\partial t} + \mathbf{v} \cdot \nabla \mathcal{E} \right) + \nabla \cdot (\mathcal{E}\mathbf{v}) = \frac{\partial}{\partial t} (\rho \mathcal{E}) + \nabla \cdot (\rho \mathcal{E}\mathbf{v}) \\
 &= \frac{\partial}{\partial t} (\rho \mathcal{E}) + \nabla \cdot (\rho \mathcal{E}\mathbf{v}) + \nabla \cdot (\rho \mathbf{v}\mathcal{E}) - \nabla \cdot (\rho \mathbf{v}\mathcal{E}) \\
 &= \frac{\partial}{\partial t} (\rho \mathcal{E}) + \nabla \cdot (\rho \mathcal{E}\mathbf{v} + \rho \mathbf{v}\mathcal{E}) - \rho \mathbf{v} \cdot \nabla \mathcal{E}
 \end{aligned}$$

SCRAMJET INTAKE





DOUBLE COMPRESSION RAMP



HYPERSONIC FLOW THEORY



Section 30-3.3: Self
Similarity Transformation
in Laminar Boundary
Layer

Jan. 28, 2019, Sriharisharan
20204

AMARITA

NS EQUATIONS AS APPLIED TO BL - 2D, STEADY

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0$$

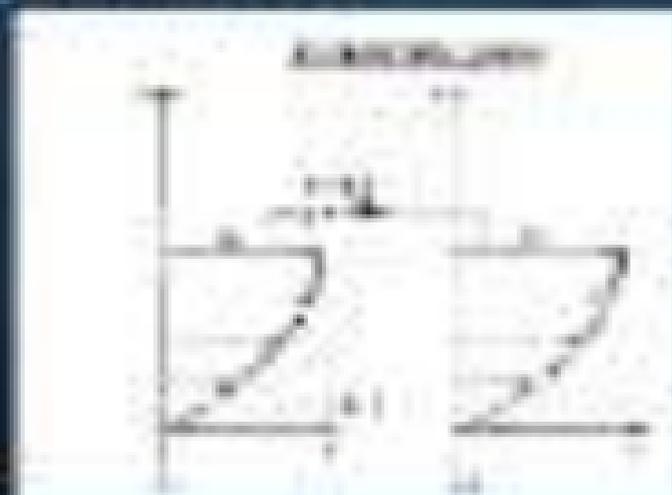
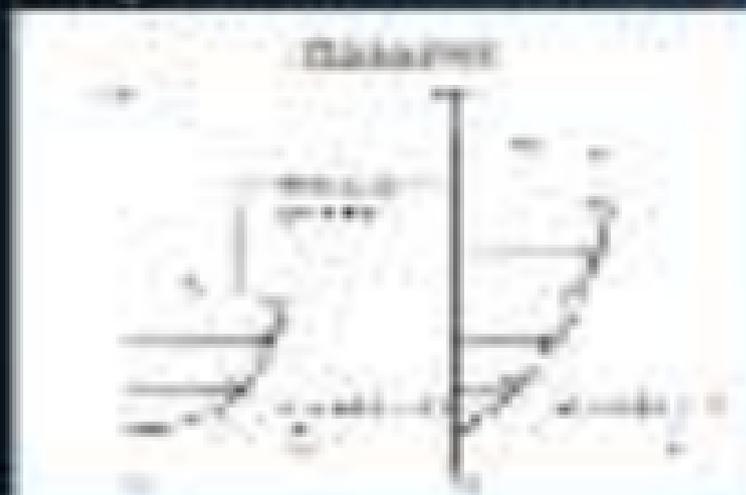
$$\rho \left(u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} \right) = \mu \frac{\partial^2 u}{\partial y^2}$$

$$\rho \left(u \frac{\partial v}{\partial x} + v \frac{\partial v}{\partial y} \right) = \mu \frac{\partial^2 v}{\partial y^2}$$

$$\rho \left(u \frac{\partial \theta}{\partial x} + v \frac{\partial \theta}{\partial y} \right) = k \frac{\partial^2 \theta}{\partial y^2}$$

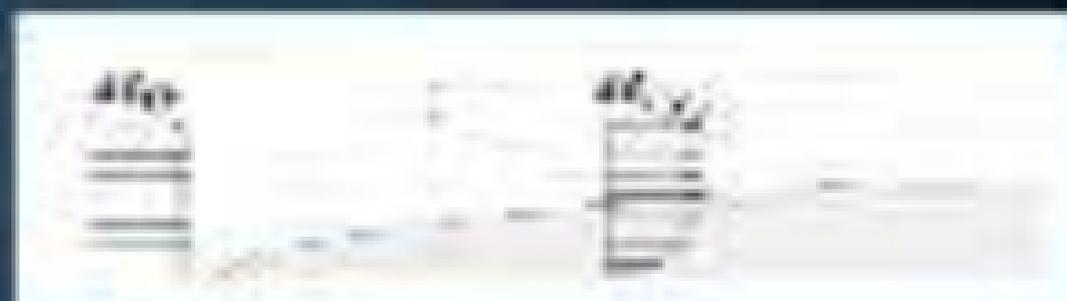
THE TRANSFORMATION: OBJECTIVE

- Transform the governing PDE to ODE which are easier to solve
- Such transformations lead to solutions that are functions of profiles that are independent of location along the surface

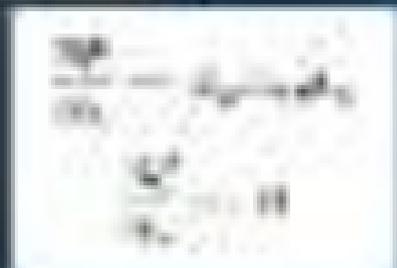


Boundary layer flows are characterized by the presence of a thin layer of fluid adjacent to the surface where the velocity gradient is large. The boundary layer is the region where the velocity is not yet fully developed and is still influenced by the surface. The boundary layer is the region where the velocity is not yet fully developed and is still influenced by the surface.

THE TRANSFORMATION



Message **Example** only



- Using the definition of stress tensor

$$\sigma_{ij} = p_{ij} - \rho \mathbf{v}_i \mathbf{v}_j$$

is symmetric for inviscid and irrotational motions

- The \mathbf{x} component of momentum is

$$\begin{aligned} \frac{1}{\rho} \left[\rho v_x \frac{dv_x}{dt} + \left(\frac{\partial v_x}{\partial t} \right) \rho \right] &= \left[\rho v_x \frac{dv_x}{dt} + \left(\frac{\partial v_x}{\partial t} \right) \rho \right] \frac{1}{\rho} \\ &= -\frac{1}{\rho} \frac{\partial p}{\partial x} - \frac{1}{\rho} \left(\frac{\partial \rho v_x v_x}{\partial x} + \frac{\partial \rho v_x v_y}{\partial y} + \frac{\partial \rho v_x v_z}{\partial z} \right) \end{aligned}$$

- In order to transform the dependent variables, define f and g such that

$$f = \int_{\mathcal{C}_1} \mathbf{c}_1 \cdot d\mathbf{r}$$



$$g = \int_{\mathcal{C}_2} \mathbf{c}_2 \cdot d\mathbf{r}$$

$$d\mathbf{r} = dx\mathbf{e}_1 + dy\mathbf{e}_2$$

TRANSFORMED GOVERNING EQUATIONS

$$\left[\frac{1}{r} \frac{\partial}{\partial r} \left(r \frac{\partial}{\partial r} \right) + \frac{1}{r^2} \frac{\partial^2}{\partial \theta^2} \right] \frac{u}{r} = \frac{1}{r} \frac{\partial}{\partial r} \left(r \frac{\partial}{\partial r} \right) \frac{v}{r} + \frac{1}{r^2} \frac{\partial^2}{\partial \theta^2} \frac{v}{r}$$

$\frac{1}{r} \frac{\partial}{\partial r} \left(r \frac{\partial}{\partial r} \right) \frac{u}{r}$

$$\frac{u}{r} = 0$$

$$\left[\frac{1}{r} \frac{\partial}{\partial r} \left(r \frac{\partial}{\partial r} \right) + \frac{1}{r^2} \frac{\partial^2}{\partial \theta^2} \right] \frac{v}{r} = \frac{1}{r} \frac{\partial}{\partial r} \left(r \frac{\partial}{\partial r} \right) \frac{v}{r} + \frac{1}{r^2} \frac{\partial^2}{\partial \theta^2} \frac{v}{r}$$

$\frac{1}{r} \frac{\partial}{\partial r} \left(r \frac{\partial}{\partial r} \right) \frac{v}{r}$

BC'S IN TERMS OF THE TRANSFORMED VARIABLES

At the end of each slide, all variables are

normalized to the range $[-1, 1]$

$$x = \frac{x_1 - x_2}{x_1 + x_2}, \quad y = \frac{y_1 - y_2}{y_1 + y_2}, \quad z = \frac{z_1 - z_2}{z_1 + z_2}$$

is applied to each

variable of the function
for the two layers of the system

$$z = 0$$

$$x = x_1$$

$$y = y_1$$

$$z = z_1$$

$$z = \frac{z_1 - z_2}{z_1 + z_2} \left[\frac{z_1 + z_2}{2} \right]$$

$$\frac{z_1 - z_2}{z_1 + z_2} = \frac{z_1 + z_2}{2}$$

$$z = \frac{z_1 - z_2}{z_1 + z_2} \left[\frac{z_1 + z_2}{2} \right]$$

are normalized to the range $[-1, 1]$

Navigation icons: back, forward, search, etc.

