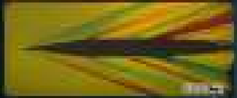


A MOOC Course on Gas Dynamics

COMPRESSIBLE FLUID FLOW: AN INTRODUCTION TO THE THEORY AND THE APPLICATIONS

Introduction to the Course

Faculty Introduction

- Srikrishnan A.R.
- With Amrita: 2012 June-
- Prior to that: ANSYS Inc. ~16 years
  - Director, Development @ ANSYS India, Pune
  - 1996-2012
- PhD. at IIT Madras, 1996
- Current Research Areas: Propulsion, High speed Flows, Heat Transfer, Combustion, ...

OVERVIEW

- Course Objective & Outcomes
- Syllabus, Text & References
- Sessions & Mode of Delivery
- Evaluation

COURSE OBJECTIVE

Introduce students to the basics of compressibility of flow and the related phenomena. Enable them to analyse subsonic and supersonic flows with simplified analytical models. Provide an introduction to the applications of compressible flow analysis in aerospace engineering.

OVERVIEW OF COURSE CONTENT

- Effects of Compressibility
- Shocks & Expansion Fans
- Compressible flow with heat transfer & friction
- Flows through nozzles

Text Book(s)

Reference

SYLLABUS

Unit 1

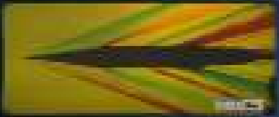
[illegible]

Unit 2

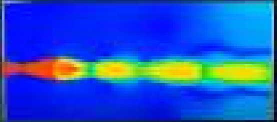
[illegible]

Unit 3

[illegible]

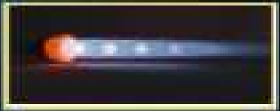


AMRITA



WHY DO WE DISCUSS APPLICATIONS FIRST ?!

- Highlight the importance and the utility
- State the main element that differentiates the subject
- Why CFF/Gas Dynamics is a **CORE** course for Aerospace ?


Density changes significantly along the flow !

$\rho \neq$ Constant !

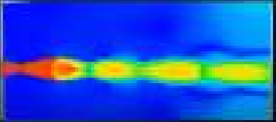
- Changes the flow physics
- The form of governing equations change dramatically
- Leads to certain phenomena that are *totally unexpected* as per Incompressible flow analysis
- These demand an in-depth analysis of the related flow phenomena

THE COUPLING OF ENERGY & VELOCITY

- Energy: Thermodynamics
- Velocity: Fluid Dynamics
- We need both
- The physics is coupled & hence the **Governing Equations** are coupled

INCOMPRESSIBLE FLOW CAN'T EXPLAIN MOST OF WHAT YOU SEE IN THIS VIDEO

- https://www.youtube.com/watch?v=[illegible]



23AES201 COMPRESSIBLE FLUID FLOW

The Governing Equations

$\rho \neq$ Constant !

- Changes the flow physics
- The form of governing equations change dramatically
- Leads to certain phenomena that are *totally unexpected* as per incompressible flow analysis
- These demand an in-depth analysis of the related flow phenomena

THE COUPLING OF ENERGY & VELOCITY

- Energy: Thermodynamics
- Velocity: Fluid Dynamics
- We need both
- The physics is coupled & hence the **Governing Equations** are Coupled

GOVERNING EQUATIONS

- The governing equations include the following conservation laws of physics:
  - Conservation of mass.
  - Newton's second law: the change of momentum equals the sum of forces on a fluid particle.
  - First law of thermodynamics (conservation of energy): rate of change of energy equals the sum of rate of heat addition to and work done on fluid particle.
- The fluid is treated as a continuum. For length scales of, say, 1μm and larger, the molecular structure and motions may be ignored.

WHY THE GENERAL FORM ?

- What form of the equations are we considering now ?
  - The most general form as applicable to unsteady viscous flows
- Can we handle the complete, general form of equations in the analysis in this course ?
  - NO !
- Then WHY do we still consider them ?
  - In order to understand the applicability
  - In order to appreciate the complexity & realize the role of assumptions
  - In order to gain insight to the analytical approach

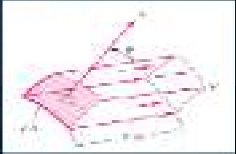
CONTINUITY EQUATION

- *Mass is conserved*
  - In a given control volume the influx and the efflux of the mass will be in such a way that the net flux will be equal to the rate of change within the volume

CONTROL VOLUME

FLOW RATE ACROSS THE SURFACE

- Volume flow rate $(\mathbf{V}.\mathbf{n})\, ds = \mathbf{V}.\mathbf{ds}$
- Mass flow rate $= \rho\, \mathbf{V}.\mathbf{ds}$



INTEGRAL FORM OF CONTINUITY EQUATION

- The net mass flow into the control volume through the entire control surface S:

$$-\oiint_S \rho \mathbf{V} \cdot d\mathbf{S}$$

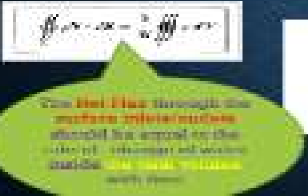
- Rate of change of mass inside the control volume:

$$\frac{\partial}{\partial t}\iiint_V \rho \, dV$$

- Applying the Continuity Principle:

$$\oiint_S \rho \mathbf{V} \cdot d\mathbf{S} = -\frac{\partial}{\partial t}\iiint_V \rho \, dV$$

VISUALIZE CONTINUITY EQUATION…



INTEGRAL TO DIFFERENTIAL FORM

- Rewriting the equation:

$$\oiint_S \rho \mathbf{V} \cdot d\mathbf{S} = -\iiint_V \frac{\partial \rho}{\partial t}\, dV$$

- Recall Divergence theorem:

$$\oiint_S \mathbf{A} \cdot d\mathbf{S} = \iiint_V (\nabla \cdot \mathbf{A})\, dV$$

- Combining:

$$\iiint_V \left[\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \mathbf{V})\right] dV = 0$$

DIFFERENTIAL FORM

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \mathbf{V}) = 0$$

MOMENTUM EQUATION

- Principle: The Time Rate of Change of Momentum of a Body Equals the Net Force Exerted on It

$$\frac{d}{dt}(m\mathbf{V}) = \mathbf{F}$$

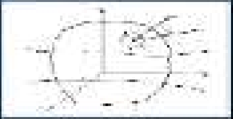
THE FORCES ?

- Body forces: Acting on the volume
  - Ex: Gravitational forces
- Surface forces: Acting on the surfaces
  - Pressure forces
  - Viscous forces (Shear force)
- INVISCID flow: The assumption of no viscosity – No viscous forces
- Hence the total forces are: Body forces + Pressure Forces

$$F = \iiint_V \rho f \, dV - \oiint_S p \, dS$$

- Total forces acting on the CV

$$F = \iiint_V \rho f \, dV - \oiint_S p \, dS$$



- Net momentum flux across:

$$F_x = \oiint_S (\rho V \cdot dS) V$$

TRANSIENT TERM

- Momentum inside the CV: $\mathbf{P} = \iiint_{V} \rho \mathbf{V}\, d\mathcal{V}$

- Rate of change of momentum within the CV due to unsteady flow effects:

$$\Lambda_t = \frac{\partial}{\partial t} \iiint_{V} \rho \mathbf{V}\, d\mathcal{V} = \iiint_{V} \frac{\partial(\rho \mathbf{V})}{\partial t}\, d\mathcal{V}$$

- Recall the fundamental relation: $\frac{d}{dt}(mV) = F$
- $\frac{d}{dt}(mV)$ [illegible]

$$\oiint (\rho \mathbf{V}\cdot d\mathbf{S})\mathbf{V} + \iiint_{\mathcal{V}} \frac{\partial(\rho \mathbf{V})}{\partial t}\, d\mathcal{V} = \iiint_{\mathcal{V}} \rho \mathbf{f}\, d\mathcal{V} - \oiint p\, d\mathbf{S}$$

DIFFERENTIAL FORM OF MOMENTUM EQUATION [illegible]

[illegible]

[illegible]

[illegible]

ENERGY EQUATION: THE COUPLING OF ENERGY & VELOCITY

- Energy: Thermodynamics
- Velocity: Fluid Dynamics
- We need both
- The physics is coupled & hence the **Governing Equations** are Coupled

ENERGY EQUATION, THERMODYNAMICS: WORK & HEAT

- "The science of Energy"
- Differentiates between WORK and HEAT

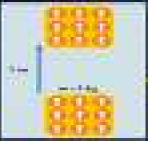
Heat Requires Temperature difference	Work Requires Force and Displacement

Heat is thermal energy transfer, while work is mechanical energy transfer across the system boundary

WORK & HEAT

- Both are energy interactions.
- Both are boundary phenomena.
- Both represent energy crossing the boundary of the system.
- They are not the property of the system.
- Both are path functions.

THE "GRADE" OF ENERGY
A POSSIBLE INTERPRETATION



"Ordered" Vs *"Disorderly"* movement

The explanation is not applicable to all processes & instances – but only if an approximation

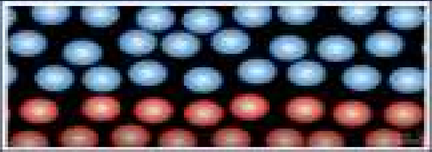
Entropy: a measure of the molecular disorder, or randomness, of a system

THERMODYNAMIC PROPERTIES

- Point function:
  - Value is based on the STATE of the system
  - NOT on the process that leads it to the state
- Path function
  - The value depends on the process
- Ex:

HEAT – A *MICROSCOPIC* VIEW

https://www.youtube.com/watch?v=[illegible]



- https://www.youtube.com/watch?v=[illegible]

INTERNAL ENERGY & ENTHALPY

- Internal Energy of an ideal gas: the sum of the kinetic energies of the particles in the gas.

INTERNAL ENERGY & ENTHALPY

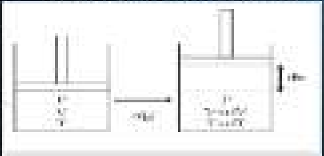
- Enthalpy: The sum of the internal energy of the system plus the product of the pressure of the gas in the system times the volume of the system.
  - *Enthalpy is the amount of energy that is transferred across a system boundary by a moving fluid.*
    - *This energy is composed of two parts: the internal energy of the fluid (u) and the flow work (pv) associated with pushing the mass of fluid across the system boundary*

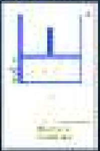
$$h = u + Pv$$

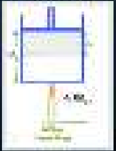
- $H_{sys} = E_{sys} + PV$

CP VS CV: WHY CP > CV

- Specific heat: Thermal energy added per unit mass, for 1 K rise in temperature
  - A material property
  - Depends on whether the process is at constant pressure or constant volume







ENERGY EQUATION

- Concept: Conservation of Energy
- Statement as applicable to flow systems:

Rate of heat added to the fluid inside the control volume from the Surroundings + Rate of work done on the fluid inside the control volume = Rate of change of the energy of the fluid as it flows through the control volume

FIRST LAW OF THERMODYNAMICS WITH REYNOLDS TRANSPORT THEOREM

$$\frac{dQ}{dt} - \frac{dW}{dt} = \frac{dE}{dt}$$

- Volumetric addition of thermal energy due to chemical reactions etc.

$$-\iiint q \rho \, dV$$

- [illegible]

[illegible]

- *NOTE: WE Are* ***not*** *considering* ***MECHANICAL work*** *or* ***shaft work*** *which is present in applications like turbine & compressor*

- Rate of change of energy as fluid flows through the CV = Transient change of energy within the CV due to unsteady effects + Net energy flux across the control surface

$$\frac{\partial}{\partial t}\iiint_V \rho\left(e+\frac{V^2}{2}\right)dV + \oiint_S \rho \mathbf{V}\cdot d\mathbf{S}\left(e+\frac{V^2}{2}\right)$$

ENERGY EQUATION: INVISCID FLOW, WITHOUT SHAFT WORK

$$\iiint_{\mathcal{V}} \dot{q}\rho \, d\mathcal{V} - \oiint_{S} p\mathbf{V} \cdot d\mathbf{S} + \iiint_{\mathcal{V}} \rho(\mathbf{f} \cdot \mathbf{V}) \, d\mathcal{V}$$

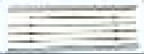
$$= \iiint_{\mathcal{V}} \frac{\partial}{\partial t}\left[\rho\left(e + \frac{V^2}{2}\right)\right] d\mathcal{V} + \oiint_{S} \rho\left(e + \frac{V^2}{2}\right) \mathbf{V} \cdot d\mathbf{S}$$

