



G. A. VARTIA
UNIVERSITY OF CALIFORNIA, BERKELEY

A MOOC Course
on Gas Dynamics



**COMPRESSIBLE
FLUID FLOW:
AN INTRODUCTION TO THE THEORY AND THE
APPLICATIONS**

UNIVERSITY OF CALIFORNIA, BERKELEY

Introduction to the Course



Faculty Introduction

- Krishnamoorti, N. R.
- **With Ansys: 2012 June-**
- Formerly Head, **Automotive Dept., IIT Bombay**
 - Director, Development of ANSYS India, Pune
 - 1996-2012
- PhD. at IIT Madras, 1986
- Current Research Areas: **Propulsion, High speed Flows, Heat Transfer, Combustion,**



Ansys



OVERVIEW

- Course Objective & Outcomes
- Syllabus, Text & References
- Sessions & Mode of Delivery
- Evaluation

COURSE OBJECTIVE

Introduce students to the basics of compressibility of flow and the related phenomena. Enable them to analyse subsonic and supersonic flows with simplified analytical models. Provide an introduction to the applications of compressible flow analysis in aerospace engineering.

SYLLABUS

Unit 1

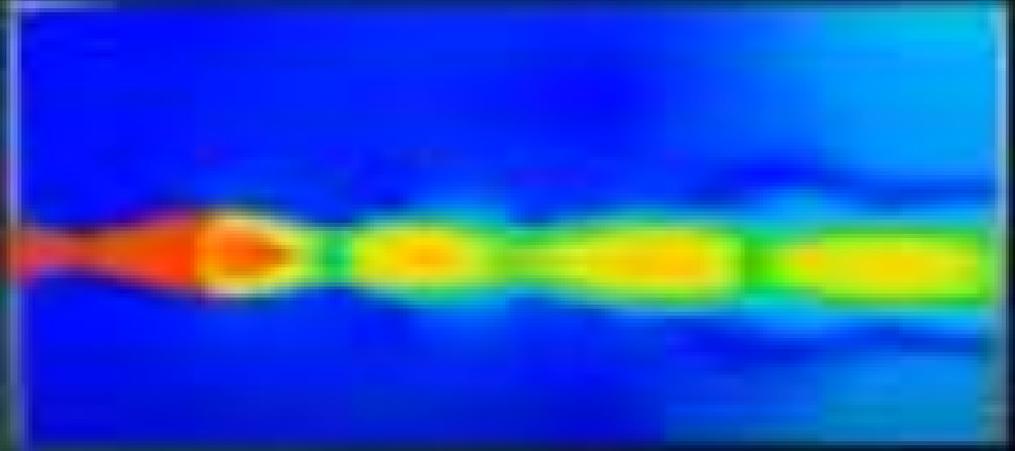
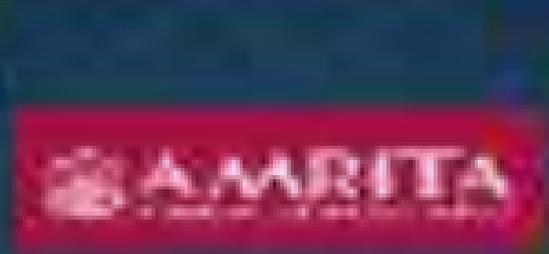
Review of (1) **Introduction**, (2) **Unit 1**, (3) **Unit 2**, (4) **Unit 3**, (5) **Unit 4**, (6) **Unit 5**, (7) **Unit 6**, (8) **Unit 7**, (9) **Unit 8**, (10) **Unit 9**, (11) **Unit 10**, (12) **Unit 11**, (13) **Unit 12**, (14) **Unit 13**, (15) **Unit 14**, (16) **Unit 15**, (17) **Unit 16**, (18) **Unit 17**, (19) **Unit 18**, (20) **Unit 19**, (21) **Unit 20**, (22) **Unit 21**, (23) **Unit 22**, (24) **Unit 23**, (25) **Unit 24**, (26) **Unit 25**, (27) **Unit 26**, (28) **Unit 27**, (29) **Unit 28**, (30) **Unit 29**, (31) **Unit 30**, (32) **Unit 31**, (33) **Unit 32**, (34) **Unit 33**, (35) **Unit 34**, (36) **Unit 35**, (37) **Unit 36**, (38) **Unit 37**, (39) **Unit 38**, (40) **Unit 39**, (41) **Unit 40**, (42) **Unit 41**, (43) **Unit 42**, (44) **Unit 43**, (45) **Unit 44**, (46) **Unit 45**, (47) **Unit 46**, (48) **Unit 47**, (49) **Unit 48**, (50) **Unit 49**, (51) **Unit 50**, (52) **Unit 51**, (53) **Unit 52**, (54) **Unit 53**, (55) **Unit 54**, (56) **Unit 55**, (57) **Unit 56**, (58) **Unit 57**, (59) **Unit 58**, (60) **Unit 59**, (61) **Unit 60**, (62) **Unit 61**, (63) **Unit 62**, (64) **Unit 63**, (65) **Unit 64**, (66) **Unit 65**, (67) **Unit 66**, (68) **Unit 67**, (69) **Unit 68**, (70) **Unit 69**, (71) **Unit 70**, (72) **Unit 71**, (73) **Unit 72**, (74) **Unit 73**, (75) **Unit 74**, (76) **Unit 75**, (77) **Unit 76**, (78) **Unit 77**, (79) **Unit 78**, (80) **Unit 79**, (81) **Unit 80**, (82) **Unit 81**, (83) **Unit 82**, (84) **Unit 83**, (85) **Unit 84**, (86) **Unit 85**, (87) **Unit 86**, (88) **Unit 87**, (89) **Unit 88**, (90) **Unit 89**, (91) **Unit 90**, (92) **Unit 91**, (93) **Unit 92**, (94) **Unit 93**, (95) **Unit 94**, (96) **Unit 95**, (97) **Unit 96**, (98) **Unit 97**, (99) **Unit 98**, (100) **Unit 99**, (101) **Unit 100**.

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COMPRESSIBLE FLOW

WIT.CIT.??

WHY DO WE DISCUSS APPLICATIONS FIRST ?!

- Highlight the importance and the utility
- State the main element that differentiates the subject
- Why CPP/Class Dynamics is a **CORE** course for Aerospace ?







1918



There is also a slight rise in temperature between the two points. This is due to the fact that the water is being heated by the sun and the air is being heated by the sun.