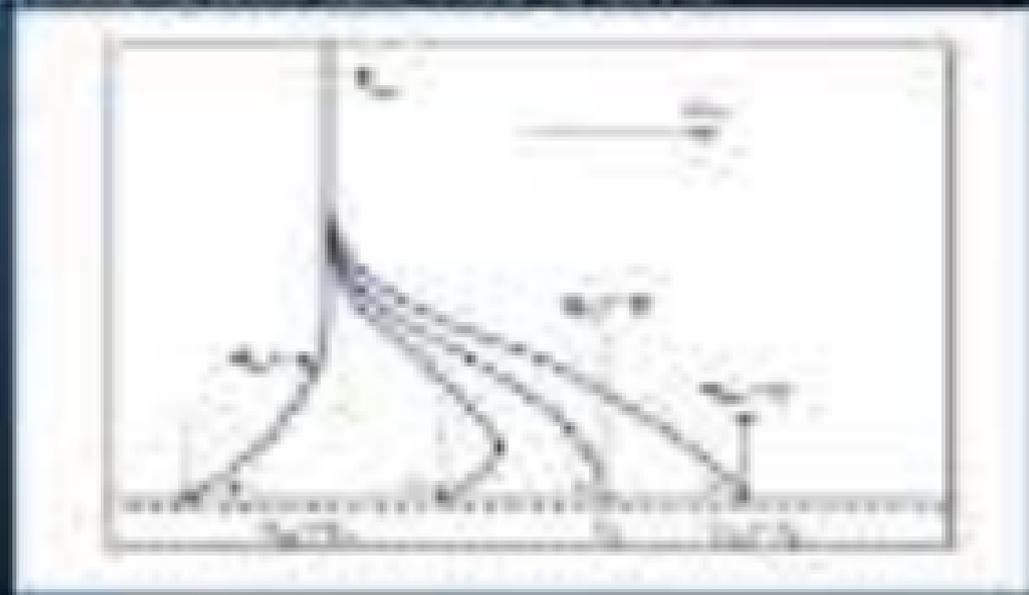
THERMAL BOUNDARY CONDITIONS: ADIABATIC WALL TEMPERATURE

- T_{aw} is the temperature of the wall when heat transfer into the wall is zero.

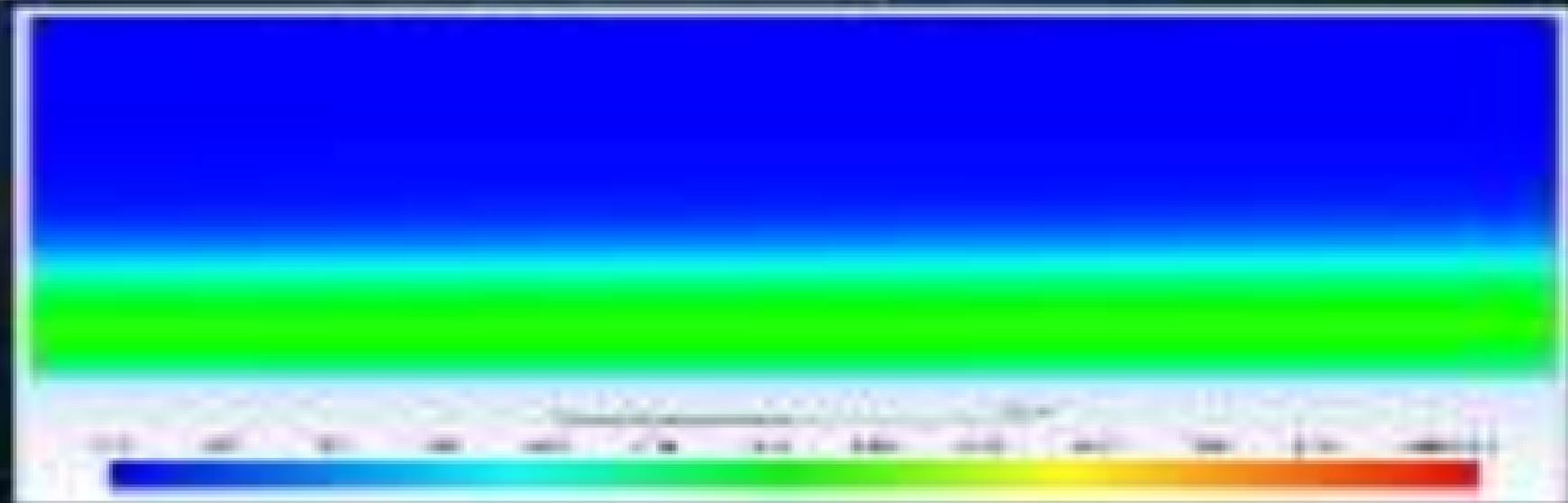
Orange box: T_{aw} is the temperature of the wall when heat transfer into the wall is zero.

Green box: Flow expands from the wall into the fluid.

Blue box: Flow contracts from the fluid to the wall.

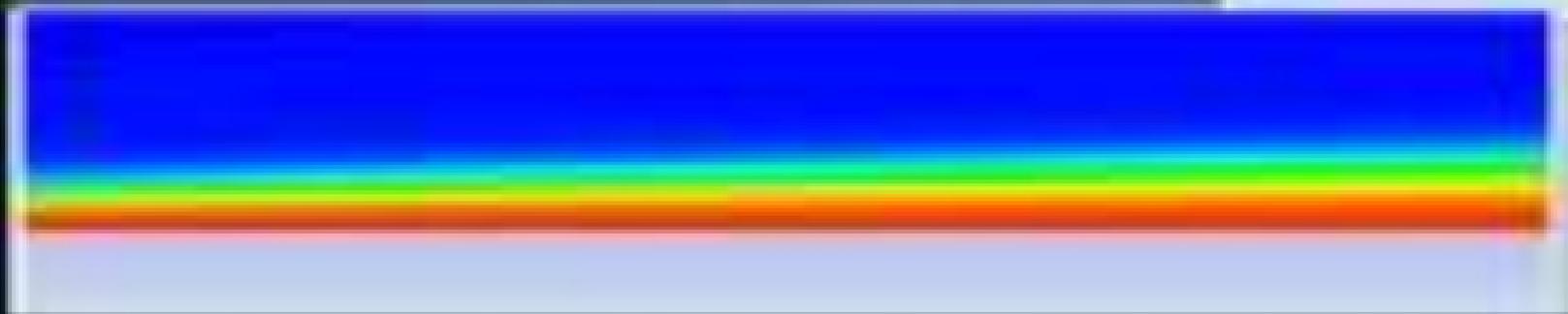








ADLABATIC
WALL



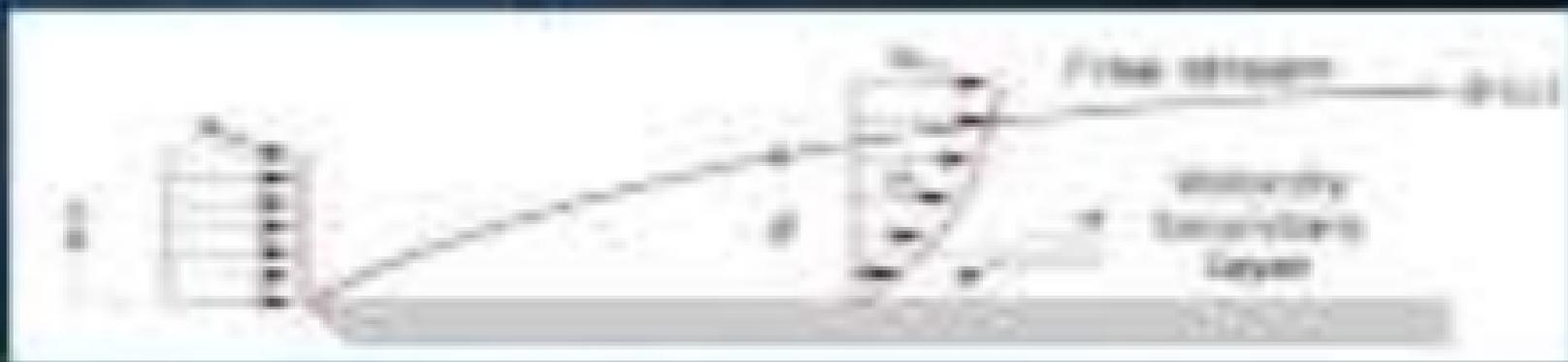
FLOW OVER A FLAT PLATE

→ The boundary

$$u = u(x, y) \quad v = v(x, y) \quad w = 0$$

$$u = u(x, y)$$

$$\frac{\partial u}{\partial x} = 0$$



$$\frac{\partial u}{\partial x} = 0 \quad \frac{\partial u}{\partial y} = 0 \quad \frac{\partial u}{\partial z} = 0$$

$$C = \rho_1 \omega_1 + \rho_2 \omega_2$$

$$Q_{in} = \left[A \frac{U \Delta T}{R_{in}} \right]_{in}$$

$$Q_{out} = \frac{T_{in} - T_{out}}{R_{out}}$$

$$T_{out} = \left[A \left(\frac{U \Delta T}{R_{out}} \right) \right]_{out}$$

NUSSOLT NUMBER & STANTON NUMBER

$$Nu = \frac{h_c X}{k_f}$$

$$St = \frac{h_c}{\rho_f u_f (c_{p,f} - c_{p,s})}$$

CONDUCTION VS CONVECTION



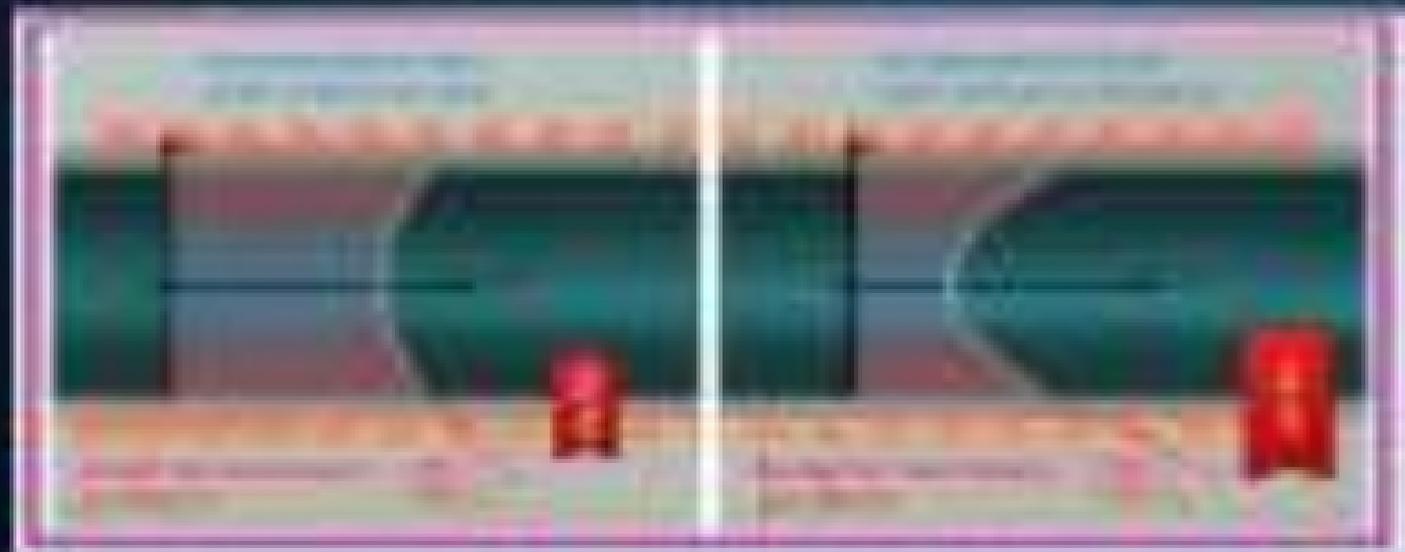
Conduction is transfer of heat through direct contact.