STEADY ONE-DIMENSIONAL FLOW

- Why ?
  - Analysis becomes easier
- Provides valuable information and insight about critical key phenomena that govern the flow
- Extremely useful as a design tools
- Most realistic practical systems can be explained/analyzed using the resulting equations

ONE-DIMENSIONALITY OF FLOW

- Flow in which parameters (velocity, pressure, density, viscosity and temperature) vary only in one direction and the flow is a function of only one co-ordinate Axis
  - The flow field is represented by streamlines which are straight and parallel.
- FLOW PROPERTIES ARE CONSTANT ACROSS THE SECTION



ONE-DIMENSIONAL CONTINUITY EQUATION

- Recall the general form:

$$-\oiint_S \rho \mathbf{V} \cdot d\mathbf{S} = \frac{\partial}{\partial t} \iiint_V \rho \, dV$$

- Steady assumption removes the volume integral (as there is NO variation with time)
- The surface integrals can be replaced by products of area (due to uniformity)
- Incorporating sign conventions for flow inlet & exit of the respective control surfaces:

$$-\rho_1 u_1 A + \rho_2 u_2 A = 0 \quad \Longrightarrow \quad \boxed{\rho_1 u_1 = \rho_2 u_2}$$

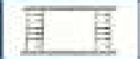
MOMENTUM EQUATION

$$\oiint_S (\rho \mathbf{V} \cdot d\mathbf{S})\mathbf{V} + \iiint_V \frac{\partial(\rho \mathbf{V})}{\partial t} dV = \iiint_V \rho \mathbf{f} \, dV - \oiint_S p \, d\mathbf{S}$$

- The steady flow without body forces

$$\oiint_S (\rho \mathbf{V} \cdot d\mathbf{S})\mathbf{V} = -\oiint_S p \, d\mathbf{S}$$

- The one dimensional assumption reduces the vector equation



ONE-DIMENSIONAL MOMENTUM EQUATION

STEADY, INVISCID FLOW WITHOUT BODY FORCE IN CONSTANT AREA PASSAGE

$$p_1 + \rho_1 u_1^2 = p_2 + \rho_2 u_2^2$$

THE ENERGY EQUATION

$$\iiint_V \dot{q}\rho \, dV - \oiint_S pV \cdot dS + \iiint_V \rho (f \cdot V) \, dV$$

$$-\iiint_V \frac{\partial}{\partial t}\left[\rho\left(e + \frac{V^2}{2}\right)\right] dV + \oiint_S \rho\left(e + \frac{V^2}{2}\right) V \cdot dS$$

- Steady flow without body forces:

$$\dot{Q} - \oiint pV \cdot dS = \oiint \rho\left(e + \frac{V^2}{2}\right)V \cdot dS$$

- Using 1-D assumption, the surface integrals get transformed as:

$$\dot{Q} + p_1u_1A_1 - p_2u_2A_2 = -\rho_1\left(e_1 + \frac{u_1^2}{2}\right)u_1A_1 + \rho_2\left(e_2 + \frac{u_2^2}{2}\right)u_2A_2$$

- Dividing by

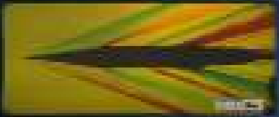
$$h_1 + \frac{u_1^2}{2} + q = h_2 + \frac{u_2^2}{2}$$

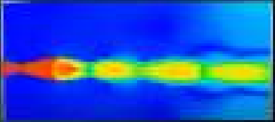
THE ONE-DIMENSIONAL EQUATIONS FOR STEADY, INVISCID COMPRESSIBLE FLOW WITHOUT BODY FORCES IN CONSTANT AREA DUCT

$$\rho_1 u_1 = \rho_2 u_2$$

$$p_1 + \rho_1 u_1^2 = p_2 + \rho_2 u_2^2$$

$$h_1 + \frac{u_1^2}{2} + q = h_2 + \frac{u_2^2}{2}$$





COMPRESSIBLE FLUID FLOW

Laws of Thermodynamics &
isentropic process

THE CONTEXT

- Reminder: Coupling between flow & energy in compressible (high velocity) flows
- Flow: Governed by the conservation equations
- Energy: Analysed using the tools of thermodynamics
- Conservation equations are considered already
- Next: A quick review of how the first & second law lead to certain important relations that govern compressible flow physics

THE FIRST & THE SECOND LAWS OF TD

- First law as applied to an open system: Considering incremental processes, the heat added to, work done on and the change in energy of a system are related as:

$$dq + dw = de$$

- Second law leads to the definition of entropy, as $ds = \frac{dq_{rev}}{T}$
- For real processes,

$$ds = \frac{dq}{T} + ds_{gen}$$

REVERSIBILITY & DISSIPATIVE PHENOMENA

- Reversible process–one in which no dissipative phenomena occur
  - Where the effects of viscosity, thermal conductivity, and mass diffusion are absent
- Viscosity & friction are REAL & are dissipative in nature
- Hence for all real processes

$$\Delta s_{gen} > 0$$

- In general:

$$ds \geq \frac{\delta q}{T}$$

- And for ADIABATIC processes: $\delta q = 0$ therefore, $ds \geq 0$

COMBINING THE FIRST & THE SECOND LAWS…

- First law: $du = \delta q - \delta w$
- For reversible process (where $\delta q_{rev} = T\,ds$) with no shaft work

$$T\,ds - p\,dv = du$$

$$T\,ds = du + p\,dv$$

$$dh = du + p\,dv + v\,dp$$

$$T\,ds = dh - v\,dp$$

FOR PERFECT GAS..

- The implication of perfect gas assumption: Cp is constant. Hence: $dh = C_p dT$
- This leads to $ds = c_p \frac{dT}{T} - \frac{v\,dp}{T}$
- Using $pv = RT$

  $ds = c_p \frac{dT}{T} - R \frac{dp}{p}$
- Integrating: $s_2 - s_1 = \int_1^2 c_p \frac{dT}{T} - R \ln \frac{p_2}{p_1}$ $s_2 - s_1 = c_p \ln \frac{T_2}{T_1} - R \ln \frac{p_2}{p_1}$

ISENTROPIC PROCESSES

- Entropy remains Constant: ds = 0

$$0 = c_v \ln \frac{T_2}{T_1} + R \ln \frac{v_2}{v_1}$$

$$\ln \frac{T_2}{T_1} = -\frac{R}{c_v} \ln \frac{v_2}{v_1}$$

$$\frac{T_2}{T_1} = \left(\frac{v_1}{v_2}\right)^{R/c_v}$$

$$\frac{T_2}{T_1} = \left(\frac{P_2}{P_1}\right)^{(k-1)/k}$$

REVERSIBLE AND ADIABATIC

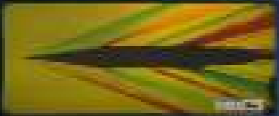
PHYSICAL RELEVANCE

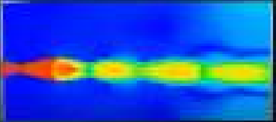
- What is a Reversible Adiabatic Process ?
- When is a process reversible ?
- When is a process adiabatic ?
- Why is it important in PRACTICAL APPLICATIONS ?
- Where will WE use these in this Course ?

VIDEO: ENTROPY !

- https://www.youtube.com/watch?v=YM-uykVfq_E

Order Order !!!

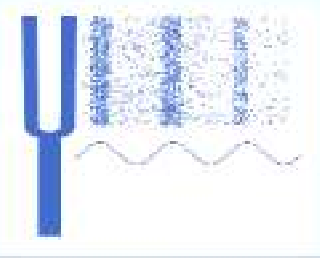


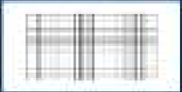


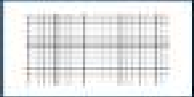
COMPRESSIBLE FLUID FLOW

Session-05

Compressibility & The Velocity of Sound







COMPRESSIBILITY

- **Compressibility** is a measure of the relative volume change of fluid or solid as a response to a pressure (or mean stress) change
  - the ability of molecules in a fluid to be compacted or compressed (made more dense) and their ability to bounce back to their original density

$$\tau = -\frac{1}{v}\frac{dv}{dp}$$

In terms of density

$$\tau = \frac{1}{\rho}\frac{d\rho}{dp}$$

  - *Note: v is volume here (NOT Velocity)*
  - The definition refers to the isentropic process of compression
  - Negative sign indicates the decrease in volume with increase in pressure
  - The inverse of compressibility is Bulk Modulus

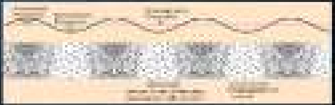
COMPRESSIBILITY: GASES VS LIQUIDS

Water	$5 \times 10^{-10}\ m^2/N$
Air	$10^{-5}\ m^2/N$

Implies the significant variation to density with pressure in gases

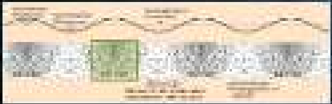
PROPAGATION OF SOUND

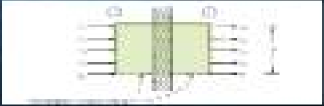
- Sound travels as **LONGITUDINAL** waves



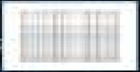
ONE-DIMENSIONAL PROPAGATION

- Variations across the control volume:





VARIATIONS ACROSS THE WAVE



- Consider 1-dimensional propagation of a pressure wave
- Reversible, adiabatic propagation
- The sound wave moves at a velocity a through the gas
- The passage of the wave leads to change in properties of the gas. Also induces a velocity dv in the gas
- Viewing from the reference frame of the wave, the gas moves with a velocity a and accelerates to a+da
  - Other gas properties also vary accordingly

- Assuming constant area, the continuity equation yields:
- Neglecting products of differential terms:
- Momentum equation gives:

- Substituting [illegible]
- & solving for a² [illegible]
- Since the whole process is considered isentropic

[illegible]

$a^2 = \left(\frac{\partial p}{\partial \rho}\right)_s$

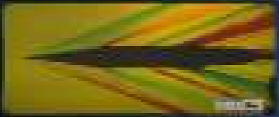
FOR ISENTROPIC FLOW OF A PERFECT GAS

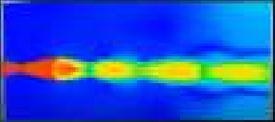
- For isentropic process: $\frac{p}{\rho^\gamma} = Constant, C$ → [illegible] = C*γρ^(γ-1)

$$a = \sqrt{\frac{\gamma p}{\rho}}$$

$$a = \sqrt{\gamma R T}$$

The Velocity of Sound in a Perfect Gas





COMPRESSIBLE FLUID FLOW

Session-05

Mach Number

COMPRESSIBILITY & VELOCITY OF SOUND

- Remember the definition of compressibility: $\tau = \frac{1}{\rho}\frac{d\rho}{dp}$
- Remember the expression that we derived for the velocity of sound:

$$a^2 = \left(\frac{\partial p}{\partial \rho}\right)_s$$

- Hence: $a^2 = \frac{1}{\rho\tau}$ *Note the impact of compressibility on the speed of sound through the medium*

MACH NUMBER

- The **MOST** important parameter in Compressible flow analysis
- Definition: Velocity of the fluid/Velocity of sound through the fluid

[illegible]

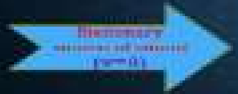
- The physical significance ? : *A measure of the relative magnitude of kinetic energy and the internal energy*

ERNST MACH

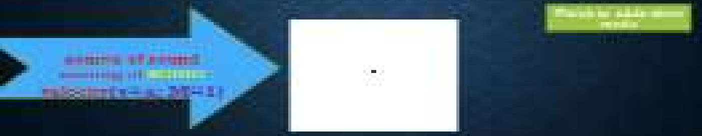
- Austrian Physicist, Philosopher
- Pioneering studies in compressible flows

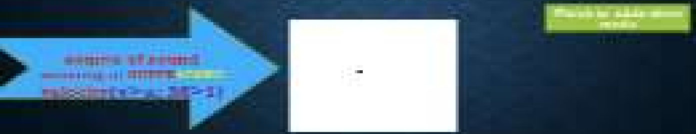
SUBSONIC SONIC SUPERSONIC

FLUID VELOCITY VS SOUND VELOCITY



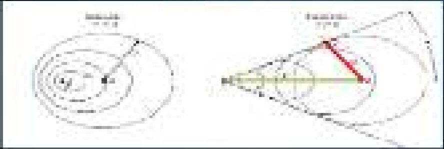
FLUID VELOCITY VS SOUND VELOCITY



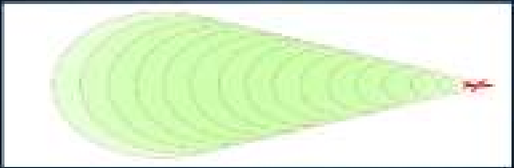


MACH CONE & MACH ANGLE

- Consider the reach of waves created by a body moving at supersonic velocity.



$$\mu = \sin^{-1}\left(\frac{1}{Ma}\right)$$

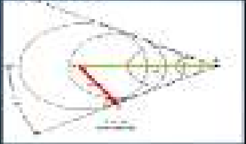


THE RANGE: CAN YOU HEAR THE NOISE ?