Density changes significantly during the flow.

$\rho \neq \text{Constant!}$

- Changes the flow physics
- The form of governing equations change dramatically
- Leads to certain phenomena that are *initially unexpected* as per *incompressible* flow analysis
- These demand an in-depth analysis of the related flow phenomena

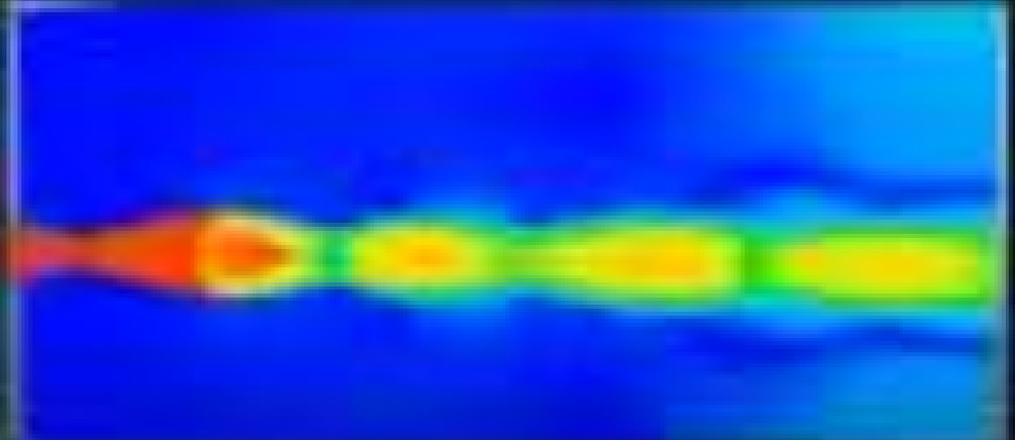
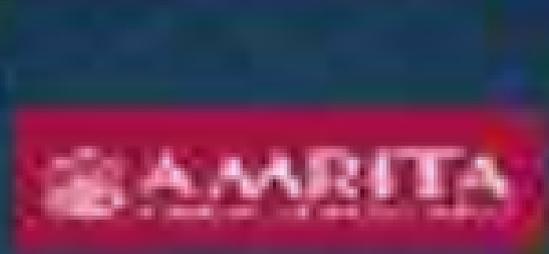
THE COUPLING OF ENERGY & VELOCITY

- Energy: Thermodynamics
- Velocity: Fluid Dynamics
- We need both
- The physics is coupled & hence the Governing Equations are Coupled

INCOMPRESSIBLE FLOW ~~WAVES~~ EXPLAIN MOST OF WHAT YOU SEE IN THIS VIDEO

<https://www.youtube.com/watch?v=KwE1u3h0C0E>





23AES201
COMPRESSIBLE FLUID
FLOW

The Governing Equations

$\rho \neq \text{Constant!}$

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THE COUPLING OF ENERGY & VELOCITY

- Energy: Thermodynamics
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GOVERNING EQUATIONS

- The governing equations include the following conservation laws of physics:
 - Conservation of mass
 - Newton's second law: the change of momentum equals the sum of forces on a fluid particle.
 - First law of thermodynamics (conservation of energy): rate of change of energy equals the sum of heat added and work done on fluid particle.
- The fluid is treated as a continuum. For length scales of, say, $1\ \mu\text{m}$ and larger, the molecular structure and motions may be ignored.

WHY THE GENERAL FORM ?

- What form of the equations are we considering now ?
 - The most general form an equation can be considered takes.
- Can we handle the complete general form of equations by the analysis we did earlier ?
 - NO !
- Then WHY do we go to such a length ?
 - In order to understand the equationality
 - In order to appreciate the complexity & realize the role of assumptions
 - In order to gain insight to the analytical approach

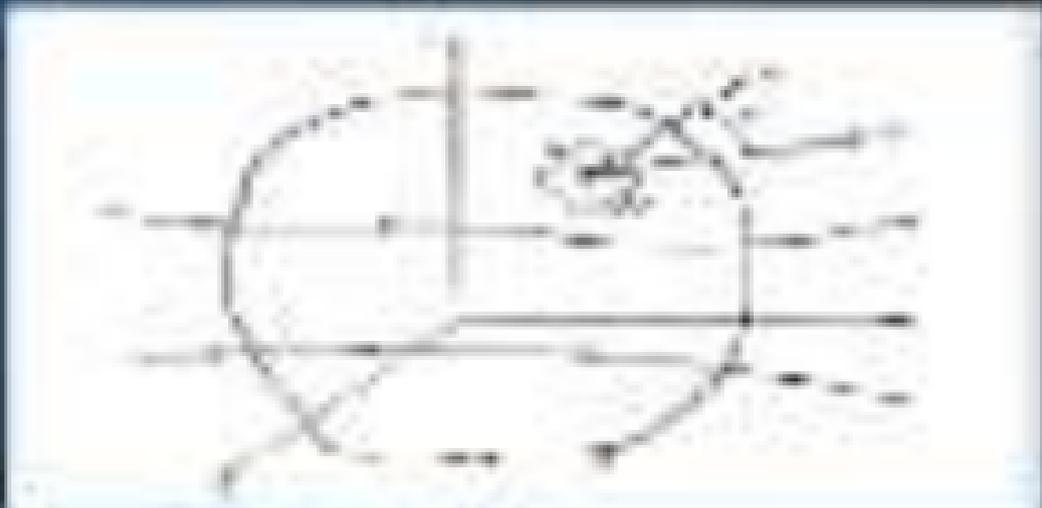
CONTINUITY EQUATION

- *Mass is conserved*

- In a given control volume the inflow and the outflow of the mass will be in such a way that the net flux will be equal to the rate of change within the volume:

CONTROL VOLUME

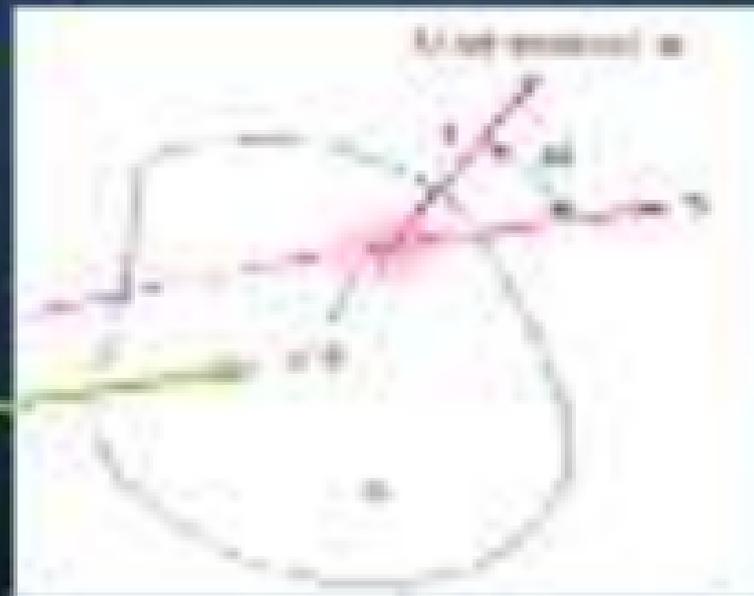
- **Fluid control volume is arbitrary volume**
- Its electrical area dA characterized by the unit normal vector \mathbf{n} **$d\mathbf{R} = \mathbf{n} dA$**
- **Flow of mass through the elemental area dA is $\rho \mathbf{v} \cdot \mathbf{n} dA$ per unit time**



$\mathbf{r} = x\mathbf{i} + y\mathbf{j} + z\mathbf{k}$ $dV = dx dy dz$ $d\mathbf{R} = \mathbf{n} dA$

FLOW RATE ACROSS THE SURFACE

- Volume flow rate: $(\mathbf{v} \cdot \mathbf{n}) \, d\mathbf{a} = \mathbf{v} \cdot d\mathbf{a}$ [Kinf. follows absolute system]
- Mass flow rate = $\rho \mathbf{v} \cdot d\mathbf{a}$



INTEGRAL FORM OF CONTINUITY EQUATION

- The net mass flow into the control volume through the mass inflow surface S_1

$$-\int_{S_1} \rho \mathbf{u} \cdot \mathbf{n} \, dS$$

- Sum of change of mass inside the control volume:

$$\frac{d}{dt} \int_{CV} \rho \, dV$$

- Applying the Continuity Principle:

$$\int_{S_1} \rho \mathbf{u} \cdot \mathbf{n} \, dS + \frac{d}{dt} \int_{CV} \rho \, dV = 0$$

VISUALIZE CONTINUITY EQUATION...

$$\rho_1 A_1 v_1 = \rho_2 A_2 v_2 = \frac{\rho}{\Delta t} \Delta V = \rho Q$$

Flow **Rate** **Flow** through the pipe is **constant** because **mass** is **conserved**. The **volume** of **fluid** that **enters** the pipe **must** be **equal** to the **volume** of **fluid** that **leaves** the pipe. **Continuity** **Equation**



INTEGRAL TO DIFFERENTIAL FORM

• Rewriting the equation:

$$\frac{d}{dt} \int_V \rho \mathbf{v} \cdot d\mathbf{V} = \frac{d}{dt} \int_V \rho \mathbf{v} \cdot \mathbf{v} \cdot d\mathbf{V}$$

• Recall Divergence Theorem:

$$\frac{d}{dt} \int_V \rho \mathbf{v} \cdot \mathbf{v} \cdot d\mathbf{V} = \int_V \rho \nabla \cdot (\mathbf{v} \otimes \mathbf{v}) \cdot d\mathbf{V}$$

• Combining:

$$\frac{d}{dt} \int_V \rho \mathbf{v} \cdot \mathbf{v} \cdot d\mathbf{V} = \int_V \rho \nabla \cdot (\mathbf{v} \otimes \mathbf{v}) \cdot d\mathbf{V}$$

DIFFERENTIAL FORM

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \mathbf{V}) = 0$$

MOMENTUM EQUATION

- Principle: The Time-Rate of Change of Momentum of a Body Equals the Net Force Exerted on It.

$$\frac{d}{dt}(m\mathbf{V}) = \mathbf{F}$$

THE FORCES ?

- **Body forces:** Acting on the volume
 - Ex: Gravitational forces
- **Surface forces:** Acting on the surfaces
 - Pressure forces
 - Viscous forces (Shear forces)
- **INVISCID flow:** The assumption of no viscosity – No viscous forces
- Hence the total forces are: **Body forces** + Pressure Forces

$$F = \iiint_V \rho \mathbf{f} \, dV + \iint_S \rho \mathbf{v} \, dA$$

- Total force acting on the CV

$$\mathbf{F} = \int_{CS} \rho \mathbf{u} \mathbf{u} \cdot d\mathbf{A} - \int_{CV} \rho \mathbf{g} dV$$



- Net momentum flux across:

$$\mathbf{F}_s = \int_{CS} \rho \mathbf{u} \mathbf{u} \cdot d\mathbf{A}$$

TRANSIENT TERM

- Momentum inside the CV

$$M = \int_{CV} \rho \mathbf{V} dV$$

Control Volume (CV) is a fixed region in space through which fluid flows. It is bounded by a control surface (CS).

• Rate of change of momentum within the CV due to unsteady flow effects:

$$\dot{M}_c = \frac{d}{dt} \int_{CV} \rho \mathbf{V} dV = \int_{CV} \frac{\partial(\rho \mathbf{V})}{\partial t} dV$$



$\frac{d}{dt} \int_V \rho \mathbf{v} \, dV$

$\frac{d}{dt} \int_V \rho \mathbf{v} \, dV$ on \mathbf{P}

Recall the fundamental relation:

$$\frac{d}{dt} \int_V \rho \mathbf{v} \, dV = \int_V \rho \frac{d\mathbf{v}}{dt} \, dV + \int_V \frac{d\rho}{dt} \mathbf{v} \, dV = \int_V \rho \frac{d\mathbf{v}}{dt} \, dV + \int_V \frac{d\rho}{dt} \mathbf{v} \, dV$$

$$\int_V \rho \frac{d\mathbf{v}}{dt} \, dV + \int_V \frac{d\rho}{dt} \mathbf{v} \, dV = \int_V \rho \frac{d\mathbf{v}}{dt} \, dV + \int_V \frac{d\rho}{dt} \mathbf{v} \, dV$$

Continuity equation
Momentum equation
Fundamental flow equations

DIFFERENTIAL FORM OF MOMENTUM EQUATION

J. F. FURNESS

$$\frac{d(\rho u)}{dt} + \nabla \cdot (\rho u \mathbf{v}) = -\frac{\partial p}{\partial x} + \mu \nabla^2 u$$

$$\frac{d(\rho v)}{dt} + \nabla \cdot (\rho v \mathbf{v}) = -\frac{\partial p}{\partial y} + \mu \nabla^2 v$$

$$\frac{d(\rho w)}{dt} + \nabla \cdot (\rho w \mathbf{v}) = -\frac{\partial p}{\partial z} + \mu \nabla^2 w$$

ENERGY EQUATION: THE COUPLING OF ENERGY & VELOCITY

- Energy: Thermodynamics
- Velocity: Fluid Dynamics
- We need both
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ENERGY EQUATION, THERMODYNAMICS: WORK & HEAT

- The science of Energy
- Differences between WORK and HEAT

Heat is thermal energy transfer, while work is mechanical energy transfer across the system boundary.