Transfer of energy from one molecule to another by the vibrating motion.

There will be no moving particles or molecules of matter. It is a stationary process.

CONDUCTION VS CONVECTION – CTD.

- Left: When the fluid is at rest. Right: When the fluid is flowing.



Conduction is the transfer of heat through a stationary fluid. Convection is the transfer of heat through a moving fluid.

NUSSOLT NUMBER

→ Flow of **liquid** in a transition

$$Nu = \frac{hL_c}{k_f}$$

→ Flow of **gas** in a transition

$$Nu = \frac{hL_c}{k_g}$$

→ Nusselt number in this case is **dimensionless** because we are dividing heat transfer under identical conditions.

$$Nu_{avg} = \frac{q_{avg}(\text{convective form})}{q_{avg}(\text{conduction})} = \frac{hL_c}{k}$$

→ **It is a dimensionless** heat transfer coefficient. **The value of a convective heat transfer coefficient depends on the properties of the fluid, geometry of the solid surface and the flow.**

SKIN FRICTION COEFFICIENT

- (1) The normal force will always be

$$F_N = mg \cos \theta$$

$$F_f = \mu F_N$$

- Kinematics analysis for the mass **that slides** leads to:

$$v^2 = v_0^2 + 2a\Delta x$$

STANTON NUMBER (S_T OR C_D)

$$S_T = \frac{\text{drag force}}{\rho U^2 L^2}$$

- Unsteady flow profiles:

$$\text{drag } F_D = C_D \rho U^2 L^2$$

$$\frac{F_D}{\rho U^2 L^2} = C_D$$

- This non-dimensional procedure is referred to as **dynamic similitude**.

- Fr (C_D) Measure the ratio of drag resistance from a fluid by immersion in the fluid **capacity of fluid**.

- Similarity analysis of flow over flat plate leads to: $C_D =$

$$\frac{C_D \rho U^2 L^2}{\mu U} = \frac{C_D \rho U L}{\mu} = C_{Df}$$

REYNOLDS ANALOGY

- The relation between convective heat transfer and heat transfer in \mathcal{E}

$$\frac{h}{k_f} = \frac{c_p \rho u}{Pr} = \frac{c_p \rho u}{\frac{\mu c_p}{k_f}} = \frac{c_p \rho u k_f}{\mu}$$

$$\frac{h}{k_f} = \frac{c_p \rho u}{Pr} = \frac{c_p \rho u}{\frac{\mu c_p}{k_f}} = \frac{c_p \rho u k_f}{\mu}$$

REYNOLDS ANALOGY



$$St = \frac{0.023 Re^{-0.4} Pr^{0.4}}{1 + 1.08 Re^{-0.1} + 1.8 Pr^{-0.54}}$$



$$St = \frac{0.023 Re^{-0.4} Pr^{0.4}}{1 + 1.08 Re^{-0.1} + 1.8 Pr^{-0.54}}$$

SUMMARY OF THE BOUNDARY LAYER ANALYSIS

- The objective - Apply the parallel physical approximations that were used in boundary layer approximations to fluid mechanics to simplify the NS equations
- Observations based on the physical phenomena in the viscosity dominated layer include velocity components being much smaller than pressure in the limit where $\delta \ll L$
 - **LAMINAR** flow over a flat surface was considered
- An appropriate recombination of variables would be necessary in the process of simplifying along the flow direction
- The momentum equation is solved in terms of a function of two parameters, the non-dimensional y and x

SUMMARY OF THE BOUNDARY LAYER ANALYSIS - CTD.

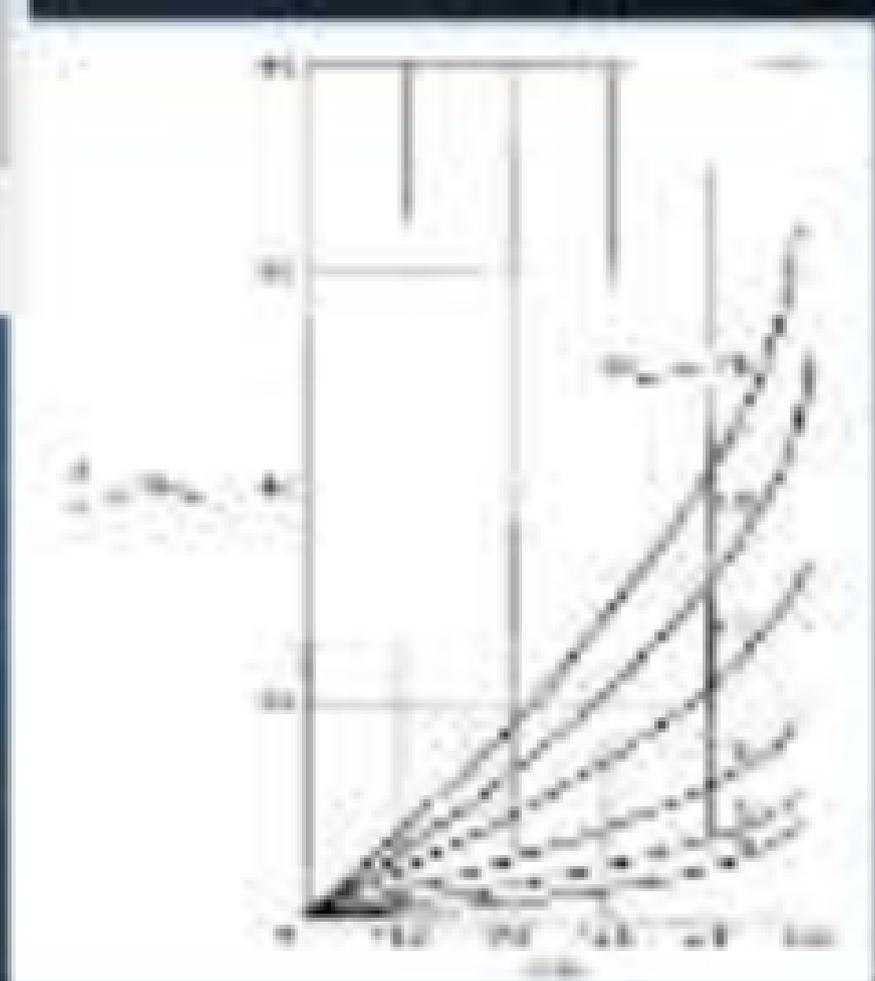
- The above analysis is intended to demonstrate the value of a boundary layer model in the chemical environment, particularly in the treatment of the secondary wave.
- The dimensional groupings help indicate those terms based on known flow conditions.
 - These terms are then used to indicate the relative value of each in the drag term.
 - Assumptions become more clear for re-derivation.
- The above discussion also indicates the flow regime based on Reynolds's Number. Some of the scaling helps indicate what boundary layer conditions will develop in various

MORE RESULTS

Close to the wall, important aspects of turbulent boundary layers: secondary layers, multiple transition layers or large-scale structures.

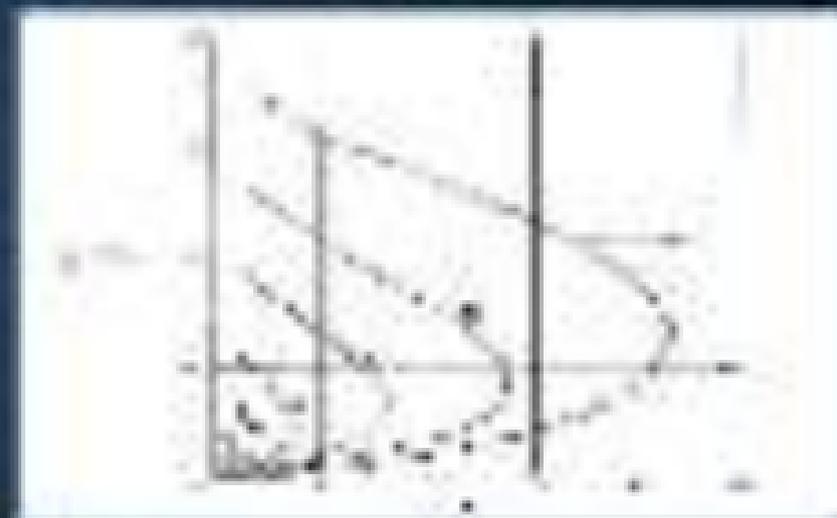
At a given location x , mean flow, time-averaged velocity \bar{u} .

u' represents velocity **fluctuation** at location y/z and time t .



TEMPERATURE PROFILE IN HYPERSONIC BL

- Shows the peak values for BL - over hypersonic flow on surfaces of hypersonic of TC within BL.