THE SONIC BARRIER & SONIC BOOM

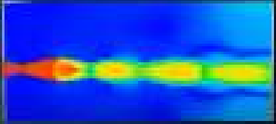
- https://www.youtube.com/watch?v=[illegible]
- https://www.youtube.com/watch?v=[illegible]

THE DISTANCE, THE ALTITUDE, THE MACH CONE & THE MACH NUMBER



NUMERICAL PROBLEM

AMRITA



COMPRESSIBLE FLUID FLOW

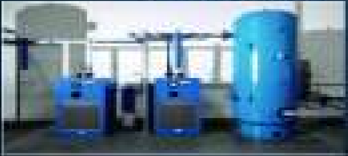
Sessions-01

Stagnation Properties &
Definition of COMPRESSIBLE FLOW

THE STAGNATION STATE

- Corresponds to the state when the flow is brought to zero velocity without dissipation of energy
  - Kinetic energy is completely converted into internal energy
- Stagnation properties are defined as those corresponding to the stagnation state of a fluid under the specified conditions
  - Like those of air in a storage tank

STAGNATION STATE & STORAGE CONDITIONS



STAGNATION PROPERTIES VS STATIC PROPERTIES

- Static pressure: Pressure (P) at a point in a flow where the velocity is V
- Stagnation pressure (Po): The pressure that would result **if** the flow is brought to rest in a reversible, adiabatic process
- Static & Stagnation properties will be SIGNIFICANTLY DIFFERENT in high speed flows

ENERGY EQUATION

- Steady, one-dimensional form in the absence of mechanical work & heat transfer:

$$h_1 + \frac{u_1^2}{2} = h_2 + \frac{u_2^2}{2}$$

- For perfect gas:

$$c_p T_1 + \frac{u_1^2}{2} = c_p T_2 + \frac{u_2^2}{2}$$

$$\frac{\gamma R T_1}{\gamma - 1} + \frac{u_1^2}{2} = \frac{\gamma R T_2}{\gamma - 1} + \frac{u_2^2}{2}$$

STAGNATION TEMPERATURE & MACH NUMBER

- If state 2 corresponds to stagnation (ie u2 = 0 in the above equation) → T2 is stagnation temperature

$$c_p T + \frac{u^2}{2} = c_p T_0$$

$$\frac{T_0}{T} = 1 + \frac{u^2}{2c_p T} = 1 + \frac{u^2}{2\gamma R T/(\gamma - 1)} = 1 + \frac{u^2}{2a^2/(\gamma - 1)}$$

$$\frac{T_0}{T} = 1 + \frac{\gamma - 1}{2} M^2$$

STAGNATION PRESSURE

- For an isentropic process of deceleration from u to 0 velocity:

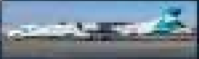
$$\frac{p_0}{p} = \left(\frac{\rho_0}{\rho}\right)^{\gamma} = \left(\frac{T_0}{T}\right)^{\gamma/(\gamma-1)}$$

- Hence:

$$\frac{p_0}{p} = \left(1 + \frac{\gamma-1}{2}M^2\right)^{\gamma/(\gamma-1)}$$

$$\frac{\rho_0}{\rho} = \left(1 + \frac{\gamma-1}{2}M^2\right)^{1/(\gamma-1)}$$

STATIC VS STAGNATION: HOW DO THE VALUES COMPARE ?



- A car moving at [illegible]km/hr
- A low speed passenger aircraft
- A Dreamliner (Boeing 787)
- A supersonic fighter – RAFALE

FEEL THE DIFFERENCE !

Calculations for Static Temperature T = 300 K & Static Pressure P = 1 atm

V, km/hr	V, m/s	[illegible]	Stagnation Temperature, K	Stagnation pressure, atm
80	22	[illegible]	300.2	1.008
180	50	[illegible]	300.4	1.005
370	100	[illegible]	[illegible]	1.107
1080	300	[illegible]	[illegible]	1.019
[illegible]	650	1.8722	510	[illegible]
[illegible]	[illegible]	[illegible]	800	[illegible]
[illegible]	2000	[illegible]	2291	[illegible]

THE DENSITY

- Recall

$$\frac{\rho_0}{\rho} = \left(1 + \frac{\gamma - 1}{2} M^2\right)^{1/(\gamma-1)}$$

"COMPRESSIBLE" FLOW: Conventionally categorized as M > 0.3

M	% Variation
0.0634	0.20
0.0864	0.37
0.3024	4.64
0.4808	10.96
0.8641	41.02
1.8722	277.37
3.0243	1246.45
5.7606	16016.80

NOTES ON COMPRESSIBLE FLOW

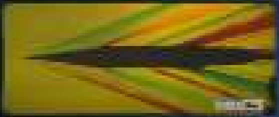
- The effects of compressibility gradually increase with Mach number
- Compressible flow at M>0.3 is conventionally followed for convenience
  - Nothing happens dramatically at M=0.3, its just a conventionally considered limit
- Note the difference in the implication of terminology: Subsonic Vs Incompressible

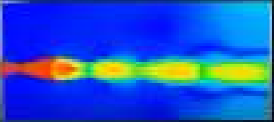
CHARACTERISTIC TEMPERATURE

- Consider a flow at some velocity V, pressure P, Temperature T etc.
- What if the flow is accelerated/decelerated, adiabatically to SONIC velocity ($V = a$)
  - The Mach number becomes unity ($M = 1$)
- The resulting static temperature is called characteristic temperature T^*
- The corresponding velocity of sound is a^*
- That is $a^* = \sqrt{\gamma R T^*}$
- A NEW Mach number is defined as $M^* = V/a^*$
- Note the difference between the definition of M (local Mach number) and M^* (characteristic Mach number)

CALCULATION OF M*: EXAMPLE

- Determine the Mach number (M) and the critical Mach number (M*) for the flow of air at a velocity of 850 m/s and static temperature = 300 K.
  - M = 850/SQRT(1.4*287*300) =
  - M* = 850/SQRT(1.4*287*T*)
  - T*/T = (1+(gama-1)*0.5*M^2)/(1+(gama-1)*0.5)
  - Calculate T* and then M* = V/SQRT(gama*R*T*)





COMPRESSIBLE FLUID FLOW

Sessions-01

Stagnation Properties &

Definition of COMPRESSIBLE FLOW

THE STAGNATION STATE

- Corresponds to the state when the flow is brought to zero velocity without dissipation of energy
  - Kinetic energy is completely converted into internal energy
- Stagnation properties are defined as those corresponding to the stagnation state of a fluid under the specified conditions
  - Like those of air in a storage tank

STAGNATION STATE & STORAGE CONDITIONS

STAGNATION PROPERTIES VS STATIC PROPERTIES

- Static pressure: Pressure (P) at a point in a flow where the velocity is V
- Stagnation pressure (Po): The pressure that would result **if** the flow is brought to rest in a reversible, adiabatic process
- Static & Stagnation properties will be SIGNIFICANTLY DIFFERENT in high speed flows

ENERGY EQUATION

- Steady, one-dimensional form in the absence of mechanical work & heat transfer:

$$h_1 + \frac{V_1^2}{2} = h_2 + \frac{V_2^2}{2}$$

- For perfect gas:

$$c_p T_1 + \frac{V_1^2}{2} = c_p T_2 + \frac{V_2^2}{2}$$

$$\frac{\gamma R T_1}{\gamma - 1} + \frac{V_1^2}{2} = \frac{\gamma R T_2}{\gamma - 1} + \frac{V_2^2}{2}$$

$$c_p T_1 + \frac{u_1^2}{2} = c_p T_2 + \frac{u_2^2}{2}$$

STAGNATION TEMPERATURE & MACH NUMBER

- If state 2 corresponds to stagnation (ie u2 = 0 in the above equation) → T2 is stagnation temperature

$$c_p T + \frac{u^2}{2} = c_p T_0$$

$$\frac{T_0}{T} = 1 + \frac{u^2}{2c_p T} = 1 + \frac{u^2}{2\gamma RT/(\gamma - 1)} = 1 + \frac{u^2}{2a^2/(\gamma - 1)}$$

$$\frac{T_0}{T} = 1 + \frac{\gamma - 1}{2} M^2$$

STAGNATION PRESSURE

- For an isentropic process of deceleration from u to 0 velocity:

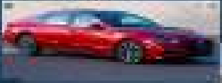
$$\frac{p_0}{p} = \left(\frac{\rho_0}{\rho}\right)^{\gamma} = \left(\frac{T_0}{T}\right)^{\gamma/(\gamma-1)}$$

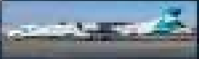
- Hence:

$$\frac{p_0}{p} = \left(1 + \frac{\gamma-1}{2}M^2\right)^{\gamma/(\gamma-1)}$$

$$\frac{\rho_0}{\rho} = \left(1 + \frac{\gamma-1}{2}M^2\right)^{1/(\gamma-1)}$$

STATIC VS STAGNATION: HOW DO THE VALUES COMPARE ?





- A car moving at 100km/hr
- A [illegible] aircraft [illegible]
- A [illegible] (Boeing 787)
- A supersonic fighter - RAFALE

FEEL THE DIFFERENCE !

Calculations for Static Temperature T = 300 K & Static Pressure P = 1 atm

V, km/hr	V, m/s	M	Stagnation Temperature, K	Stagnation pressure, atm
80	22	[illegible]	300.2	[illegible]
[illegible]	[illegible]	[illegible]	[illegible]	1.005
576	160	[illegible]	[illegible]	1.127
1080	300	[illegible]	[illegible]	1.020
[illegible]	650	1.8722	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
[illegible]	2000	[illegible]	[illegible]	[illegible]

THE DENSITY

- Recall

$$\frac{\rho_0}{\rho} = \left(1 + \frac{\gamma - 1}{2} M^2\right)^{1/(\gamma - 1)}$$

"COMPRESSIBLE" FLOW: Conventionally categorized as M > 0.3

M	% Variation
0.0634	0.20
0.0864	0.37
0.3024	4.64
0.4808	10.56
0.8641	41.02
1.8722	277.47
3.0243	1246.45
5.7606	16016.80

NOTES ON COMPRESSIBLE FLOW

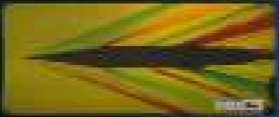
- The effects of compressibility gradually increases with Mach number
- Compressible flow at M>0.3 is conventionally followed for convenience
  - Nothing happens dramatically at Mach 0.3, its just a conventionally considered limit
- Note the difference in the implication of terminology: Subsonic Vs Incompressible

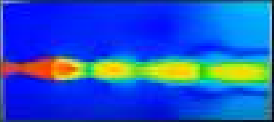
CHARACTERISTIC TEMPERATURE

- Consider a flow at some velocity V, pressure P, Temperature T etc.
- What if the flow is accelerated/decelerated, adiabatically to SONIC velocity ($V = a$).
  - The Mach number becomes unity ($M = 1$)
- The resulting static temperature is called characteristic temperature T^*
- The corresponding velocity of sound is a^*
- That is $a^* = \sqrt{\gamma R T^*}$
- A NEW Mach number is defined as $M^* = V/a^*$
- Note the difference between the definition of M (local Mach number) and M^* (characteristic Mach number)

CALCULATION OF M*: EXAMPLE

- Determine the Mach number (M) and the critical Mach number (M*) for the flow of air at a velocity of 850 m/s and static temperature = 300 K.
  - M = 850/SQRT(1.4*287*300) =
  - M* = 850/SQRT(1.4*287*T*)
  - T*/T = (1+(gama-1)*0.5*M^2)/(1+(gama-1)*0.5)
  - Calculate T* and then M* = V/SQRT(gama*R*T*)





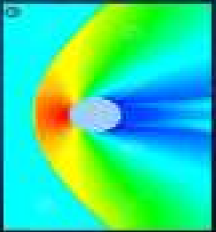
COMPRESSIBLE FLUID FLOW

Normal SHOCKs

WHAT HAPPENS WHEN THE FLOW PROCEEDS FASTER THAN SOUND (M>1) ?

CAN THIS HAPPEN IN SUPERSONIC FLOW ?