Heat Requires
Temperature
differences

Work Requires
Force and
Displacement

WORK & HEAT

Both are energy interactions.

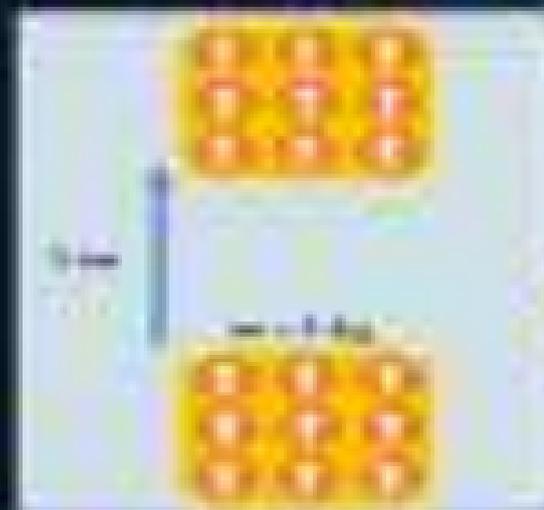
Both are boundary phenomena.

Both represent energy crossing the boundary of the system.

They are not the property of the system.

Both are path functions.

THE "GRADE" OF ENERGY A POSSIBLE INTERPRETATION



Ordered vs
"Disorderly"
movement



This explanation is not applicable to all processes & reactions – just only to the organization.

Entropy is measure of the molecular disorder, or randomness, of a system.

THERMODYNAMIC PROPERTIES

- Point functions:
 - Value is based on the STATE of the system
 - NOT on the process that leads it to that state
- Path functions:
 - This value depends on the process
- Ex:

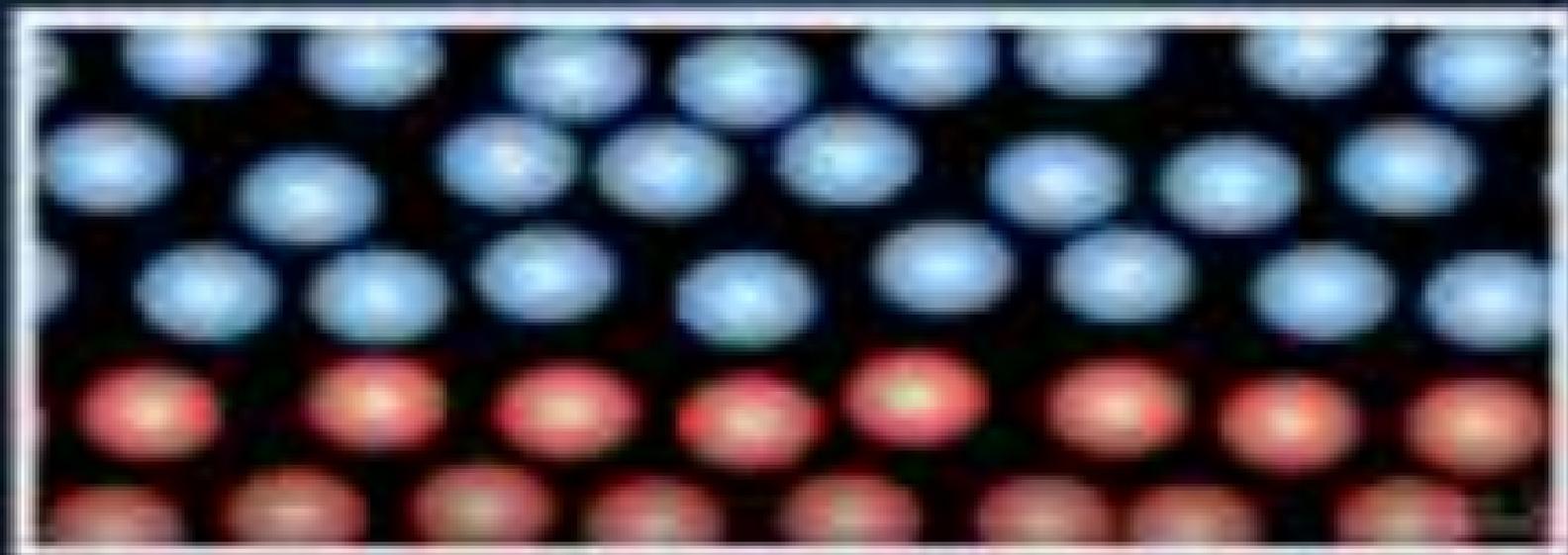
Work done
expansion

Heat added
expansion



HEAT – A MICROSCOPIC VIEW

<https://www.youtube.com/watch?v=5A3mM0QNA>



• <https://www.youtube.com/watch?v=3LQW7Xv01GA>

INTERNAL ENERGY & ENTHALPY

- Internal Energy of an ideal gas: the sum of the kinetic energies of the particles in the gas.



INTERNAL ENERGY & ENTHALPY

- Enthalpy: The sum of the **internal energy** of the system **plus** the **product of the pressure of the gas in the system times the volume of the system.**
 - Enthalpy is the amount of energy that is transferred into a system at constant **boundary by a moving line.**
 - This energy is composed of two parts: the internal energy of the fluid (u) and the **flow work** (pv) associated with pushing the mass of fluid across the system boundary.