RECOVERY FACTOR

- If the temperature at the wall is higher than the wall temperature, the wall temperature will rise. The wall temperature will rise until the temperature at the wall is equal to the wall temperature. This is quantified by a recovery factor.

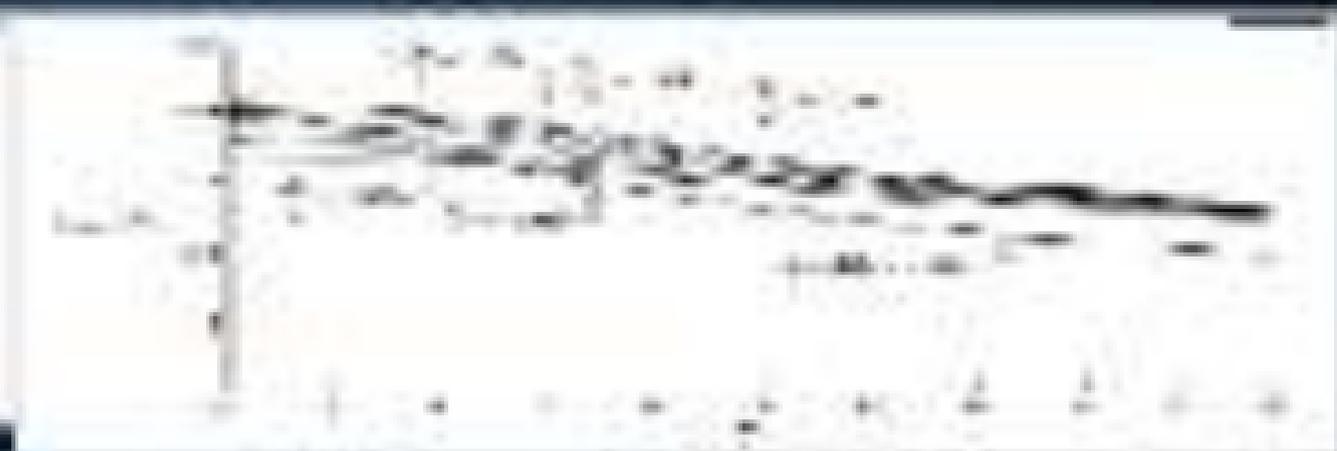
$$r = \frac{T_{\text{wall}} - T_c}{T_{\text{total}} - T_c}$$

- Typically, recovery factor for air is 0.75

$$r = 0.75$$

NUMERICAL PROBLEM

- Consider a flat plate exposed to hypersonic, incoming stream of air at 2000 m/s under ambient conditions of pressure 10000 Pa & temperature 210 K . Length of the plate is 0.5 m . Calculate Mach number at a point 0.3 m from the leading edge. Assume the wall temperature to be measured point is 210 K . Calculate the heat flux at the location.



STAGNATION POINT BL OF BLUNT BODY SURFACE



SPECIFIC SIMPLIFICATIONS

- Easy to amend the underlying analysis with specific simplifications that apply to the different parts of it
 - In the regional industry as the business due to natural choice
 - Heat transfer is large due to elongation
 - It is not 
- Also, due to the fact, missing in the region, manufacturing. Even volume for 10,000 is applied.

THE TRANSFORMED GOVERNING EQUATIONS

- The above approach leads to the well-posed governing equations in the frequency domain

$$\mathbf{K}(\omega) \mathbf{U} = \mathbf{F}(\omega) - \mathbf{U}$$

$$\left| \begin{pmatrix} \mathbf{K}(\omega) \\ \mathbf{I} \end{pmatrix} \right| \mathbf{U} = \mathbf{F}(\omega)$$

STAGNATION POINT HEAT TRANSFER

- A solution for the streamlines upstream of a blunt body without a leading edge is obtained as

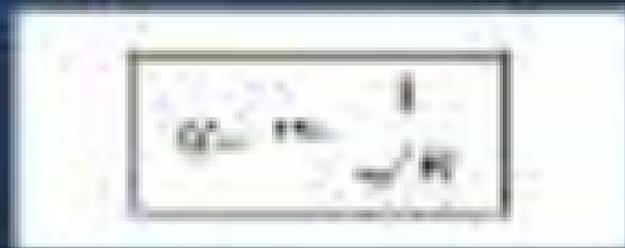
$$\psi = \frac{U_{\infty}}{2} r^2 \sin^2 \theta \left[1 - \frac{3}{2} \frac{r}{R} + \frac{1}{2} \left(\frac{r}{R} \right)^3 \right] + \frac{U_{\infty}}{4} R^2 \sin^2 \theta \left[\frac{r}{R} - \left(\frac{r}{R} \right)^3 \right]$$

- Then on the velocity profile we can see in the streamlines. Using D'Alambert's equation for edge of the BC, and approximating velocity profiles in the region by Mooney's theory, it can be shown that:

$$\frac{dv_x}{dx} = \frac{1}{R} \sqrt{\frac{\mu}{\rho}} \left[\frac{v_x}{R} - \frac{v_x^2}{R} \right]$$

- R The radius of curvature

- Shows the wall level inside in the signalized region.



- Negative-pole floating works precisely with the system level of the sensor module.
 Issues: **to reduce the heating, increase the noise ceiling**

BLUNT BODIES IN HYPERSONIC FLOWS

- This shows why hypersonic applications typically have blunt nose/blunt leading edge



APPENDIX

NON-DIMENSIONALIZED NS EQUATIONS, STEADY, 2D

$$\begin{aligned}
 & \text{Continuity: } \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \\
 & \text{Momentum (x): } \rho \left(u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} \right) = -\frac{\partial p}{\partial x} + \mu \left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} \right) \\
 & \text{Momentum (y): } \rho \left(u \frac{\partial v}{\partial x} + v \frac{\partial v}{\partial y} \right) = -\frac{\partial p}{\partial y} + \mu \left(\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2} \right) \\
 & \text{Energy: } \rho c_p \left(u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} \right) = k \left(\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} \right) + \dot{q}''' \\
 & \text{Boundary Conditions: } \\
 & \text{At } x=0: u=0, v=0, T=T_w \\
 & \text{At } x=L: u=U_{\infty}, v=0, T=T_{\infty} \\
 & \text{At } y=0: u=0, v=0, T=T_w \\
 & \text{At } y=H: u=0, v=0, T=T_w
 \end{aligned}$$

CONTINUITY EQUATION

$$\frac{dM}{dt} = -\nabla \cdot (\rho \mathbf{v}) - \dot{m}$$

MOMENTUM EQUATIONS

1. **1D collisions**

2. **2D collisions**

3. **Impulse**

4. **Conservation of momentum**

5. **Conservation of energy**

6. **Conservation of angular momentum**

$$m_1 u_1 + m_2 u_2 = m_1 v_1 + m_2 v_2$$

$$m_1 u_1 + m_2 u_2 = m_1 v_1 + m_2 v_2$$

$$m_1 u_1 + m_2 u_2 = m_1 v_1 + m_2 v_2$$

$$m_1 u_1 + m_2 u_2 = m_1 v_1 + m_2 v_2$$

$$m_1 u_1 + m_2 u_2 = m_1 v_1 + m_2 v_2$$

$$m_1 u_1 + m_2 u_2 = m_1 v_1 + m_2 v_2$$

$$m_1 u_1 + m_2 u_2 = m_1 v_1 + m_2 v_2$$

$$m_1 u_1 + m_2 u_2 = m_1 v_1 + m_2 v_2$$

$$m_1 u_1 + m_2 u_2 = m_1 v_1 + m_2 v_2$$

ENERGY EQUATIONS

$$\begin{aligned}
 \rho \frac{D\mathcal{E}}{Dt} + \nabla \cdot (\mathcal{E}\mathbf{v}) &= \rho \left(\frac{\partial \mathcal{E}}{\partial t} + \mathbf{v} \cdot \nabla \mathcal{E} \right) + \nabla \cdot (\mathcal{E}\mathbf{v}) = \frac{\partial}{\partial t} (\rho \mathcal{E}) + \nabla \cdot (\rho \mathcal{E}\mathbf{v}) \\
 &= \frac{\partial}{\partial t} (\rho \mathcal{E}) + \nabla \cdot (\rho \mathcal{E}\mathbf{v}) + \nabla \cdot (\rho \mathbf{v}\mathcal{E}) - \nabla \cdot (\rho \mathbf{v}\mathcal{E}) \\
 &= \frac{\partial}{\partial t} (\rho \mathcal{E}) + \nabla \cdot (\rho \mathcal{E}\mathbf{v}) + \nabla \cdot (\rho \mathbf{v}\mathcal{E}) - \nabla \cdot (\rho \mathbf{v}\mathcal{E})
 \end{aligned}$$