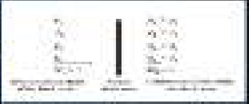
- The flow travels faster than the pressure disturbance (M >1)
- So the pressure waves cannot travel upstream
- The Consequence: **THE FLOW IS [illegible]ED !**



SUBSONIC VS SUPERSONIC

NORMAL SHOCK RELATIONS

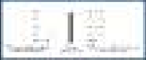
- What happens across the shock ?
- If we know properties upstream of the shock, can we calculate those downstream ?



- How do we design anticipating the impact of shock formation ?

VARIATIONS ACROSS..

- Typically in all applications the optimum properties are known



THE GOVERNING EQUATIONS ACROSS A NORMAL SHOCK

- Steady, 1 dimensional, adiabatic flow in constant area duct – for a perfect gas

$$\rho_1 u_1 = \rho_2 u_2$$

$$p_1 + \rho_1 u_1^2 = p_2 + \rho_2 u_2^2$$

$$h_1 + \frac{u_1^2}{2} = h_2 + \frac{u_2^2}{2}$$

$$p = \rho R T$$

$$h = c_p T$$

[illegible]

ENERGY EQUATION IN TERMS OF SOUND VELOCITY

PRANDTL'S RELATION

- Simplifying:

[illegible]

[illegible]

UPSTREAM TO DOWNSTREAM: THE MACH NUMBER RELATION

$$M_2^2 = \frac{1 + [(\gamma - 1)/2]M_1^2}{\gamma M_1^2 - (\gamma - 1)/2}$$

DENSITY CHANGE ACROSS THE NORMAL SHOCK

[illegible]

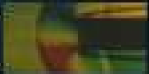
[illegible]

STATIC PRESSURE & STATIC TEMPERATURE

- An expression for the pressure ratio can be derived using the momentum equation:

$$p_2 - p_1 = \rho_1 u_1^2 - \rho_2 u_2^2$$

$$\frac{p_2}{p_1} = 1 + \frac{2\gamma}{\gamma + 1}\left(M_1^2 - 1\right)$$



Applying ideal gas law:

$$\frac{T_2}{T_1} = \left(\frac{p_2}{p_1}\right)\left(\frac{\rho_1}{\rho_2}\right)$$

$$\frac{T_2}{T_1} = \frac{h_2}{h_1} = \left[1 + \frac{2\gamma}{\gamma - 1}\left(M_1^2 - 1\right)\right]\left[\frac{2 + (\gamma - 1)M_1^2}{(\gamma + 1)M_1^2}\right]$$

LIMITING VALUES FOR PROPERTY VARIATION ACROSS NORMAL SHOCK AS M→∞

- By substituting $M_1 \to \infty$ in the respective relations

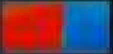
$$\lim_{M_1 \to \infty} M_2 = \sqrt{\frac{\gamma - 1}{2\gamma}} = 0.378$$

$$\lim_{M_1 \to \infty} \frac{\rho_2}{\rho_1} = \frac{\gamma + 1}{\gamma - 1} = 6$$

$$\lim_{M_1 \to \infty} \frac{p_2}{p_1} = \infty$$

$$\lim_{M_1 \to \infty} \frac{T_2}{T_1} = \infty$$

ACROSS THE NORMAL SHOCK:



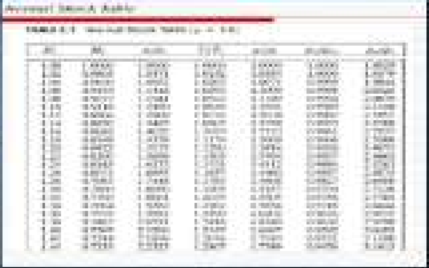
- [illegible] becomes subsonic
- The static pressure increases: COMPRESSION
- Density & Static temperature increase
- Stagnation pressure drops
- Stagnation Temperature REMAIN CONSTANT: Remember energy equation
- Entropy Increases

SOME PRACTICAL ISSUES IN AEROSPACE APPLICATIONS

TABULAR PRESENTATION OF NORMAL SHOCK RELATIONS

- All the important parameters and ratios are tabulated as functions of upstream Mach numbers
- These tabulations are just the solutions of the relations that we have derived
- Values are listed in different tables for [illegible]
- [illegible]
- The tabulated ratios can be used with the conditions/known upstream conditions to calculate the downstream properties

Normal Shock Table

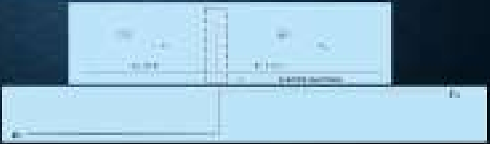


NORMAL SHOCK TABLES: ONLINE APPLET

- [illegible]

NUMERICAL PROBLEM

- Consider normal shock formed in a constant area duct in which air is flowing at Mach [illegible]. If the static pressure & temperature before the shock are [illegible] & [illegible] K respectively calculate the following after the shock.
  - Static pressure
  - Stagnation pressure
  - Stagnation temperature
  - Mach number



NUMERICAL PROBLEM

- Consider a normal shock formed inside a Supersonic wind tunnel in which air is being supplied from a storage tank. The pressure inside the tank is 10 atm and the temperature inside the tank is 300 K. If a normal shock is formed at a constant area section within the tunnel where the Mach number is 2.0, calculate the static pressure & static temperature downstream of the shock

- Conditions inside the tank are
- P0 & M1 → P1; T0 & M1 → T1
- Use relations/tables for p2/p1 & T2/T1
- Calculate p2, T2

NUMERICAL PROBLEM
GATE 2019

- [illegible]

[illegible]

[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]

NON-ISENTROPIC NATURE VS STAGNATION PRESSURE LOS

- [illegible]
- The shock by itself is a very thin region: thickness of a few [illegible]

- [illegible]
  - [illegible]
  - [illegible]
- Recall for perfect gas:

$$s_2 - s_1 = c_p \ln \frac{T_2}{T_1} - R \ln \frac{p_2}{p_1}$$

- In terms of stagnation states:
-

$$s_2 - s_1 = -R \ln \frac{p_{02}}{p_{01}}$$

[illegible]

HUGONIOT RELATIONS

- The relations for compression waves in terms of pressure and specific volume ($v = 1/\rho$)
  - Significance: Normal shock relations are in terms of Mach number, velocity. They are flow properties
- P & v are fluid properties
  - Thermodynamic variables
  - Appropriate description of shock as a compression process

TRANSFORMING THE EQUATIONS IN TERMS OF STATE VARIABLES *(THERMODYNAMIC PROPERTIES)*

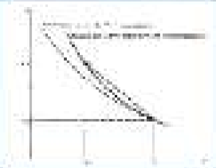
- Continuity equation yields: [illegible]
- Using Momentum equation: [illegible] [illegible]
- Energy equation in the form: [illegible] [illegible]

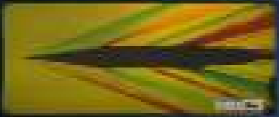
$$e_1 - e_0 = \frac{p_0 + p_1}{2}(v_0 - v_1)$$

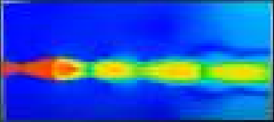
– *the change in internal energy equals the mean pressure across the shock times the change in specific volume*

SHOCK ON A PV DIAGRAM

- A compression process which is more drastic than isentropic compression
  - Generates more entropy as well.

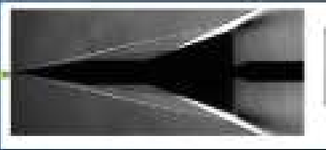


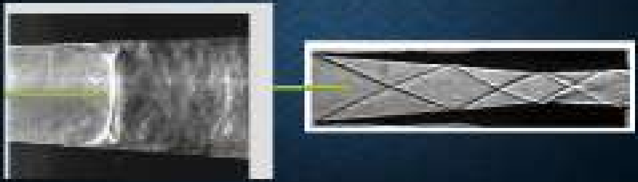




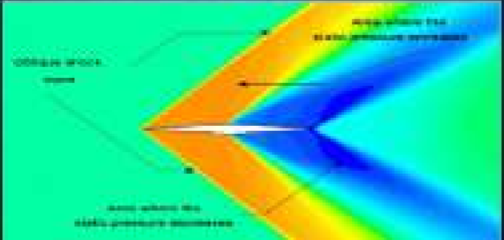
COMPRESSIBLE FLUID FLOW

Oblique SHOCKS





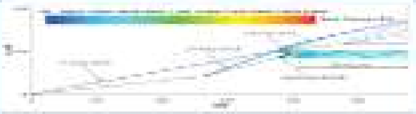
OBLIQUE SHOCKS

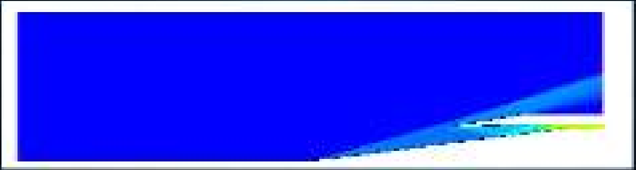


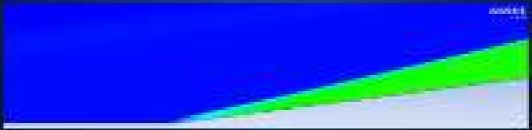
SCRAMJET CONFIGURATION

Compression by forebody and inlet shock system

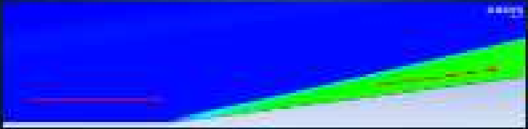
HOW DO SHOCKS "LOOK LIKE"?



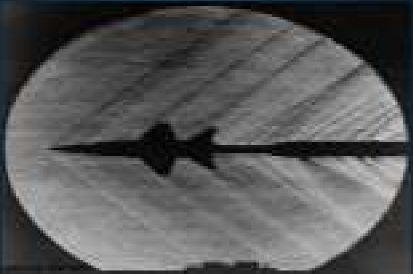




AFTER THE SHOCK, THE (INVISCID) FLOW BECOMES **PARALLEL** TO THE **DEFLECTING SURFACE**...



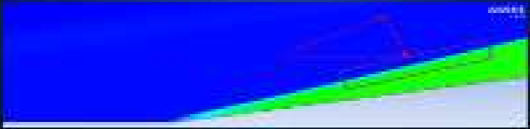
FLOW TURNING

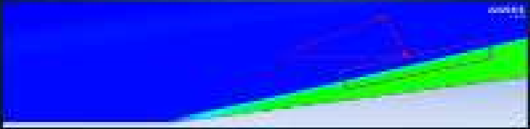


"WEDGE" AND "CORNER"

Note the difference:
Surface angle θ and
shock angle β

THE TWO-DIMENSIONALITY:
INCORPORATED BY VELOCITY COMPONENTS



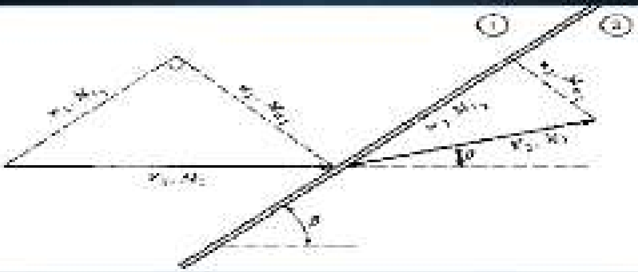


OBLIQUE SHOCK RELATIONS

- Analytical Relations from Governing equations
  - same approach as for normal shocks
  - use conservation equations and state equations
- Conservation Equations
  - [illegible], [illegible] and [illegible]
  - this time 2 momentum equations - [illegible] for a 2-d oblique shock
- *Assumptions*
  - steady flow (stationary shock), inviscid except inside shock, adiabatic, no work but flow work

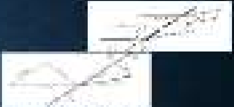
OBLIQUE SHOCK GEOMETRY

- $u_{1n} = V_1 \sin\beta$; $u_{1t} = w_1 = V_1 \cos\beta$
- $u_{2n} = V_2 \sin(\beta - \theta)$; $u_{2t} = w_2 = V_2 \cos(\beta - \theta)$
- $M_{1n} = M_1 \sin\beta$; $M_{1t} = M_1 \cos\beta$
- $M_{2n} = M_2 \sin(\beta - \theta)$; $M_{2t} = M_2 \cos(\beta - \theta)$