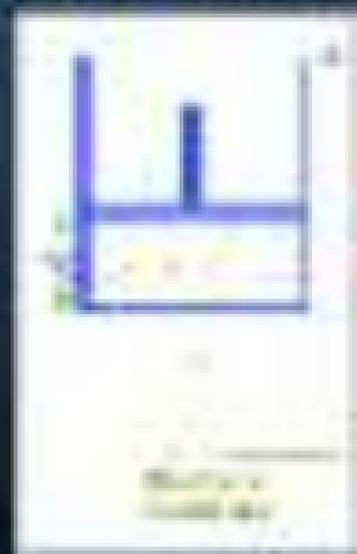
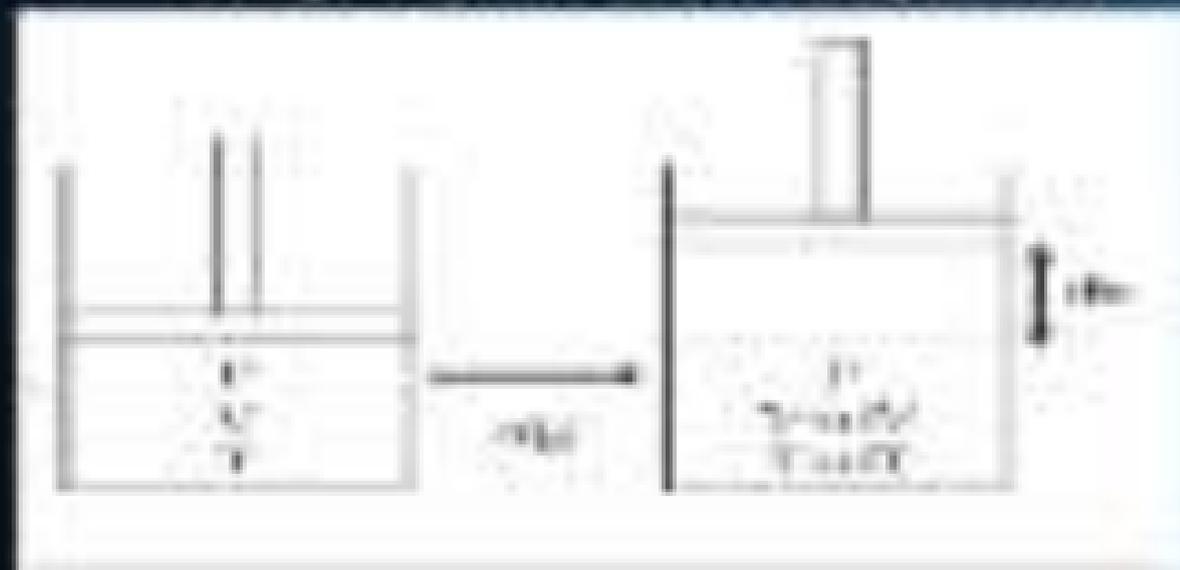
$$h = u + pv$$

$$H_{\text{sys}} = E_{\text{sys}} + PV$$



CP VS CV: WHY $CP > CV$

- **Specific heat:** Thermal energy added per unit mass, for 1 K rise in temperature
 - A material property
 - Depends on whether the process is at constant pressure or constant volume



ENERGY EQUATION

- Concept: **Conservation of Energy**

- Statement as applicable to flow systems:

Rate of **heat added to the fluid** inside the control volume from the surroundings + Rate of **work done on the fluid** inside the control volume = Rate of change **of the energy of the fluid** as it flows through the control volume

FIRST LAW OF THERMODYNAMICS WITH REYNOLDS TRANSPORT THEOREM

$$\frac{dQ}{dt} - \frac{dW}{dt} = \frac{dE}{dt}$$

- Volumetric addition of thermal energy due to chemical reactions etc.

$$- \int_{T_1}^{T_2} q_p dT$$

- **Slide 107** - Work done by the shaft is just present form of the mechanical energy and body force



- **NOTE:** WE ARE **NOT** considering **MECHANICAL work** or **shaft work** which is present in applications like turbine & compressor

- Rate of change of energy as fluid flows through the CV = Total heat change of energy within the CV due to molecular effects + Net energy flux across the control surface.

$$\frac{d}{dt} \int_{CV} \rho e \left(v + \frac{v^2}{2} \right) dV + \int_{CS} \rho \mathbf{v} \cdot \mathbf{N} \cdot h \left(v + \frac{v^2}{2} \right)$$

$$e + \frac{1}{2} v^2$$

Source: <http://www.khanacademy.org/a/thermodynamics-101>

ENERGY EQUATION: INVISCID FLOW, WITHOUT SHEAR WORK

$$\begin{aligned}
 \frac{d}{dt} \int_V \rho e dV &= \frac{d}{dt} \int_V \rho v \cdot dS + \frac{d}{dt} \int_V \rho (e + \frac{v^2}{2}) dV \\
 &= \frac{d}{dt} \int_V \frac{\partial}{\partial t} \left[\rho \left(e + \frac{v^2}{2} \right) \right] dV + \frac{d}{dt} \int_V \rho \left(e + \frac{v^2}{2} \right) v \cdot dS
 \end{aligned}$$

STEADY ONE-DIMENSIONAL FLOW

- Why?

- Analyze processes easier
- Provides useful information and insight about processes, **parameters that govern the flow**
- Commonly used as a design tool
- More analysis, particularly by **engineers** than for replacement analysis, using **flow-mass-balance equations**

ONE-DIMENSIONALITY OF FLOW

- **Flow** in which parameters (velocity, pressure, density, viscosity and temperature) vary only in one direction and the flow is a function of only one coordinate (x or z). The flow field is approximated by streamlines that are straight and parallel.
- **flow velocity, no component normal to section**



ONE-DIMENSIONAL CONTINUITY EQUATION

- Recall the general form:

$$\frac{d}{dt} \int_{CV} \rho \, dV + \int_{CS} \rho \mathbf{V} \cdot d\mathbf{A} = \int_{CV} \dot{\rho} \, dV$$

- **Steady assumption** removes the volume integral (as there is no variation with time):
- The surface integral can be replaced by **pointwise mass flow** (due to uniformity):
- In one-dimensional flow, the volume flow rate is the same at all cross-sectional areas:

$$\rho_1 u_1 A_1 = \rho_2 u_2 A_2$$

$$\rho_1 u_1 = \rho_2 u_2$$

MOMENTUM EQUATION

$$\frac{d}{dt} \int_{CV} \rho \mathbf{V} dV = \sum_{CS} \rho \mathbf{V} \mathbf{V} \cdot \mathbf{n} dA + \sum_{CS} \rho \mathbf{V} \mathbf{V} \cdot \mathbf{n} dA$$