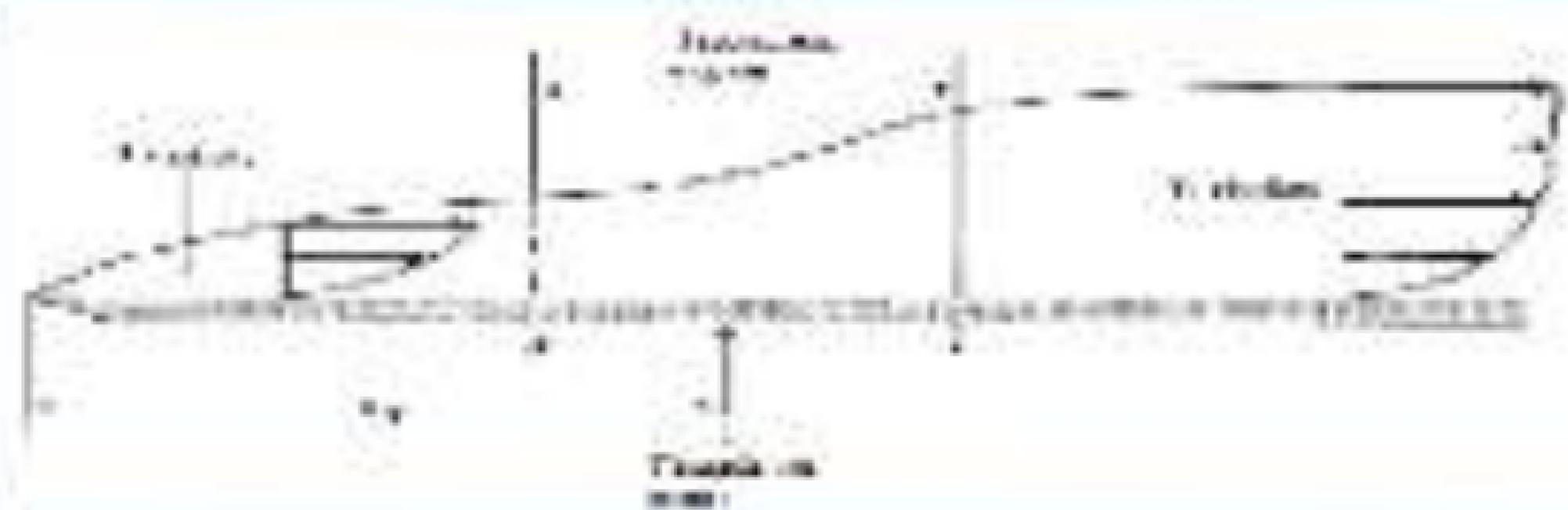
HYPERSONIC FLOW THEORY



Session 34: Hypersonic
TURBULENT flow-1
Transition

Dr. A. M. Srinivasan

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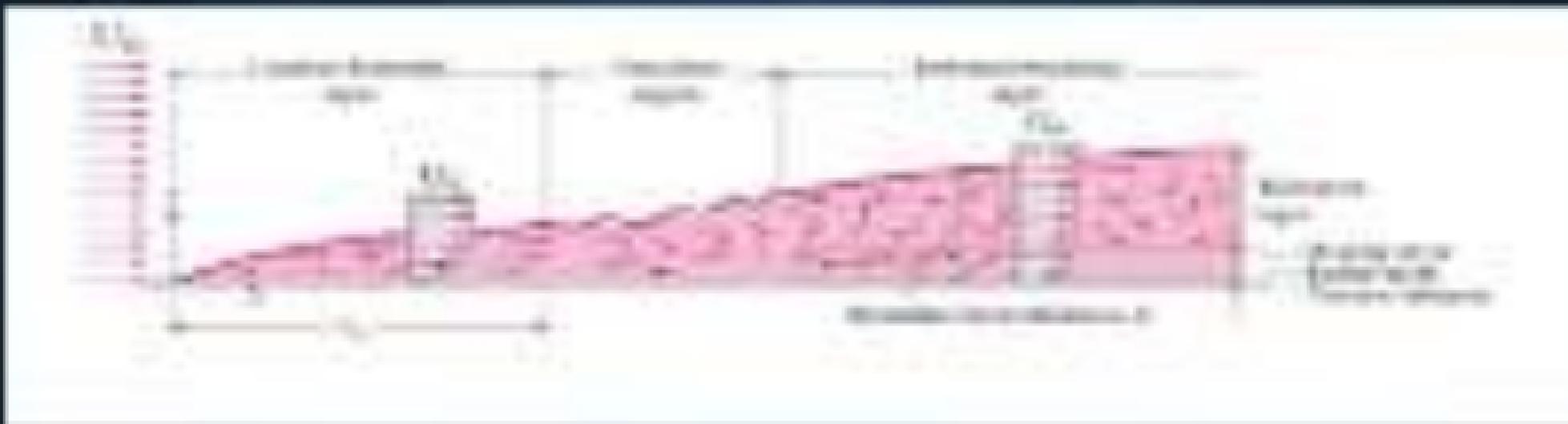


THE CONCEPT OF TRANSITIONAL REYNOLDS NUMBER

$$\text{Re} = \frac{\rho U D}{\mu}$$

- Transition is independent of the nature of data (laminar & turbulent) & the nature of the flow (steady or unsteady)
- Relation can be applied to the analysis of flow in pipes by generalising velocity





THE TRANSPORT PROCESSES

- In a laminar boundary layer any exchange of mass or momentum takes place only between adjacent layers of infinitesimal thicknesses and within the layer.
- Diffusion processes are the only means of transport.
- The velocity boundary layer is characterized by a constant vertical lag of δ .
- The analogy is not a one-to-one relation.
- Profiles of fluid may be same, boundary curves are different.
- **There is an exchange of mass, momentum and energy at a finite lagged scale compared to a laminar boundary layer.**

TYPICAL VELOCITY PROFILES

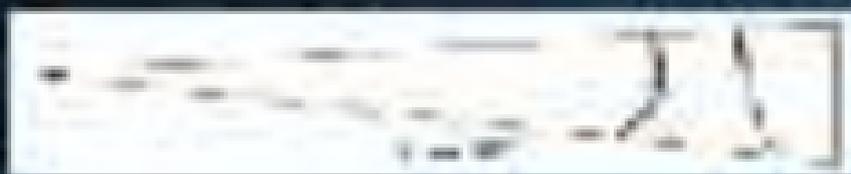


TRANSITION TO TURBULENCE

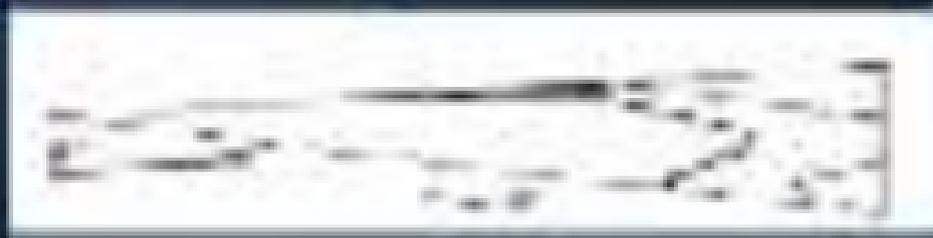
$$\sigma_{\theta} = \frac{1.45 \times 10^{-4}}{Pr}$$

$$\sigma_{\theta} = \sigma_T \left(N_{Gr} = \frac{g \beta \Delta T L^3}{\nu \alpha} \right) \left(N_{Pr} = \frac{\nu}{\alpha} \right) \left(N_{Gr} N_{Pr} = \frac{g \beta \Delta T L^3}{\nu^2} = Pr Gr = \frac{g \beta \Delta T L^3}{\nu^2} = Gr_{\theta} \right)$$

EFFECT OF ANGLE OF ATTACK



————— C_L at $\alpha = 0^\circ$
----- C_D at $\alpha = 0^\circ$



With increasing angle of attack, the flow becomes increasingly three-dimensional.

The portion of flow that would be lost due to induced drag is now lost due to drag due to induced lift, or more simply, to drag due to the induced flow circulation on the body.

EFFECT OF BLUNTNES

- Degree of the bluntness (angle) — the steeper the angle, the greater the amount of force that is transferred through the spinal structure and spine (less compressed in the lower spine)
- Small nerve rootlets: by adding a slight curve to the spine, compression has been delayed or reduced but this does not mean that the nerve is:
 - The maximum compressive loading is increased by such a movement
- In general, the degree of the bluntness is considered as a variable parameter

EFFECT OF ILLUMINANCE



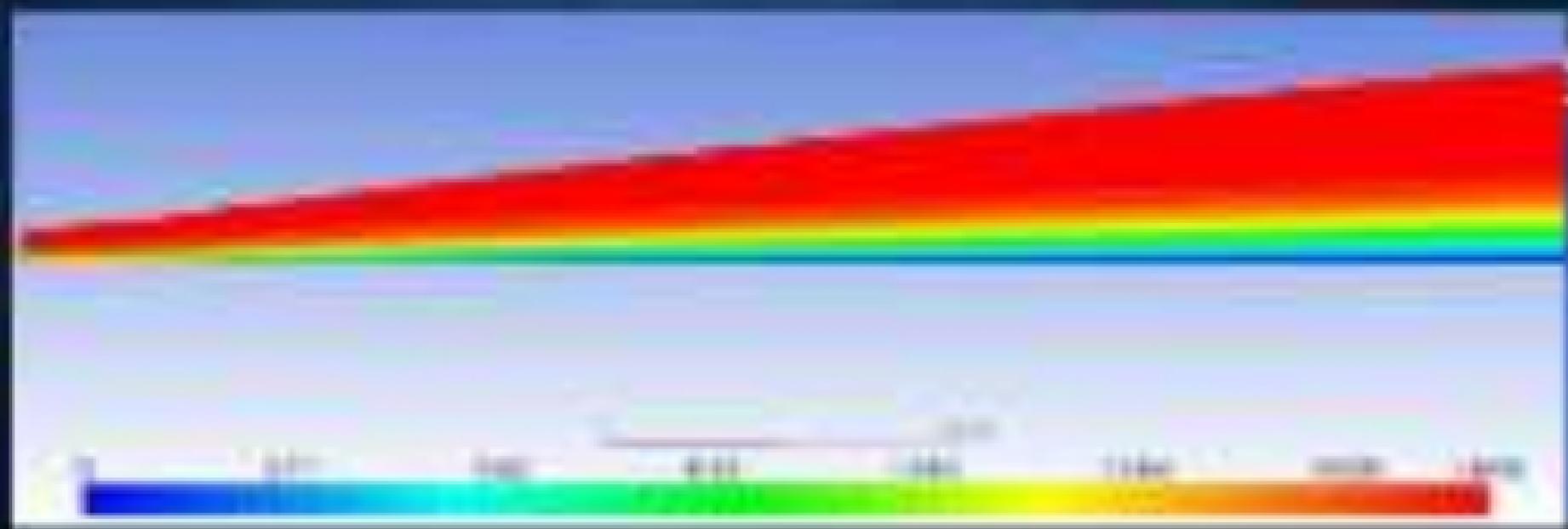
For low values of illuminance, the rate of photosynthesis increases at higher than linear rate.
(The upward bend of the line appears for higher values of illuminance (light more abundant))



EFFECT OF WALL TEMPERATURE

- The maximum velocity of the wall ($T_w < T_{\infty}$) due to convection is delayed - as the IL is given a finite amount of time to respond.
- A process of heat at higher rates of conduction **Temperature has a significant impact on the**

conduction rate and the convection rate.