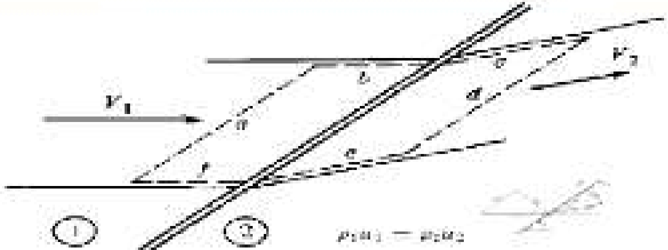
1
2
θ
β

CONTINUITY EQUATION ACROSS OS



- *Consider flow across the control volume.*
- *Note that the tangential component of velocity does not [illegible] along the upstream & the downstream faces.*
- *The side faces (b-c-d & e-f) are parallel to V in the respective regions (1 & 2) and hence have no component of u that contributes to the mass flux.*
- *Hence the continuity equation boils down to the mass flux balance due to the normal velocity (u):*

$$\rho_1 u_1 = \rho_2 u_2$$



V_1
V_2
L
a
d
e
f
1
2

MOMENTUM EQUATION – *TANGENTIAL DIRECTION*

- Momentum equation: Rate of change of momentum = net forces acting on CV

  (Neglecting body forces & viscous forces)

- Consider tangential components: [tangential component of the surface force =0]
  - They cancel on f & b
  - They cancel on c & e
  - They are zero on a & d
- Hence rate of change of momentum due to tangential velocity = 0

$$(-\rho_1 u_1)w_1 + (\rho_2 u_2)w_2 = 0$$

$$(-\rho_1 u_1) w_1 + (\rho_2 u_2) w_2 = 0$$

- Recall continuity equation: $\rho_1 u_1 = \rho_2 u_2$
- These show that: $w_1 = w_2$

The tangential component of velocity remain unchanged across the oblique shock

Hence, all the changes across the oblique shock are restricted to those due to change in normal component of velocity

NORMAL DIRECTION

- Considering forces in the NORMAL direction

$$(-\rho_1 u_1)u_1 + (\rho_2 u_2)u_2 = -(-p_1 + p_2)$$

$$p_1 + \rho_1 u_1^2 = p_2 + \rho_2 u_2^2$$

ENERGY EQUATION

- [illegible]
- [illegible]

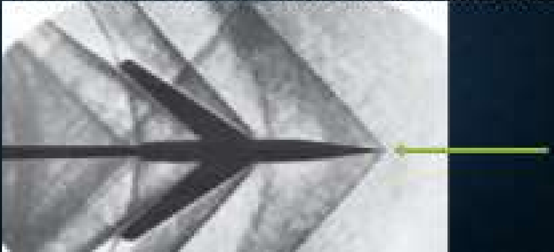
[illegible]

[illegible]

[illegible]

$$h_1 + \frac{u_1^2}{2} = h_2 + \frac{u_2^2}{2}$$

RECALL THE CONTEXT OF ANALYSIS/APPLICATION



OVERVIEW OF THE 3 EQUATIONS

[illegible]

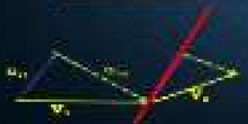
[illegible]

[illegible]

- They are ALL identical with Normal shock relations WRITTEN WITH THE NORMAL COMPONENTS OF VELOCITY FOR OBLIQUE SHOCK
- The implication of this ?

NORMAL VS OBLIQUE

- For an oblique shock with upstream velocity V_1, the normal component is u_1
- Hence the results will correspond to a normal shock with upstream velocity = u_1
- That is with upstream Mach number M_{1n}
- Recall $M_{1n} = M_1 \sin\beta$



OS RELATIONS FOLLOW FROM THE RESPECTIVE NS RELATIONS…

- [illegible]

$$[illegible]$$

$$[illegible]$$

$$[illegible]$$

$$[illegible]$$

THE θ-β-M RELATION

- The Mach number can be used to calculate the shock angle as well.

$$M_2 = \frac{M_{n_2}}{\sin(\beta - \theta)}$$

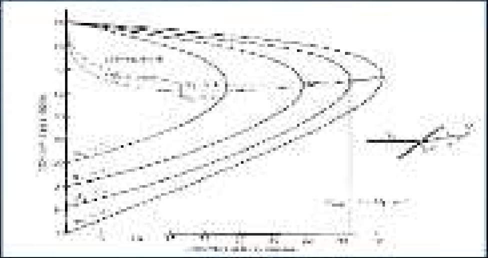
$$\tan\beta = \frac{u_1}{w_1}$$

$$\tan(\beta - \theta) = \frac{u_2}{w_2}$$

$$\frac{\tan(\beta - \theta)}{\tan\beta} = \frac{2 + (\gamma - 1)M_1^2 \sin^2\beta}{(\gamma + 1)M_1^2 \sin^2\beta}$$

$$\tan\theta = 2\cot\beta \left[\frac{M_1^2 \sin^2\beta - 1}{M_1^2(\gamma + \cos 2\beta) + 2} \right]$$

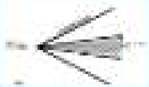
θ-β-M GRAPH

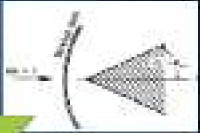


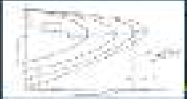
THE STRONG SHOCK & THE WEAK SHOCK

- For a given M1 & θ there can TWO [illegible] β
- [illegible]
- [illegible] value of β [illegible] formation of a WEAK Shock
- The higher value of β → STRONG shock
- [illegible]

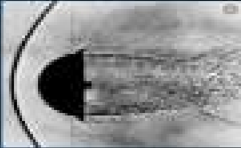
WHAT HAPPENS WITH INCREASE IN DEFLECTION ANGLE θ??





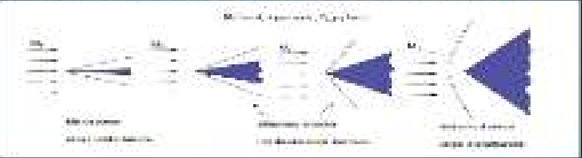


DETACHED/BOW SHOCK

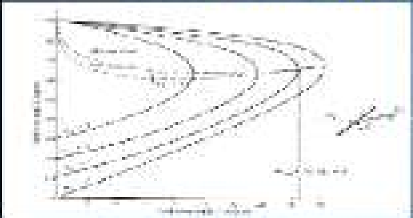


- For any Mach number M1 > 1, there is a maximum value of deflection angle (θ), which an attached oblique shock can be formed
- Beyond that value of θmax, an OBLIQUE SHOCK SOLUTION IS NOT possible
  - [illegible]
- Central region of the detached shock will be similar to a Normal shock
- As you move away, the shock reduces in strength, becomes oblique
- Stagnation pressure loss is severe in the region where the normal shock region
- A detached shock is to be avoided in most applications where stagnation pressure loss is important

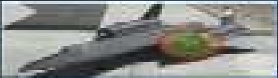
MACH WAVE TO DETACHED SHOCK – THE IMPACT OF DEFLECTION ANGLE…



SOME IMPORTANT OBSERVATIONS



APPLICATION: AN INSTANCE OF REAL OBLIQUE SHOCK FORMATION *(RAMJET INTAKE)*



OBLIQUE SHOCK TABLE

[illegible]

[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]

NUMERICAL PROBLEM

- Consider air at Mach 3.0 flowing over a wedge-shaped body with a half angle of 20 deg. The upstream values of static pressure and static temperature are 100 kPa and 300 K respectively.
  - Calculate the static pressure, static temperature and stagnation pressure downstream of the shock formed, assuming it to be a weak shock.
  - Calculate the same parameters as above if a STRONG shock is formed.

  Assume air to be perfect gas with $\gamma = 1.4$

GAS TABLE

SOLUTION FOR WEAK SHOCK

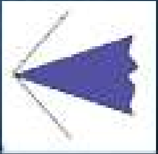
- P2/P1 = 3.771 → P2 = 377.1 kPa
- T2/T1 = 1.[illegible] → T2 = [illegible] K
- M2 = 1.[illegible]

→ Po2 = [illegible] kPa

STRONG SHOCK

[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]

STRONG SHOCK SOLN.



NUMERICAL PROBLEM - 2

- Consider supersonic flow over a supersonic [illegible] model with a ramp angle of [illegible]. If it is exposed to a Mach number of [illegible] calculate the wave angle and static pressure after the shock, assuming the upstream static pressure is 1 bar (assume the shock to be weak).
- What is the maximum ramp angle which can have an attached shock under these conditions ?

GATE 2012

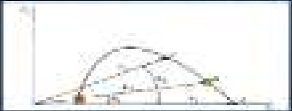
SHOCK POLAR

- Graphical representation of oblique shocks
- Possible solutions of shock functions represented in a [illegible] plane
- Useful for analysis, particularly when complex shock structures are possible under the given flow/geometry conditions
- Coordinate system aligned with the flow direction (x-axis) and the normal to it (y-axis)
  - As opposed to the normal and tangential components that we considered over the oblique shock
- This plane is called **Hodograph**

THE COMPONENTS

WEAK SHOCK & STRONG SHOCK SOLUTIONS IN HODOGRAPH PLANE

WEAK SHOCK & STRONG SHOCK SOLUTIONS IN HODOGRAPH PLANE



- Shock polars are also plotted by non-dimensionalizing velocity components using a*

WHAT HAPPENS WHEN AN OBLIQUE SHOCK HITS A SURFACE ?

- When does it happen ?

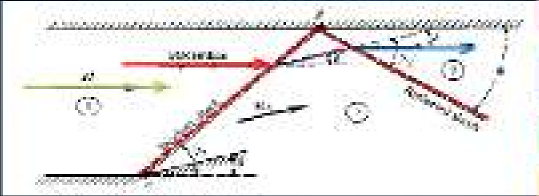
OBLIQUE SHOCK HITTING A SOLID SURFACE

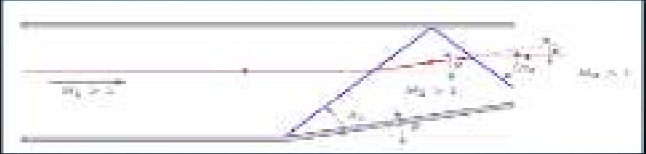
SHOCK REFLECTION

- When an oblique shock impinges at a wall, the flow across it should undergo a realignment due to the new obstruction.
- This takes place by a way of a new oblique shock that emanates from the surface being hit.
- The nature of the new shock depends on the geometry and flow conditions.
- The phenomenon is called SHOCK REFLECTION

INCIDENT & REFLECTED SHOCKS

SHOCK REFLECTION – THE SEQUENCE





SHOCK REFLECTION

- Note the terminology: INCIDENT shock & REFLECTED shock
- Note that M2 < M1. Hence, the reflected shock is formed at a lower Mach number – Hence the reflected shock is WEAKER than the incident shock
    - [illegible]
    - [illegible]

SHOCK INTERSECTION

- Families of the shocks: The two families of **Left** running & **Right** Running shocks

- What happens if in a supersonic passage, two shocks of opposite families intersect ?

SHOCK INTERSECTION

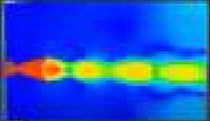
- Consider the intersection of shocks S1 and S2
- Regions 2 & 3 will have different properties
- (Mach no., pressure etc.) as S1 & S2 are not of same strength
- So new shocks S3 & S4 will be formed
- Regions 4 & 5 must have same static pressure – Physically
  - But the velocities can be different
  - A slip line can be identified across which two different and entropy may exist

NUMERICAL PROBLEM

- In a supersonic flow passage the flow [illegible] goes through a horizontal [illegible] section and faces a compression corner on the bottom wall of the passage. The corner is at 10° with the approaching flow direction. The oblique shock formed at the corner reflects from the top wall so that the flow is [illegible] to be in the horizontal direction. If the approaching flow has M = 2.8 static pressure 100 kPa, calculate the Mach number and the static pressure after the reflection. Assume weak shock formation.

SOLUTION

- **Shock 1: M1= 3, Theta1= 10 deg.**
  - [illegible]
  - [illegible] deg
  - [illegible]
- **Shock 2: M2= 2.505, Theta2= 10 deg.** [illegible]
  - [illegible]
- P3 = p1 [illegible] kPa



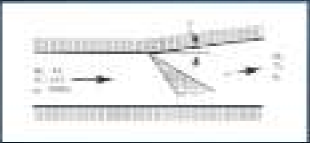
23AES212
COMPRESSIBLE FLUID FLOW

Expansion Waves

EXPANSION WAVES

APPLICATIONS..