→ The velocity \mathbf{V} can be either locally known

$$\frac{d}{dt} \int_{CV} \rho \mathbf{V} dV = \sum_{CS} \rho \mathbf{V} \mathbf{V} \cdot \mathbf{n} dA$$

- The new decomposition \mathbb{R}^n satisfies the same equation

$$\frac{d}{dt} \int_{\mathbb{R}^n} \psi \, dV + \int_{\mathbb{R}^n} \psi \operatorname{div} v = - \int_{\mathbb{R}^n} \psi \operatorname{div} u$$

To

$$\frac{d}{dt} \int_{\mathbb{R}^n} \psi \, dV + \int_{\mathbb{R}^n} \psi \operatorname{div} v = - \int_{\mathbb{R}^n} \psi \operatorname{div} u$$



$$\frac{d}{dt} \int_{\mathbb{R}^n} \psi \, dV + \int_{\mathbb{R}^n} \psi \operatorname{div} v = - \int_{\mathbb{R}^n} \psi \operatorname{div} u$$

ONE-DIMENSIONAL MOMENTUM EQUATION

STEADY, INVISCID FLOW WITHOUT BODY FORCES BY CONSTANT AREA PASSAGE

$$p_1 + \rho_1 u_1^2 = p_2 + \rho_2 u_2^2$$

THE ENERGY EQUATION

$$\frac{d}{dt} \int_{CV} \rho u^2 + \frac{1}{2} v^2 + \frac{1}{2} \omega^2 = \dot{Q} - \dot{W}_s$$

$$= \int_{CS} \rho \left(u^2 + \frac{v^2}{2} + \frac{\omega^2}{2} \right) \mathbf{V} \cdot d\mathbf{A}$$

- Steady flow without body forces:

$$\rho \frac{d}{dt} \int_V \mathbf{v} \, dV = \rho \int_V \left(\mathbf{v} + \frac{dV}{dt} \right) \cdot \mathbf{n} \, dA$$

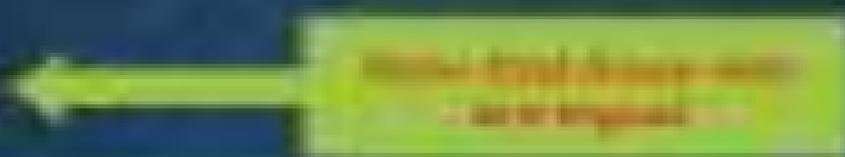
- Using 1-D assumption, the surface integrals get transformed as:

$$\rho \frac{d}{dt} \int_V \mathbf{v} \, dV = \rho \int_V \left(\mathbf{v} + \frac{dV}{dt} \right) \cdot \mathbf{n} \, dA = \rho \int_V \left(\mathbf{v} + \frac{dV}{dt} \right) \cdot \mathbf{n} \, dA$$

→ Dividing by

z^2

z^2



$$0 = \frac{r_1}{z} + r_2 + r_3 z + r_4 z^2 + r_5 z^3 + r_6 z^4 + r_7 z^5 + r_8 z^6 + r_9 z^7 + r_{10} z^8 + r_{11} z^9 + r_{12} z^{10}$$

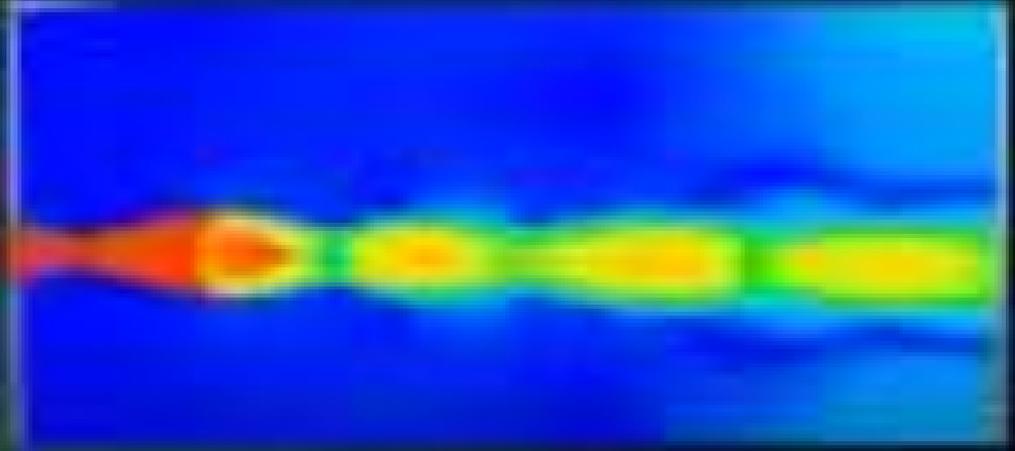
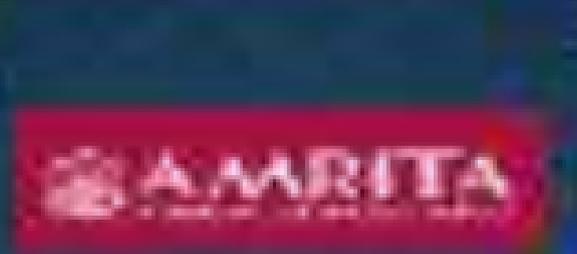
$$r_1 + \frac{r_2}{z} + r_3 + r_4 z + r_5 z^2 + r_6 z^3 + r_7 z^4 + r_8 z^5 + r_9 z^6 + r_{10} z^7 + r_{11} z^8 + r_{12} z^9 = 0$$

USE THE CONTINUITY EQUATION FOR
STEADY, INVISCID COMPRESSIBLE FLOW
WITHOUT BODY FORCES IN CONSTANT AREA
DUET

$$\rho_1 u_1 = \rho_2 u_2$$

$$p_1 + \rho_1 u_1^2 = p_2 + \rho_2 u_2^2$$

$$h_1 + \frac{u_1^2}{2} + q = h_2 + \frac{u_2^2}{2}$$



COMPRESSIBLE FLUID FLOW

Laws of Thermodynamics &
nonlinear processes

THE CONTEXT

- Reminder: **Coupling** between **flow & energy** in compressible (high velocity) **flows**
- Flow: Governed by the **conservation equations**
- Energy: Analysed using the tools of **thermodynamics**
- Conservation equations are considered already
- Next: A quick review of how the **1st & 2nd law** lead to **useful integral relations** that **govern compressible flow physics**