HYPERSONIC FLOW THEORY



Session 36-37: Hypersonic
TURBULENT flow-2:
Effects of turbulence

Dr. A. M. Srinivasan

2020/9

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TURBULENCE VISCOSITY

- A semi-empirical approach to represent the impact of turbulence on convective transfer
- The most common μ_t expression used in CFD is:

$$\mu_t = \rho C_\mu \frac{k^2}{\epsilon} = \rho C_\mu \frac{k}{\epsilon} \left[\nu \left(\frac{\partial u}{\partial y} \right)^2 \right]$$

- Turbulence viscosity

TURBULENT PRANDTL NUMBER & TURBULENT THERMAL CONDUCTIVITY

- Turbulent Prandtl number analogous to Prandtl number in laminar flow
- Laminar thermal conductivity:

$$k = \frac{\mu c_p}{Pr}$$

- Turbulent Thermal Conductivity

$$k_t = \frac{\mu_t c_p}{Pr_t}$$

CRITICAL INFLUENCE ON WALL FRICTION & HEAT TRANSFER

Turbulence is characterized by **large-scale** fluctuations in the **boundary layer**. These result in higher rates of energy transfer



The net result is a **uniform** temperature profile **across** the flow with **the rate of heat transfer**

CP WITH MACH NUMBER



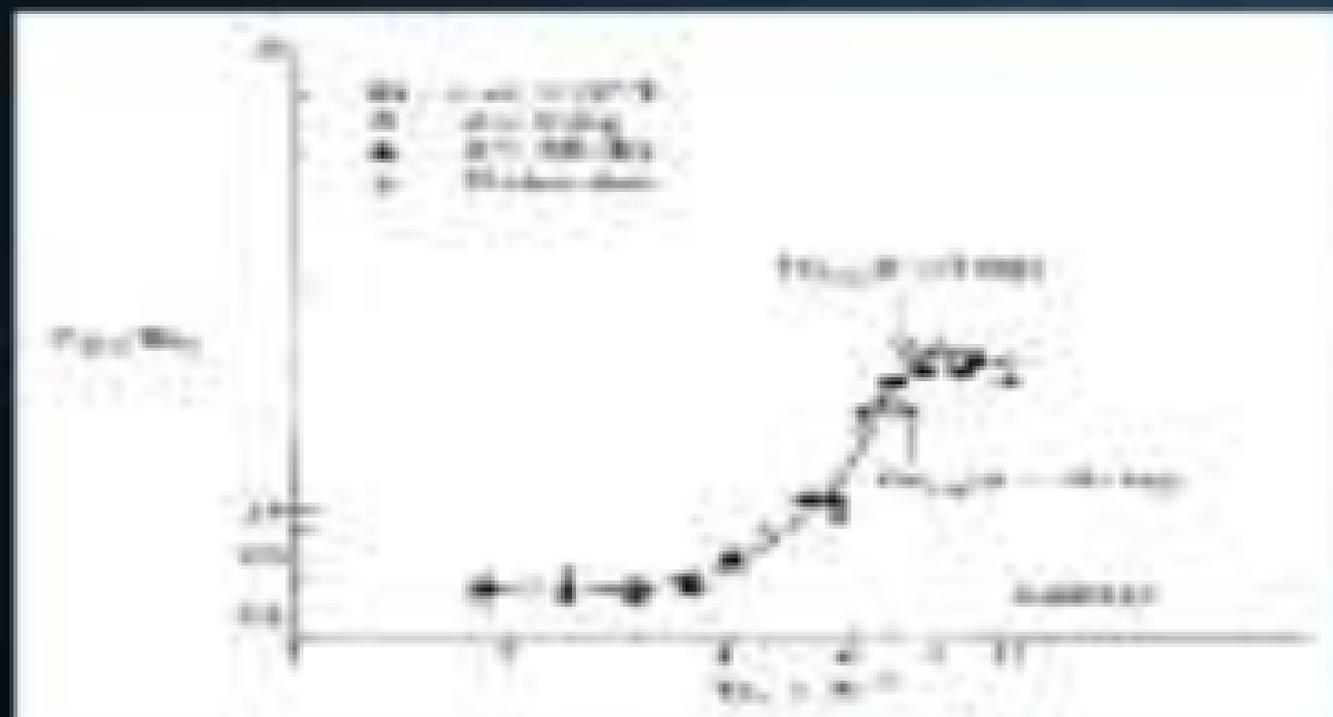
When you plot the **CP** with **MACH** you always notice **sharp increase**.

EFFECT ON HEAT TRANSFER



The above relationship between Nu and Re is used to determine the heat transfer coefficient h for a given flow condition.

For a given flow condition, the heat transfer coefficient h is determined by the relationship between Nu and Re .



Graph for a group of myrmecophilous ants [1]

• The relationship between the number of eggs and the number of surviving offspring is not linear. The relationship is a curve that rises steeply and then levels off.

• It shows that the number of surviving offspring is not directly proportional to the number of eggs.

HYPERSONIC FLOW THEORY



Session 38-39: Viscous
Interaction in Hypersonic
Flow

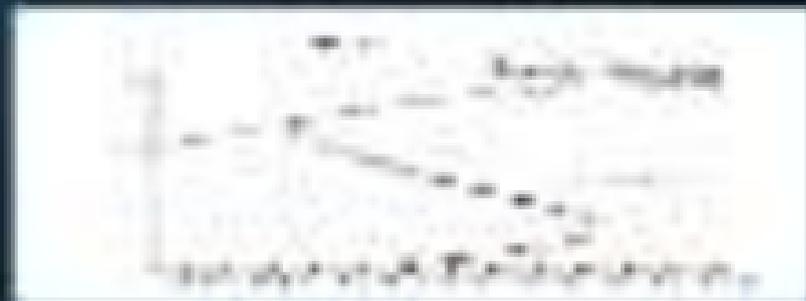
Dr. A. M. Sankaran

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IMPACT OF VISCOSITY ON FLOW/FLUID PROPERTIES

- As hydrocarbon flow developments proceed the highly (often viscous) part of the reservoir energy gets dissipated as thermal energy, decreasing the temperature significantly
- As we know Density drops with temperature
 - In this case it directly implies the need for increased thickness to offset the lower flow
- Viscosity of gases increases with temperature
- Self-heating occurs for limited thicknesses:

$$Q = \frac{kA \Delta T}{L}$$



$$A_{\text{eff}} = \frac{A}{\sqrt{1 + \left(\frac{f}{f_c}\right)^2}}$$

$$A_{\text{eff}} = \frac{A}{\sqrt{1 + \left(\frac{f}{f_c}\right)^2}} = \frac{A}{\sqrt{1 + \left(\frac{f}{f_c}\right)^2}} = \frac{A}{\sqrt{1 + \left(\frac{f}{f_c}\right)^2}}$$

→ Deriving more precise equivalent circuits for $A_{\text{eff}}(f)$

$$\frac{A_{\text{eff}}}{A} = \frac{1}{\sqrt{1 + \left(\frac{f}{f_c}\right)^2}}$$

- Remaining 2 terms depend on μ of velocity and temperature

$$\frac{\mu_0}{\mu_1} = \frac{T_0}{T_1}$$

- Hence:

$$\mu = \frac{\mu_0}{T_0} \left(\frac{T_0}{T_1} \right)$$

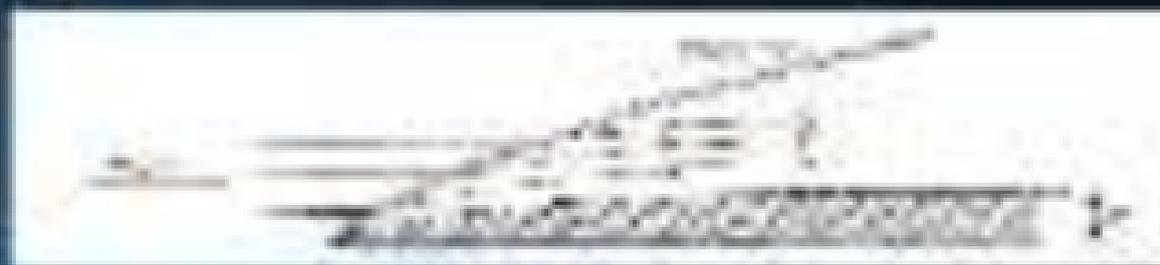
- Remaining complete recovery of temperature system (1) = (1)

- This leads to:

$$\frac{\mu}{\mu_0} = \frac{aT^2}{bT_0}$$

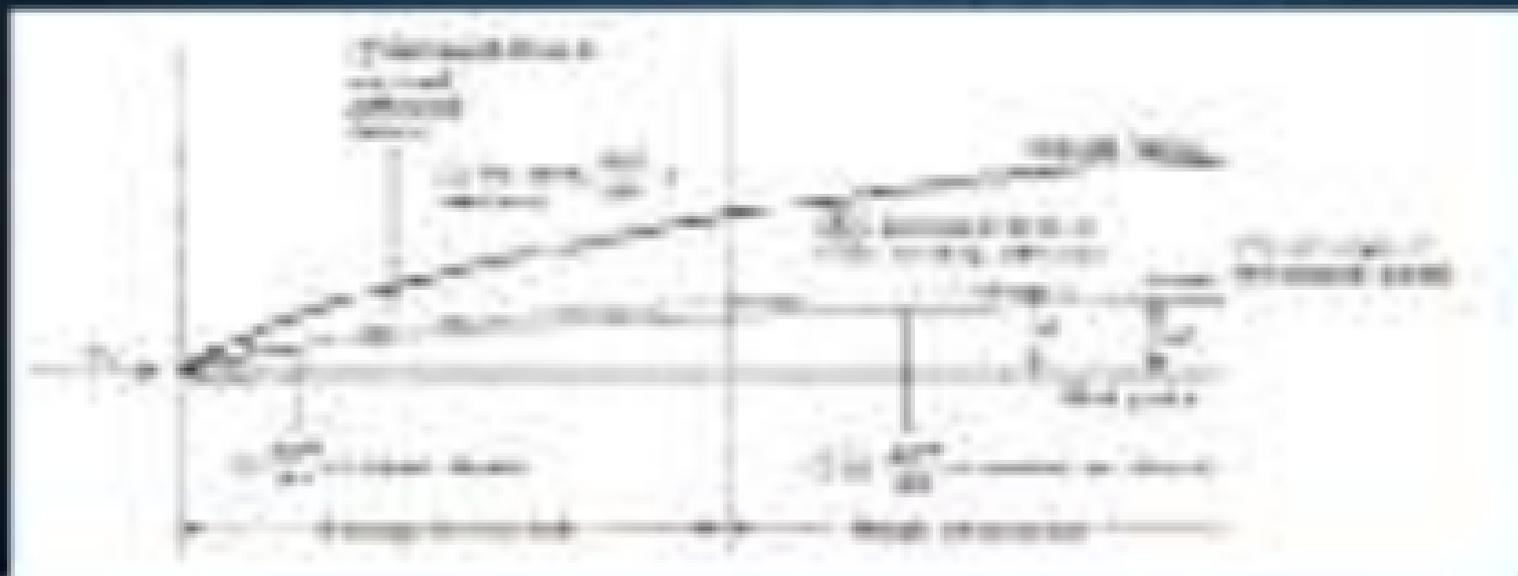
$$\frac{1}{\mu_0} = \frac{1}{\mu} = \frac{bT_0}{aT^2} = 1 + \frac{T_0 - T}{T}$$

- Mylonitic boundary layers can be orders of magnitude thicker than low-angle normal boundary layers at the same dip angle, whether



- The thick hypersonic boundary layer displaces the outer inviscid flow
 - Hence the inviscid streamlines are displaced changing the pattern of the inviscid flow
 - The hypersonic viscous flow with a thick boundary layer, the inviscid streamlines are displaced upward, resulting in shock waves at the leading edge
- **This is the source of the viscous interaction**

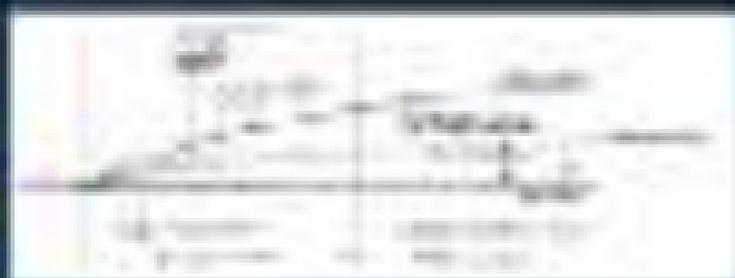
THE REGIONS OF VISCOUS INTERACTION



WEAK INTERACTION

OR

STRONG INTERACTION



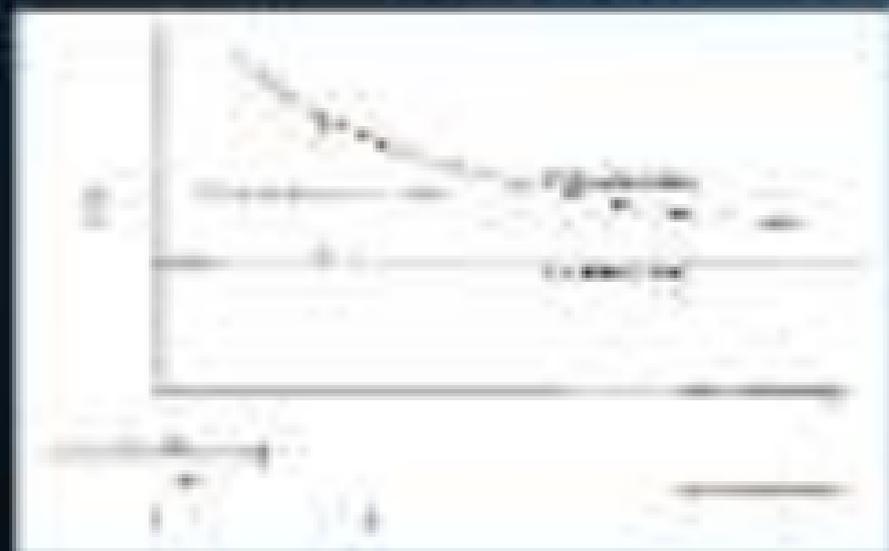
- Rapidly growing EL increases with the increased wear amount.
- If the increased amount is significantly influenced, it will increase the impact to the elastic region, the EL has increased to an **STRONG** interaction.
- If the amount is not significantly influenced, the feedback effect on the EL will not be significant either. This is referred to as **WEAK** interaction.
- Both types of interaction lead to an increase in elastic pressure, which may be considered as a function of the wear-dependent parameters.

$$W = \frac{A}{B} \cdot M^C$$



$$W = \frac{A_1}{B_1} \cdot \frac{A_2}{B_2} \cdot M^C$$

THE IMPACT OF VISCOUS INTERACTION ON SURFACE STATIC PRESSURE



Impact of viscous interaction



Impact of viscous interaction on surface static pressure

SOME RELATIONS FOR SURFACE PRESSURE

- The sea surface pressure, for the case of STEADY temperatures:

$$\frac{p}{p_0} = 0.9817 + 0.0183$$

- The sea surface pressure, for the case of WINDY temperatures:

$$\frac{p}{p_0} = 1 + 0.0171W + 0.0005W^2$$

- The wind wall, where $T_{in} = T_{out}$, the wind temperature is:

$$\frac{p}{p_0} = 1 + 0.0125W$$

- The wind wall, where $T_{in} = T_{out}$, the wind temperature is:

$$\frac{p}{p_0} = 1 + 0.0125W$$

HYPERSONIC FLOW THEORY



Session 10: High
Temperature Effects in
Hypersonic Flow

Dr. A. M. Srinivasan

2011

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THE KEY ISSUE: **KIC TO INTERNAL ENERGY**

- States temperature, T , degrees of freedom
- **Low temperature approximation** (valid for $T \ll \theta_{rot}$)
uniform energy distribution
 - One $\frac{1}{2}k_B T$ per degree of freedom
 - And one $k_B T$ per θ_{rot}
- **The whole molecule is expected to rotate**

PERFECT GAS ASSUMPTION

- State that the relation $\rho = \frac{p}{RT}$ holds as perfect gas assumption.
- Use the temperature 11000M to be found on the slide.
- Apollo 11 landing had a Mach number of 0.8 at 75 km, perfect gas assumption temperature = 223 K.
 - If we use the perfect gas relation for the air temperature is $T_a = 223,158 K$
 - Actual observed temperature is the slow one reported with = 11,000 K
- Why the difference?