A TYPICAL EXPANSION.:



EXPANSION WAVES

- Flow phenomenon that occurs in the condition when supersonic flow is turned in the opposite direction –
- A continuous expanding region called an expansion fan appears composed of an infinite number of Mach waves called Prandtl-Meyer expansion waves
- The flow does not turn suddenly as through a shock but gradually
- The Mach number downstream of the expansion increases ($Ma_2 > Ma_1$), while pressure, density, and temperature decrease

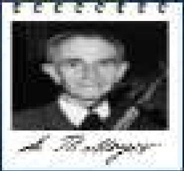
DURING EXPANSION…

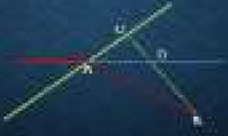
- The static pressure drops & the flow accelerates in a continuous process
- The expansion proceeds through a fan of Mach lines
- Consider expansion from M1 to M2:
  - The leading Mach wave has:
    - [illegible]
  - And the terminating Mach wave has…
    - [illegible]

PRANDTL-MEYER EXPANSION

- [illegible]
- An isentropic process of gradual expansion over an infinite number of Mach waves
- Stagnation pressure remains constant
- [illegible]
- Analysis: Determine M2, P2, T2 etc. if M1 and θ are specified
    - Or other way: Determine the deflection required to achieve a set of properties
- Steady flow, adiabatic wall

Theodor Meyer
1882-1972





CHANGES ACROSS A WEAK WAVE...

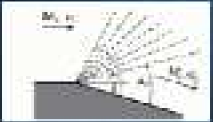
- Applying the law of sines to the triangle ABC

[illegible]

[illegible]

[illegible]

THE EXPANSION FAN



INFINITESIMAL DEFLECTION..

- As ω → 0:

[illegible]

- For E < 1:

[illegible]

Neglecting higher order [illegible]

[illegible] [illegible]

INTEGRATION ACROSS THE FINITE DEFLECTION ANGLE…

[illegible]

- Can be integrated as [illegible]

[illegible]

- Here the derivation in yellow which gives

- The analysis so far was to a small (Mach wave) across the deflection where θ varies from 0 to θ_m & Mach no. varies from M_1 to M_2:

PRANDTL-MEYER FUNCTION

- The integral: [illegible]
- Is evaluated as:

[illegible]

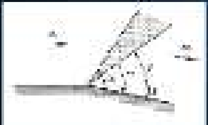
- [illegible]

[illegible]

P-M EXPANSION TABLES

MAXIMUM EXPANSION ANGLE

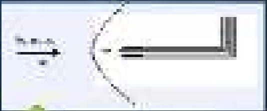
ANGLES OF THE LEADING & THE TRAILING MACH WAVES



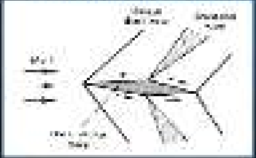
[illegible]

NUMERICAL PROBLEM

INTRUSIVE MEASUREMENTS IN SUPERSONIC FLOWS



FLOWS WITH SHOCKS AND EXPANSIONS

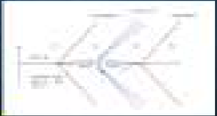


SHOCK-EXPANSION METHOD

- Several supersonic flow cases can be analyzed/solved by patching together appropriate combinations of the oblique shock wave, and the Prandtl-Meyer expansion fan
- TWO Salient aspects of the flow field:
  - The streamline ahead of the leading edge is non-deflected; Supersonic flow has no upstream influence
  - The flow streams over the upper and lower surfaces are completely independent of one another

SHOCK EXPANSION "[illegible]" [METHOD]

- An approach for calculation of aerodynamic forces on two-dimensional supersonic/hypersonic airfoils whose surfaces can be approximated as formed by straight lines.
    - Flat plate at an angle of attack
    - Diamond airfoils
- Using the oblique shock relations and the Prandtl-Meyer function
    - obtain pressures on all surfaces of the diamond wedge and flat plate and thus lift and drag forces can be calculated.
- Applicable to sharp-nosed bodies with an attached shock wave



A FLAT PLATE AT AN ANGLE OF ATTACK

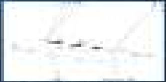
- The flow on the upper surface is turned through an expansion angle by means of a Prandtl-Meyer expansion fan attached to the leading edge of the airfoil, whereas the flow on the lower side is turned through a compression angle by means of an oblique shock.
- The flow on the upper surface is recompressed to the upstream pressure by means of an oblique shock wave attached to the trailing edge of the airfoil. Likewise, the flow on the lower surface is re-expanded to the upstream pressure by means of an expansion fan.

DRAG & LIFT

- Drag and lift forces per unit span of the flat plate

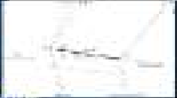
$$L = (p_2' - p_3)\, c \cos \alpha_0$$

$$D = (p_2' - p_3)\, c \sin \alpha_0$$



- What is the reason for this drag ? Note that we have assumed inviscid flow ….

WAVE DRAG



- 2D steady inviscid subsonic flow past a body gives ZERO drag (neglecting edge effects) is ZERO
- Not so in supersonic flow
  - Even inviscid flow leads to a nonzero drag due to the difference in pressure forces
  - The drag caused by shock waves & expansion waves is called "WAVE DRAG"
- Drag per unit span


plate

WAVE DRAG – TERM FROM SHIP HYDRODYNAMICS ?

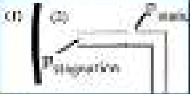
WAVE DRAG – ON FLAT PLATE

ACCURACY OF SHOCK-EXPANSION MODEL

NUMERICAL PROBLEM

[illegible]

- A Pitot-probe placed in a supersonic stream of air records a stagnation pressure of 200 kPa and a static pressure of 100 kPa. If the stagnation temperature is 400 K, determine the Mach number & velocity of the main stream flow

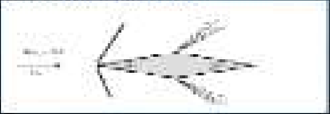


- Given: The values of Po2 & P1
  - Calculate M1
- Get M2 from tables
- [illegible]

[illegible]

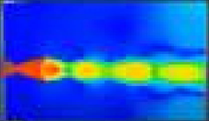
NUMERICAL PROBLEM

- Consider a symmetric diamond shaped airfoil. Air flows at $M_\infty = 3.0$ past the airfoil section, with zero angle of attack. The freestream static pressure is 1 atm. The included angle for the leading and the rear faces is (δ/2) 20deg. Determine the pressure distribution on the airfoil section.



SOLUTION

- Steps:
  - Solve for the oblique shock at leading edge: M1 & theta are given
  - Calculate P2/P1
  - Solve for the expansion fan. Use M2 & theta
  - Calculate M3
  - Po2 = Po3 (Expansion fan)
  - Use M3 & Po3 to calculate P3



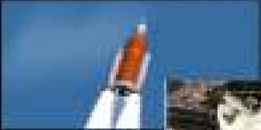
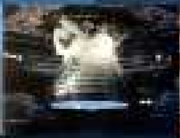
23AES213

COMPRESSIBLE FLUID FLOW

Nozzle Flow

FLOW WITH AREA CHANGE

- What is the impact of varying area on compressible flow?
- None of the phenomena that we considered so far accounted for area change.
- How does the area change impact the flow properties when the flow is COMPRESSIBLE?
- [illegible]
  - How do we obtain supersonic flow in an internal passage in the first place?
  - How do we accelerate the flow past sonic velocity?


QUASI 1-DIMENSIONAL FLOW

- Velocity is only a function of the stream-wise coordinate

CONTINUITY EQUATION

- Steady, 1-dimensional flow

[illegible]

$$\rho_1 u_1 A_1 = \rho_2 u_2 A_2$$

MOMENTUM EQUATION

$$\oiint (\rho \mathbf{V}\cdot d\mathbf{S})\mathbf{V} + \iiint \frac{\partial(\rho \mathbf{V})}{\partial t}\, d\mathcal{V} = \iiint \rho \mathbf{f}\, d\mathcal{V} - \oiint p\, d\mathbf{S}$$

- Steady flow without body forces & viscous forces

$$p_1 A_1 + \rho_1 u_1^2 A_1 + \int_{A_1}^{A_2} p\, dA = p_2 A_2 + \rho_2 u_2^2 A_2$$

ENERGY EQUATION

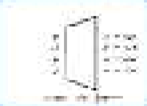
[illegible]

- [illegible]

[illegible]

$$h_1 + \frac{V_1^2}{2} = h_2 + \frac{V_2^2}{2}$$

FLOW ACROSS A DIFFERENTIAL VOLUME..



- Continuity yields:
- Momentum equation:
- Multiplying expanded continuity eqn. by u & subtracting from momentum eqn.
- Differential energy equation:

VARIATION OF VELOCITY WITH AREA

$$\frac{dA}{A} = (M^2 - 1)\frac{du}{u}$$

- Consider subsonic flow (M < 1)
  - How does A vary with u?
- Consider supersonic flow (M > 1)
  - How does A vary with u?

HOW DO WE ACCELERATE THE FLOW TO SUPERSONIC ?

- In the subsonic regime, A **convergent** section is required to accelerate the flow to Sonic Mach number.
- At M = 1 the flow requires a **divergent** section to accelerate further & become supersonic.
- Hence, A **CONVERGENT – DIVERGENT** nozzle is required for accelerating the flow from subsonic to supersonic Mach numbers.

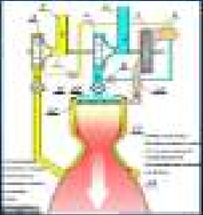
C-D NOZLLE

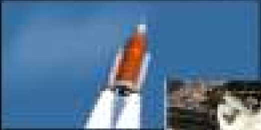
[DE LAVAL NOZZLE]

DESIGN OPERATION OF A CD NOZZLE: HOW DO THE FLOW PROPERTIES VARY ACROSS ?

- Velocity: Increases continuously
- Mach number: Increases continuously
- Static pressure: Decreases in both Conv & div sections (Expansion)
- Density: ?
- Static temperature: ?
- Stagnation temperature: Remains constant (Adiabatic, no work transfer)
- Stagnation pressure: Remains constant (Reversible adiabatic)

CD NOZZLE IN ROCKETS





NOZZLE DESIGN: HOW TO DESIGN FOR A SPECIFIED MACH NUMBER

- What should be the exit area to achieve a desired Mach number ?
- How should we determine the minimum area of cross-section (The THROAT area) ?

AREA VARIATION IN THE DIVERGENT SECTION

- Applying continuity between a general section A and the throat (A*)

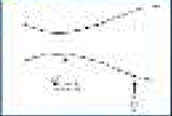
[illegible]

[illegible]

[illegible]

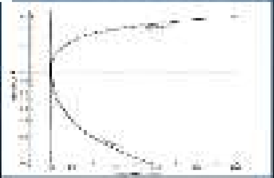
[illegible]

[illegible]

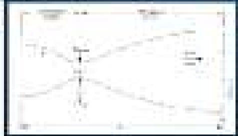


AREA-MACH NUMBER RELATION

$$\left(\frac{A}{A^*}\right)^2 = \frac{1}{M^2}\left[\frac{2}{\gamma+1}\left(1+\frac{\gamma-1}{2}M^2\right)\right]^{(\gamma+1)/(\gamma-1)}$$



STAGNATION VS STATIC CONDITIONS



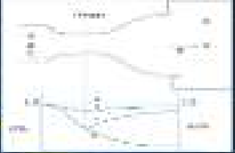
MACH NUMBER VARIATION ALONG THE NOZZLE

- Design operation: A continuous increase in Mach number to the design value at exit
- Design: A_e = [illegible]

ISENTROPIC FLOW TABLES

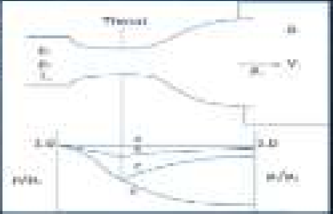
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]

THE IMPACT OF "BACK"PRESSURE



- Consider an arrangement where you can vary the Pb, the pressure to which the gas discharges from the nozzle
- The supply is from a tank maintained at Po
- What if Pb=Po to begin with?
  - No flow !

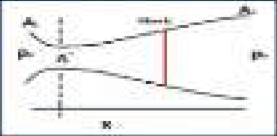
THE IMPACT OF "BACK"PRESSURE – CTD.

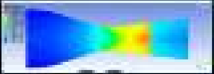


FORMATION OF SHOCK WAVES INSIDE...