THE FIRST & THE SECOND LAWS OF TD

- First law as applied to an open system: Considering incremental processes, the heat added to, work done on and the change in energy of a system are related as:

$$dq + dw = de$$

- Second law leads to the definition of entropy, as

$$dq = T ds$$

- For real processes,

$$de = \frac{dq}{T} + d\psi_{irr}$$

REVERSIBILITY & DISSIPATIVE PHENOMENA

- **Reversible process**—one in which **no dissipative phenomena occur**
 - Where the effects of entropy, thermal conductivity, and viscosity are ignored

- **Consistency of treatment with KKKL:** $\delta Q = T ds$ <https://www.youtube.com/watch?v=U1111111111>

- **Derive for all** $\delta Q = T ds$

- In general:

$$\delta Q = T ds + \sum_i \delta x_i$$

$$\delta Q_{rev} = T ds$$

- And for **ADIABATIC** processes:

$$\delta Q = 0$$

→ Hence,

$$T ds = -\sum_i \delta x_i$$

COMBINING THE FIRST & THE SECOND LAWS...

• First law:

$$dU = \delta Q - \delta W$$

• For reversible process (where $\delta W = p dV$), with no shaft work $\delta W_{sh} = 0$

Heat transfer $\delta Q = T ds$ (reversible)

$$T ds = \delta Q = dU + p dV$$



$$dU = T ds + p dV + \delta W_{sh}$$

$$T ds = dU + p dV$$

FOR PERFECT GAS...

- This simplification of perfect gas assumption (γ) is constant. Hence:
 $dh = Cv dt$

- This leads to $dh = Cv \frac{dT}{T} = \frac{Cv dT}{T}$

- Using $p v = R T$

$$dT = v \frac{dp}{p} = \frac{R}{Cv} \frac{dp}{p}$$

- Integrating: $dh = h_2 - h_1 = \int_1^2 Cv \frac{dT}{T} = Cv \ln \frac{T_2}{T_1}$

$$\left| \frac{h_2 - h_1}{Cv} = \ln \frac{T_2}{T_1} \quad \text{or} \quad \frac{h_2 - h_1}{Cv} = \ln \frac{p_2}{p_1} \right|$$

ISENTROPIC PROCESSES

- Entropy remains **Constant**: $ds = 0$

$$\begin{aligned} ds &= \frac{1}{T} du + \frac{p}{T} dv \\ ds &= \left(\frac{du}{T}\right)_{dv} \end{aligned}$$

$$0 = \frac{1}{T_2} du + \frac{p}{T_2} dv = \frac{1}{T_1} du + \frac{p}{T_1} dv$$

$$\frac{T_2}{T_1} = \left(\frac{v_2}{v_1}\right)^{\gamma} \quad \text{11}$$

REVERSIBLE AND
ADIABATIC

PHYSICAL RELEVANCE

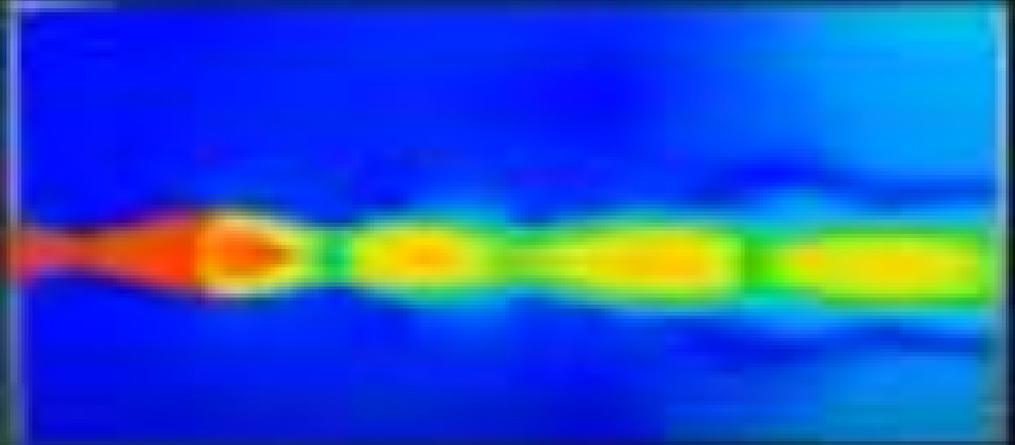
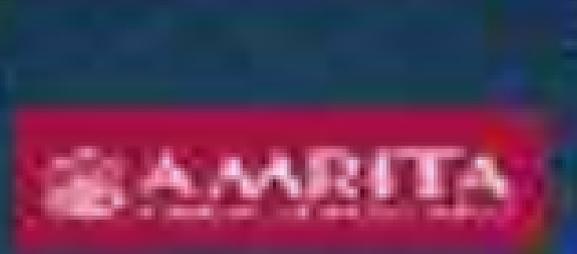
- What is a Reversible Adiabatic Process?
- When is a process reversible?
- When is a process adiabatic?
- Why is it important in MECHANICAL APPLICATIONS?
- Where will WE use them in this Course?