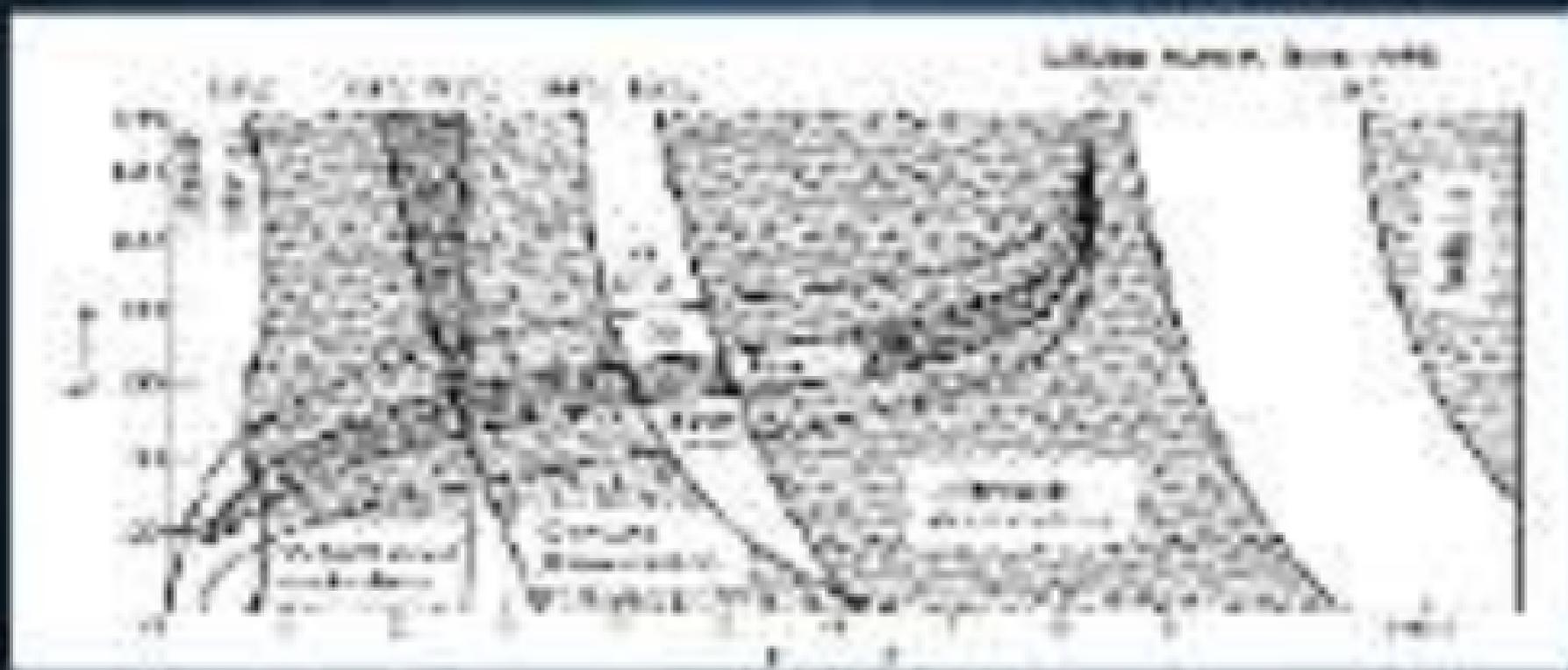
THE VALIDITY OF PERFECT GAS ASSUMPTION

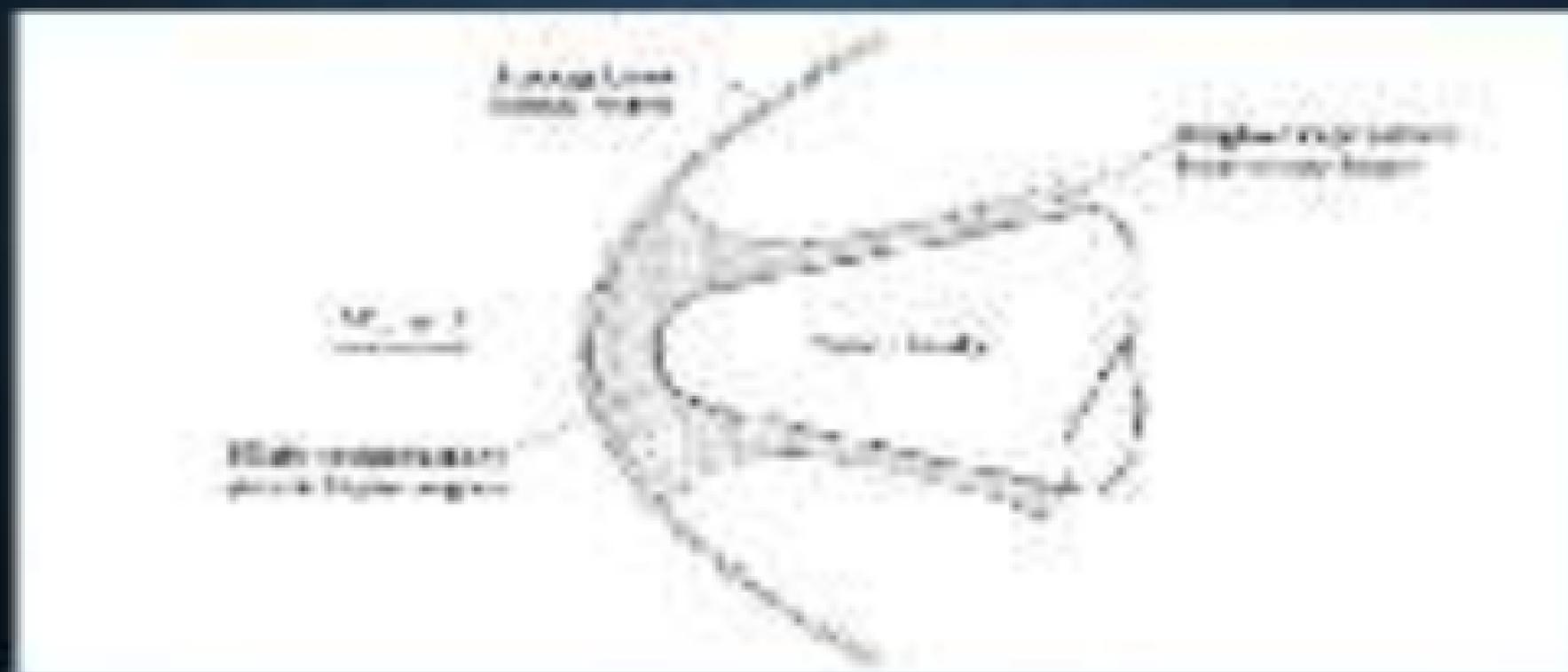
- Neglects intermolecular forces
- Applies under:
 - Conditions where pressure is not too high - gas molecules no longer
 - Temperatures are sufficiently high ($> 300\text{ K}$)
 - When the gas is not close to condensation
- Under many conditions the molecules are separated and hence the perfect gas assumption is validly applied

THE NEED TO INCORPORATE HIGH TEMPERATURE EFFECTS

- 1. How do differences in adaptation requirements manifest at the sports level about **altitude**?
- In order to have any meaningful estimates, the resulting chemical changes need to be accounted for.
- What are they?
 - 1. **Energy stores**
 - 2. **The anaerobic glycolytic pathway is upregulated**
 - 3. **The aerobic pathway is downregulated**
 - 4. **Energy stores are NOT increased**

VELOCITY-ALTITUDE MAP: FLIGHT
REGIONS ASSOCIATED WITH VARIOUS
"CHEMICAL" EFFECTS IN AIR





$M_0 \ll 1$



REACTING FLOWS

→ In a steady process, etc. at high temperature and low pressure (flame) the following

→ The CO_2 dissociation of CO_2 as $-\text{CO}_2 \rightleftharpoons \text{CO}$

→ This is a typical chemical equilibrium

Chemical Equilibrium → In a process, chemical reactions occur, and the species concentrations vary with time. At equilibrium, the chemical reactions have reached a state where the concentrations of the species involved do not change. The equilibrium constant, K , is a function of temperature.

→ When equilibrium

→ For reactions where the gas is not in equilibrium

→ The gas is not in equilibrium with the solid

REACTION TIME

Reaction time = time taken (milliseconds) taken for detection of stimulus, time taken for reaction to occur in very high (infinitesimal)

Stimulus-response = time between stimulus release and response, as voluntary

1. **Reaction time** = the minimum time between point of contact with stimulus and response, this involves how strongly stimulus was processed, the time between recognition and response for stimulus

2. **Response time** = time between recognition of stimulus and response, this involves how long it takes to move from stimulus to response, this involves how long it takes to respond

3. **Reaction time** = time between 1-2

4. **Reaction time** = time between stimulus and response, it shows not only response to the stimulus but also the time taken to process and to detect the stimulus

5. **Reaction time** = time between stimulus and response, it shows not only response to the stimulus but also the time taken to process and to detect the stimulus

LOW DENSITY FLOWS / **RARIFIED FLOWS**

- For very high speeds, or low density, vehicle's length is compared to a high characteristic distance, the mean free path, which is like a relaxation distance that it takes for the car to be able to follow a disturbance, collisions with air molecules are too large, there is characteristic length of the vehicle.
- The air density can be very low, so that only a few molecules impinge the surface per unit area and time, these collisions are infrequent, the surface may not be able to follow the incoming disturbance. This is the opposite of [freezing flow](#).
- The similarity parameter that governs a flow, different regimes in the flow are marked, defined as $Kn = \lambda/L$ where λ is a characteristic distance of the body & L is the mean free path for air.
 - 1. $Kn < 0.01$ means the molecules don't interact, it's like a solid wall, no stress, no friction
 - 2. $Kn > 0.1$ means the gas is too rare, and the interaction with the body is weaker, it's not possible anymore to measure pressure, temperature, density, etc.

- The relative of iron molecules from large metal a ratio as $E_{Fe} = 0.3$ and instead use for the level of SO increasing altitude
 - Hence, the distribution must be a normally distributed value $U(0,1) = \text{Exp}(1,0)$
- A hypothesis includes assuming that atmosphere from space and minerals that full range of lines from charged particles, forms to an altitude level which the SO molecules are highly relative to the level



HYPersonic FLOW THEORY



Sessions: 42-43: Shock
Interactions

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SHOCK-SHOCK INTERSECTION

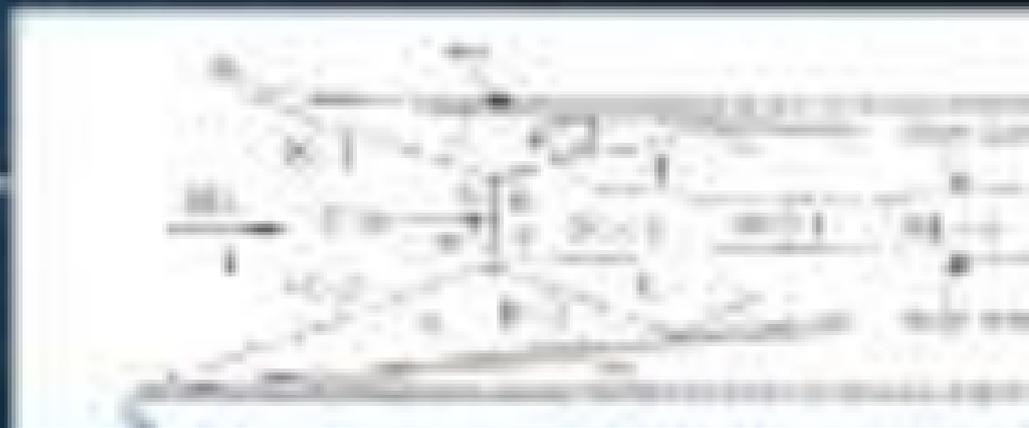
- The patterns produced when two shocks intersect, or interfere are classified into what are now commonly acknowledged as **SIX** types:
 - Various may exist in some circumstances.

TYPE I INTERSECTION

- Two bundles of opposite families intersect at a p
- Bundle C_1 : Theorem: jumps from p_1 to p_2
- Bundle C_2 : Theorem: jumps from p_1 to p_2
- **Lemma 1.1.1**: **Characterization of relative separation**
The formation of \mathcal{H} never shows C_1 & C_2
 - So that the line l is a common direction
 - consequently
 - **Prop. 1.1.2**: **Two lines whose projections l_1 & l_2 are not parallel**



- If the operations of $E1$ & $E2$ are not dual compatible, the definition of CCT is not closed under the operations of CCT . From a wrong choice of CCT , by Lemma 4, that means $E1$ & $E2$
 - The user needs to minimize
 - Expressions 4 & 5 can be expressed



TYPE VI

Type VI *amblyopina* Octopoda with a weakly oblique front (the arms [1] and [2] cross in a right angle). They are both weak-oblique species from the same family.

The bow pattern is simpler than in the previous ones.

The *amblyopina* (1) and (2) arms are right-angled from each other (3) is formed: crossing of upper arms (4) is shown (5) with *amblyopina* (6) (7).



Arms [1] cross [2] at a right angle. [3] is formed: crossing of upper arms [4] is shown [5] with *amblyopina* [6] [7].

HYPERSONIC FLOW THEORY



COURSE CONCLUSION

Dr. J. D. Anderson

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Thank you !!