NOZZLE PRESSURE RATIO & MASS FLOW RATE - CHOKING

NORMAL SHOCK INSIDE THE NOZZLE





UNDEREXPANSION & OVEREXPANSION

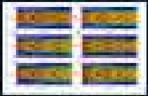
- Remember: P_e is not necessarily equal to P_b for a supersonic jet issuing from a CD nozzle
  - Recall: Pe = the static pressure of the gas at the exit (in the nozzle)
  - Pb: the ambient pressure
- UNDEREXPANDED when Pe > Pb
- OVEREXPANDED when Pe < Pb
- The Ideal/Complete/Perfect Expansion is when Pe = Pb

UNDER/OVEREXPANSION: THE CONSEQUENCES

- Flow: A series of expansion/compression waves (shocks) that eventually [illegible] equalization with the ambient conditions
- [illegible]
  - [illegible]
  - [illegible]

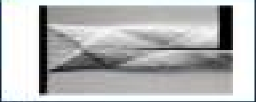
UNDEREXPANDED JET

- At the exit, the jet has a static pressure HIGHER than the ambient pressure
- The jet expands and undergoes a series of shocks & expansions subsequently



OVEREXPANDED JET

- At the exit, the jet has a static pressure LOWER than the ambient pressure
- The jet undergoes compression and emits a series of shocks & reflections subsequently

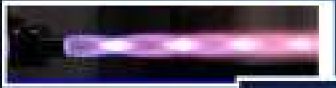


ROCKET JET

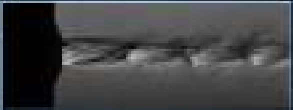
- A rocket with fixed geometry nozzle works at design condition (Pe=Pa) only at ONE altitude
  - As the atmospheric pressure keeps changing with altitude

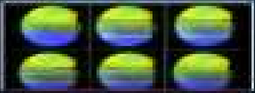

UNDER-EXPANDED
OPTIMUM (IDEAL)
SLIGHTLY OVER-EXPANDED
OVER-EXPANDED

SHOCK *DIAMONDS*



VIDEO





NUMERICAL PROBLEM – 1

- [illegible]

$$\frac{A}{A^*} = \frac{1}{M}\left[\left(\frac{2}{\gamma+1}\right)\left(1+\frac{\gamma-1}{2}M^2\right)\right]^{\frac{\gamma+1}{2(\gamma-1)}}$$

- The above eqn. can be used.
  Compare the value of A/A* for γ = [illegible]
  [illegible]

GAS TABLES FOR GAMMA = 1.4

[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]

NUMERICAL PROBLEM - 2

- A convergent divergent nozzle is to be designed for air being supplied from a tank with p_0 = 500 kPa and T_0 = 500 K. The flow Mach number should be 3.3 at nozzle exit. Determine the nozzle exit area and the design conditions if the throat area is 10 cm²?
- What will be exit pressure under these conditions?

SOLUTION

- Isentropic flow tables give, for Ma [illegible] (tables/ graph)
- [illegible] = 51.22 mm
- [illegible]
- [illegible]
- T_1 from T_0 & Ma_1
- V_1 from T_1 & Ma_1
- Calculate Mass flow [illegible]
  - = 0.825 kg/s

- Calculate [illegible]

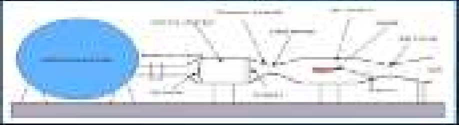
GATE 2016

[illegible]

SUPERSONIC WIND TUNNEL

- Experimental set up to provide a uniform supersonic flow at the test section
- Requires
  - A high pressure storage (tank)
  - Settling chamber
  - A CD nozzle to accelerate the flow to the required supersonic Mach number
  - Test section with the required instrumentation

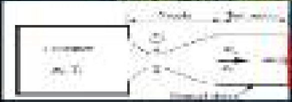
SUPERSONIC WIND TUNNEL



THE SECOND CD PASSAGE – WHY IS IT REQUIRED ? - I

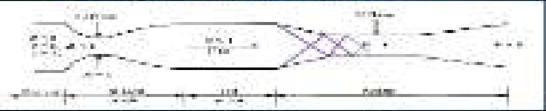
- Consider a WT with one section Mach number 3.0
- Assume the flow leaves at 3.0 to atmosphere (P_a = 1 bar)
- What is the average stagnation pressure required (P_0) ?
  - P_0 = 1/0.0272 → P_0 = 36.7 atm
- Consider that a normal shock is located before the flow discharges to atmospheric pressure

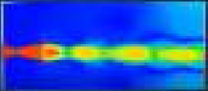
THE SECOND CD PASSAGE – WHY IS IT REQUIRED ? - II



SUPERSONIC WIND TUNNEL COMPONENTS

- Driving stagnation pressure requirement can still be reduced by having a CD Diffuser
  - The diffuser decelerates the supersonic flow through a series of oblique shocks





23AES212

COMPRESSIBLE FLUID FLOW

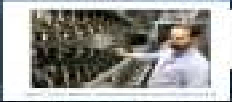
ASSIGNMENT 2

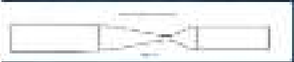
[illegible]

THE ISSUE OF NON-UNIFORMITY IN MACH NUMBER DISTRIBUTION IN SUPERSONIC NOZZLE DESIGN FOR WIND TUNNELS – A CASE STUDY

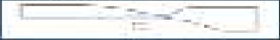
Courtesy: A Study by [illegible], Old Dominion University, VA, USA

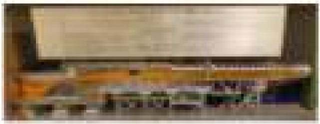
TWO DIMENSIONAL CD NOZLE FOR WIND TUNNEL





THE SLIDING BLOCK…



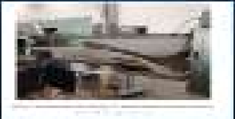


- When the nozzle moves to the left the area of the nozzle throat increases, leading to a lower nominal Mach number in the test section.

- The sliding block nozzle utilizes 'nozzle blocks' to adjust the effective geometry of the nozzle.
  - [illegible]
- The [illegible] supersonic tunnel (SST) has an asymmetric sliding block nozzle
  - [illegible]
  - [illegible]

THE PROBLEM

- The supersonic wind tunnel has issues in keeping flow uniformity in the test section, which is where the nominal Mach number is measured.
    - Sometimes Mach number is the result of unmatched expansion waves upstream in the divergent section of the wind tunnel nozzle.



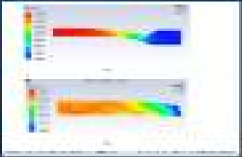
OBJECTIVE OF THE STUDY

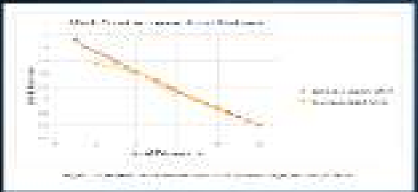
- Analyze the flow within the ODU asymmetric nozzle wind tunnel
- Suggest a method for improving the flow uniformity using the MOC and computational software
  - not used at the time of the creation of the ODU SWT
- Establish a baseline for improvement of the Mach number uniformity in the test section

THE MESH USED FOR CFD ANALYSIS

NON UNIFORMITY IN FLOW

- Non uniformity in the distribution of Mach numbers
- Cause: Pressure waves across the flow

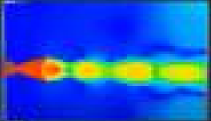




MACH NUMBER DISTRIBUTION

SUMMARY OF FINDINGS

- The study highlights the inadequacy of nozzle design as reflected by the cross-sectional non-uniformity in Mach number distribution.
- The need for appropriate use of MOC in designing the divergent section of the supersonic nozzle is demonstrated.



23AES212

COMPRESSIBLE FLUID FLOW

Sessions: 37-38-39-40

Compressible Flow with Heat Transfer

NON-ADIABATIC COMPRESSIBLE FLOW

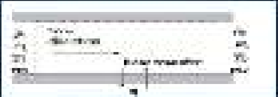
- What happens when the wall of the duct is not [illegible]
- Or by design, heat is either added or taken away from the flowing gas ?
- How does compressibility couple with the transfer of thermal energy from/to the flow ?
  - Under steady state [illegible] conditions

APPLICATIONS

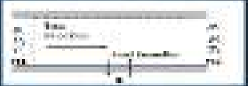
- Aircraft combustors: Heat addition by way of combustion
  - In thermodynamic analysis, this is modeled as a heat transfer across the wall
- Combustors in ramjets & SCramjets
  - Flow is compressible, supersonic in Scramjets
  - Heat addition will have a strong effect on the required [illegible] design for the passage
- Flows through heated/cooled pipes in processing plants

THE CONTROL VOLUME WITH HEAT TRANSFER

- Steady 1 dimensional flow
- Inviscid
- No body forces



THE GOVERNING EQUATIONS



$$\rho_1 u_1 = \rho_2 u_2$$

$$p_1 + \rho_1 u_1^2 = p_2 + \rho_2 u_2^2$$

$$h_1 + \frac{u_1^2}{2} + q = h_2 + \frac{u_2^2}{2}$$

HEAT ADDITION AND STAGNATION TEMPERATURE RISE

- The energy equation yields:

$$q = \left(c_p T_2 + \frac{V_2^2}{2}\right) - \left(c_p T_1 + \frac{V_1^2}{2}\right)$$

- [illegible]

$$q = c_p T_{02} - c_p T_{01} = c_p (T_{02} - T_{01})$$

[illegible]

STATIC PRESSURE RATIO

- Momentum term can be rewritten as:
- From momentum equation:

OTHER PROPERTY RATIOS

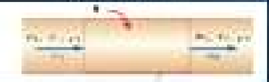
- [illegible]

RAYLEIGH FLOW

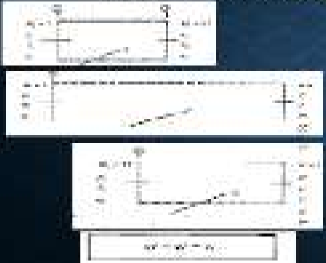
- One dimensional compressible flow with heat addition
- The analysis named after Lord Rayleigh

THE REFERENCE STATE CORRESPONDING TO M = 1

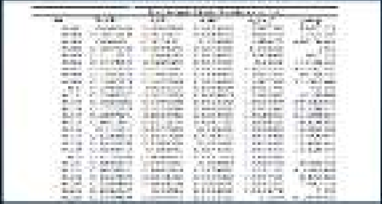
- Consider the specific case where the reference section belongs to M = 1



OTHER PROPERTY RATIOS WRT SONIC SECTION



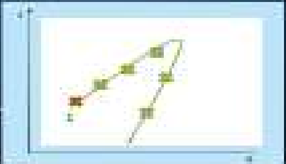
RAYLEIGH FLOW TABLES



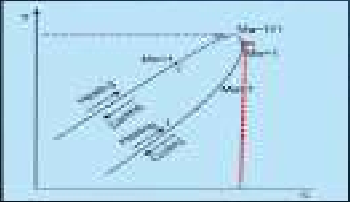
RAYLEIGH FLOW TABLES

THE EFFECT OF HEAT TRANSFER IN COMPRESSIBLE FLOW: THE TS DIAGRAM

- Consider compressible flow at state 1
- Heat added/removed [illegible]
    - [illegible]
- The flow reaches a state 2
- We know all properties at 1 (T_1, M_1, P_1 etc.)
- Consider a second point 2 with static temperature T_2
    - We can Calculate M_2, P_2 [illegible]
    - Make a plot of [illegible] for various values of [illegible]
    - Plot on TS diagram



THE RALEIGH LINE/CURVE/RAYLEIGH FLOW DIAGRAM



ON THE VARIATION OF VELOCITY WITH HEAT TRANSFER - 1

- From energy eqn as applicable to a differential process,

- Also, since,

ON THE VARIATION OF VELOCITY WITH HEAT TRANSFER - 2

- From perfect gas eqn. (since R is constant): $\frac{dP}{P} = \frac{d\rho}{\rho} + \frac{dT}{T}$

- From continuity eqn. for constant area flow (dA=0): $\frac{d\rho}{\rho} = -\frac{dV}{V}$

- From momentum eqn. for constant area flow:

$$\frac{dP}{P} = -\gamma M^2 \frac{dV}{V}$$

ON THE VARIATION OF VELOCITY WITH HEAT TRANSFER - 3

- [illegible]

[illegible]

- [illegible]

[illegible]

ON THE VARIATION OF VELOCITY WITH HEAT TRANSFER - 4

- OR

$$\frac{dq}{dV} = c_p \frac{T}{V} \qquad \frac{V}{\gamma - 1}$$

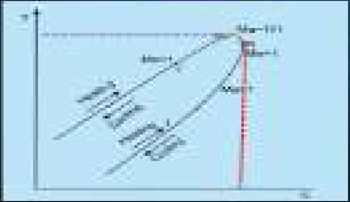
AT SONIC POINT …

- $dq/dr = 0$ requires:

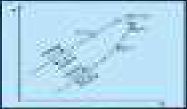
[illegible]

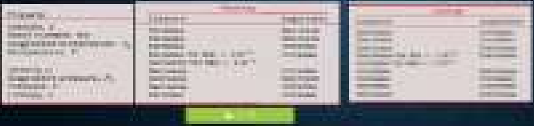
  - Happens at sonic velocity (M=1)
  - Hence M=1 will be an inflection point

THE RALEIGH LINE/CURVE/RAYLEIGH FLOW DIAGRAM



PROPERTY VARIATION IN SUBSONIC & SUPERSONIC FLOWS





NUMERICAL PROBLEM - 1

- Air enters a duct subsonically at section 1 at 1.2 kg/s. When 650 kW of heat is added, the flow chokes at the exit at p_2 = 95 kPa and T_2 = 700 K. Assuming frictionless heat addition, estimate (a) the velocity, and (b) the stagnation pressure at section 1

SOLUTION

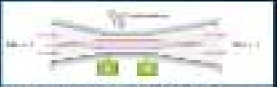
- [illegible]
- [illegible]
  - [illegible]
  - [illegible]
  - [illegible]

[illegible]

SOLUTION-CTD

- V1 = M1*SQRT(1.4RT1)
  - T1 = To1/(1+.2*M1^2) = [illegible] K
  - → V1 = [illegible] m/s
- Similarly, use Po1/Po* from table; Calculate Po* based on P*; Calculate Po1; Calculate P1 from Po1 & M1.
- → Po1 = [illegible] kPa (Check)

NUMERICAL PROBLEM -2



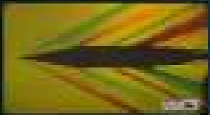
- [illegible]

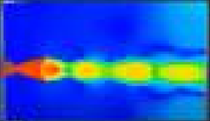
SOLUTION

- From Rayleigh flow tables:
  - [illegible]
  - [illegible]
    - [illegible]
    - [illegible]

SOLN - CTD

- Based on Mach number at [illegible]
- Hence, p^o = [illegible]
- [illegible]
- [illegible]
- [illegible]
- [illegible] (Check)





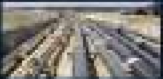
23AES212

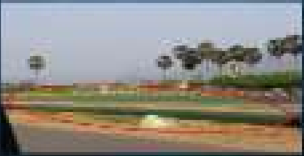
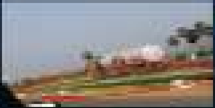
COMPRESSIBLE FLUID FLOW

Compressible Flow with Friction

GASES FLOWING IN LONG PIPE LINES….

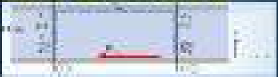
- **FRICTION:** All real flows are viscous & experience friction
- The effect of friction, coupled with compressibility, becomes important in **LONG pipe lines carrying gases**



THE ANALYTICAL MODEL : STEADY, 1-DIMENSIONAL, ADIABATIC FLOW WITH FRICTION

FANNO FLOW
THE MODELING APPROACH



- Analysis of the impact of fluid friction on compressible flow
  - [illegible]
  - Friction being modelled as a reaction shear force at the discrete opposite to the flow
    - [illegible]
- Application of continuity equation and energy equation helps express Mach number and temperature with reference to the location where the flow would generally choke in the duct

STEADY, 1-D FLOW WITH FRICTION IN ADIABATIC, CONSTANT AREA DUCT

- Momentum equation as applied over a length

[illegible]

[illegible]

[illegible]

- Using the conventional formulation of shear stress in terms of the empirically determined friction coefficient f
- The differential momentum equation along with $\tau_w = f\frac{1}{2}\rho V^2$, continuity & energy equations and the perfect gas assumption, transforms as

$$\frac{4f\,dx}{D} = \frac{2(1-M^2)}{\gamma M^2\left(1+\frac{1}{2}(\gamma-1)M^2\right)}\frac{dM}{M}$$

PROPERTY RATIOS ACROSS THE DUCT

- By the application of energy equation to the adiabatic control volume,

$$\frac{T_2}{T_1} = \frac{2+(\gamma-1)M_1^2}{2+(\gamma-1)M_2^2}$$

- Combining the common mass-continuity equation with this

$$\frac{p_2}{p_1} = \frac{M_1}{M_2}\frac{a_1}{a_2} = \frac{M_1}{M_2}\sqrt{\frac{T_2}{T_1}}$$

- That is:

$$\frac{p_2}{p_1} = \frac{M_1}{M_2}\sqrt{\frac{2+(\gamma-1)M_1^2}{2+(\gamma-1)M_2^2}}$$

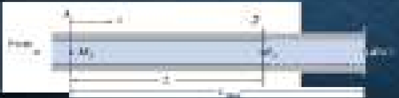
FANNO FLOW

STEADY, 1-D, ADIABATIC FLOW WITH FRICTION

- **Dr.** Gino Girolamo Fanno

HOW DO WE INCORPORATE THE EFFECT OF FRICTION ON COMPRESSIBLE FLOW ?

CHOKING LENGTH – INTERPRETATION OF THE [illegible] EXTENSION OF THE DUCT TO L_{max} OR L^*

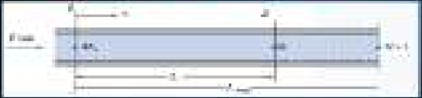


RESULT OF INTEGRATION OVER CHOKING LENGTH

- For engineering applications Fanno flow calculations are made by integrating the momentum equation from entry to the pipe (L = 0) to a **hypothetical choking length L***.
- The result of integration has been worked out as:

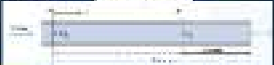
$$\frac{fL^*}{D} = \frac{1-M^2}{\gamma M^2} + \frac{\gamma+1}{2\gamma}\ln\frac{(\gamma+1)M^2}{2+(\gamma-1)M^2}$$

L ALSO IS REFERRED TO AS



FANNO FLOW SOLUTION PROCEDURE

- Typically M_2 is found by determining the choking length, L_{max} or L^*, associated with the inlet Mach number M1
  - The above length is L_1^*
  - And then comparing the length to the choke length from the actual outlet of the pipe as $L_2^* = L_1^* - L$

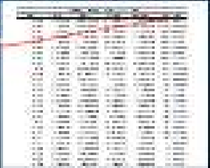


  - The outlet Mach M2 can

  Then be found inversely from L_2^*

FANNO FLOW TABLES..

choking length for
Each Mach number



FANNO FLOW ON TS DIAGRAM & PROPERTY VARIATIONS

COMPARISON WITH RAYLEIGH FLOW

NUMERICAL PROBLEM

- Air flows in an adiabatic pipe with [illegible] cm diameter and [illegible] m length. The static pressure and temperature at inlet to the pipe are [illegible] kPa and 300 K respectively. Inlet Velocity is [illegible] m/s. The pipe has an average friction factor f = [illegible]. (a) Will the flow be choked at the end of the pipe? (b) Calculate the Mach number and the static pressure at the exit of the pipe.

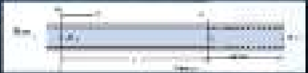
FANNO FLOW SOLUTION PROCEDURE

- [illegible]
  - [illegible]
  - [illegible]

[illegible]

- [illegible]
- [illegible]

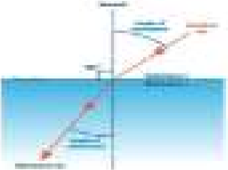
[illegible]

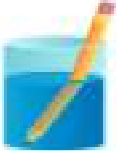


SOLUTION…

Introduction to Schlieren and Shadowgraph

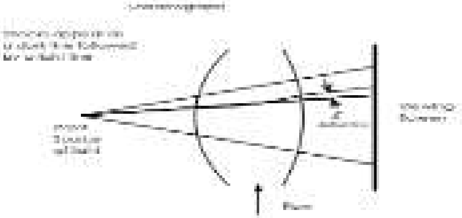
REFRACTION

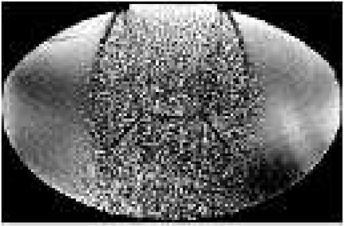


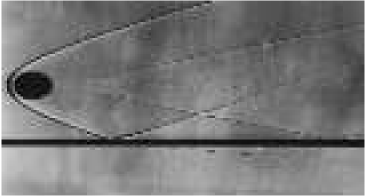


Optical Techniques for Flow Visualization

$$n = K\rho + 1$$



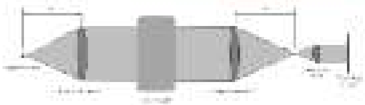


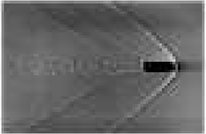


Imaging Based on Density Variations

- [illegible] band towards the region with greater RI.
  - [illegible]
  - [illegible]
- [illegible]
- Very sensitive optics are required to detect changes in density in gases.

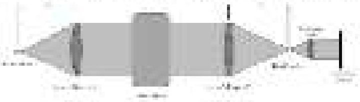
Shadowgraph

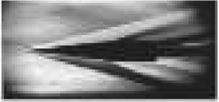


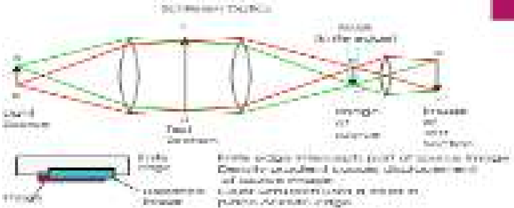


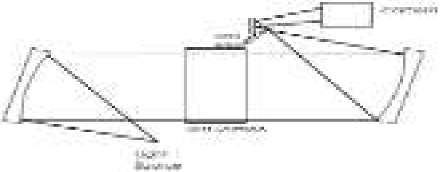
Schlieren Imaging

- Detects changes in the first derivative of density
  - Addition of a knife edge at the focal point of the second mirror







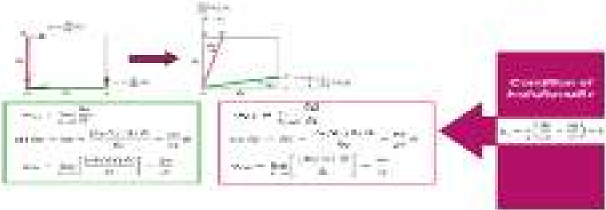


Introduction to the
Method of Characteristics

Streamline, , angles..
-Reviewing some terminology

Review of “[illegible]” of flow

Angular Motion and Deformation



Steady, inviscid flow in 2D domain

- 2D continuity equation in differential form $\frac{\partial}{\partial x}(\rho u) + \frac{\partial}{\partial y}(\rho v) = 0$
- The condition for irrotational flow: $\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y} = 0$
- Definition of velocity potential $u = \frac{\partial \phi}{\partial x}$ $v = \frac{\partial \phi}{\partial y}$
- Momentum equation in x-direction $\rho u \frac{\partial u}{\partial x} + \rho v \frac{\partial u}{\partial y} = -\frac{\partial p}{\partial x}$

- [illegible]

$$[illegible]$$

$$[illegible]$$

- [illegible]

$$[illegible]$$

A line which makes Mach angle with the stream line direction (at a point) is also a direction along which the derivative of x-component is indeterminate

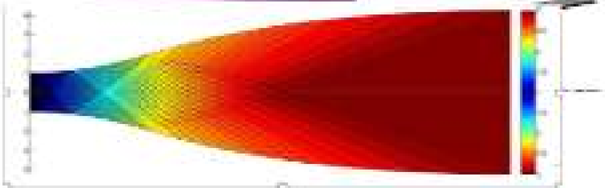
Such lines are called as ***Characteristic Lines***

Characteristics: Summary

Notes On Characteristic Lines

- *Recall that we are considering 2D, inviscid flow – The governing PDE's are hyperbolic nature, mathematically*
- ***We have identified that for such flows directions can be identified along the Mach lines (angle = μ with flow direction) in such a way that the derivatives of flow properties ($\frac{\partial u}{\partial x}$, $\frac{\partial p}{\partial y}$ etc.) are indeterminate***
- **The flow properties (u, p, T etc.) are continuous along Mach lines**

An application: Nozzle design



Small Perturbations → Linearization

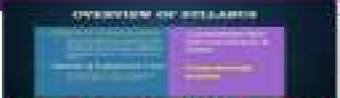
- [illegible]
- [illegible]

[illegible]

- [illegible] [illegible] and [illegible]

- [illegible]

$$(1 - M_\infty^2)\frac{\partial^2 \phi}{\partial x^2} + \frac{\partial^2 \phi}{\partial y^2} = 0$$



Thank You !!