VIDEO: **ENTROPY I**

- https://www.youtube.com/watch?v=YM-uykVtq_E



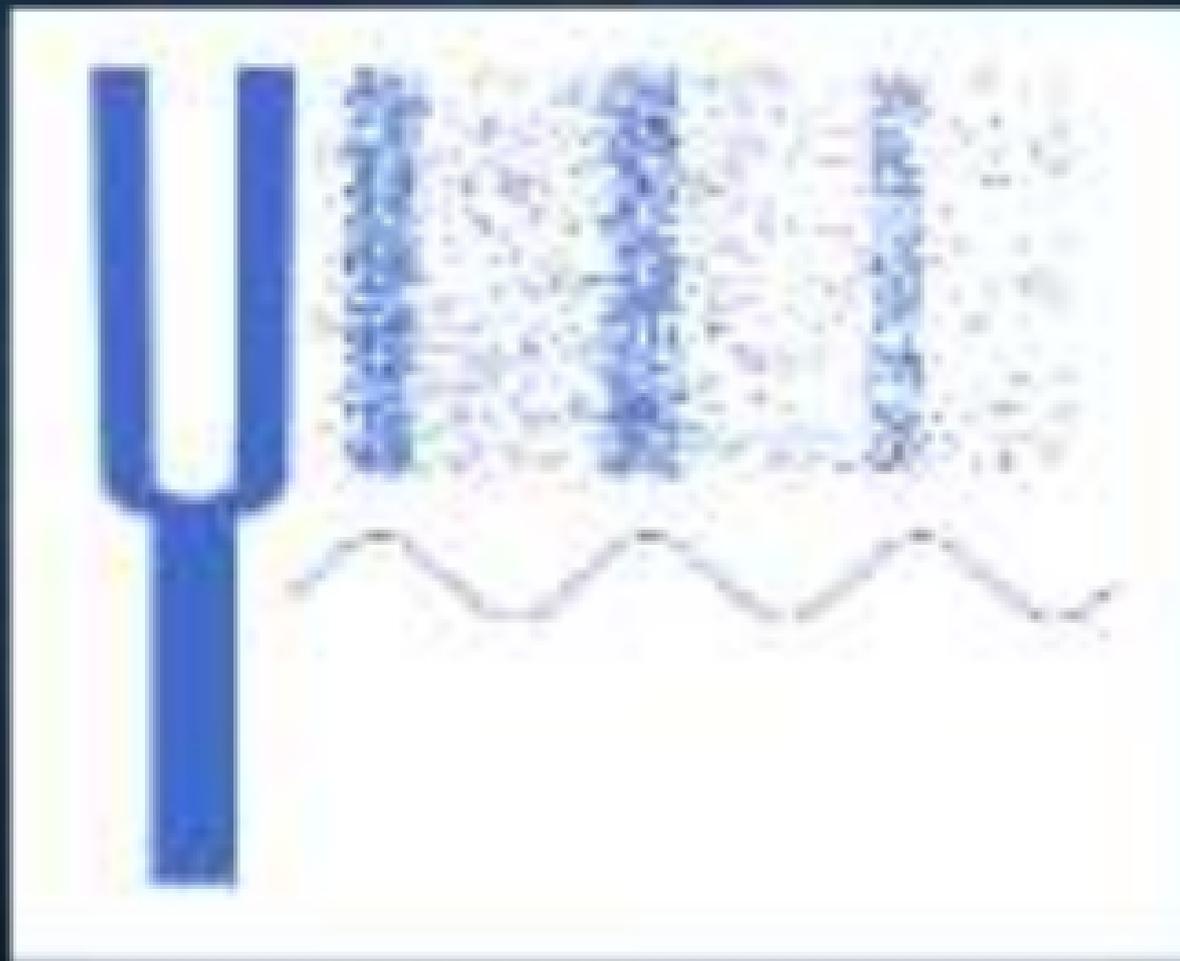
*Order
Order !!!*

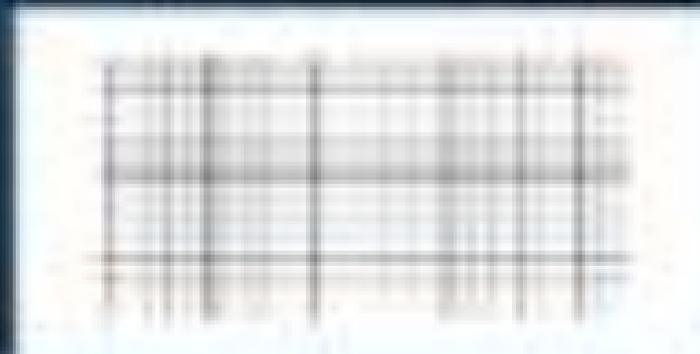
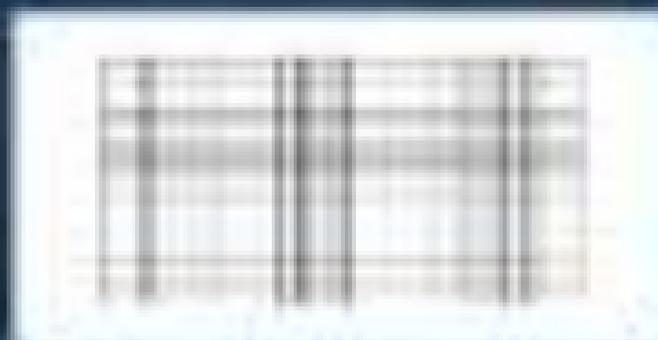


COMPRESSIBLE FLUID FLOW

Geometries-03

Compressibility & The Velocity of Sound





COMPRESSIBILITY

- **Compressibility** is a measure of the relative volume change of fluid as solid as a response to a pressure (or mean stress) change
 - the ability of material to be solid as to compressed (under mean stress) and their ability to recover back to their original density

$$\beta = -\frac{1}{v} \frac{dv}{dp}$$

The reciprocal
of density

$$\kappa = \frac{1}{\rho} \frac{d\rho}{dp}$$

- Note, v is volume here (NOT velocity)
- The definition refers to the isotropic pressure of compression
- Negative sign indicates the decrease in volume with increase in pressure
- The inverse of compressibility is **bulk modulus**

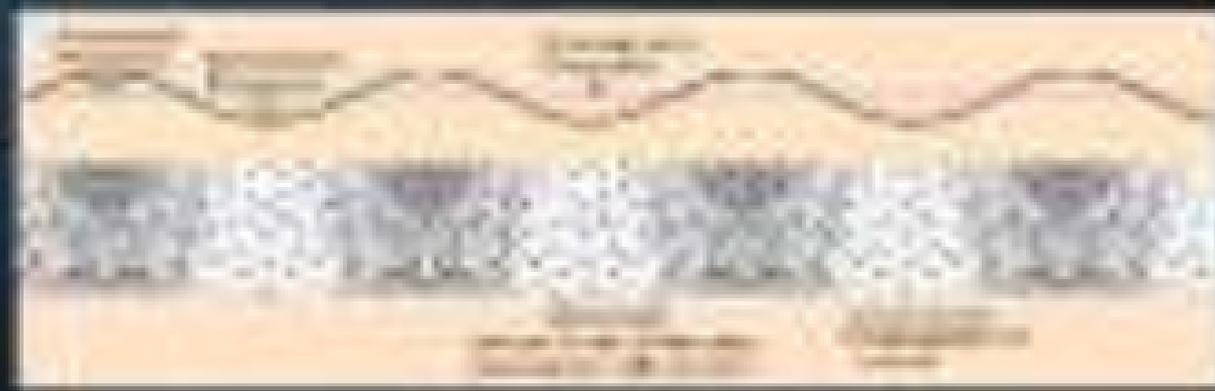
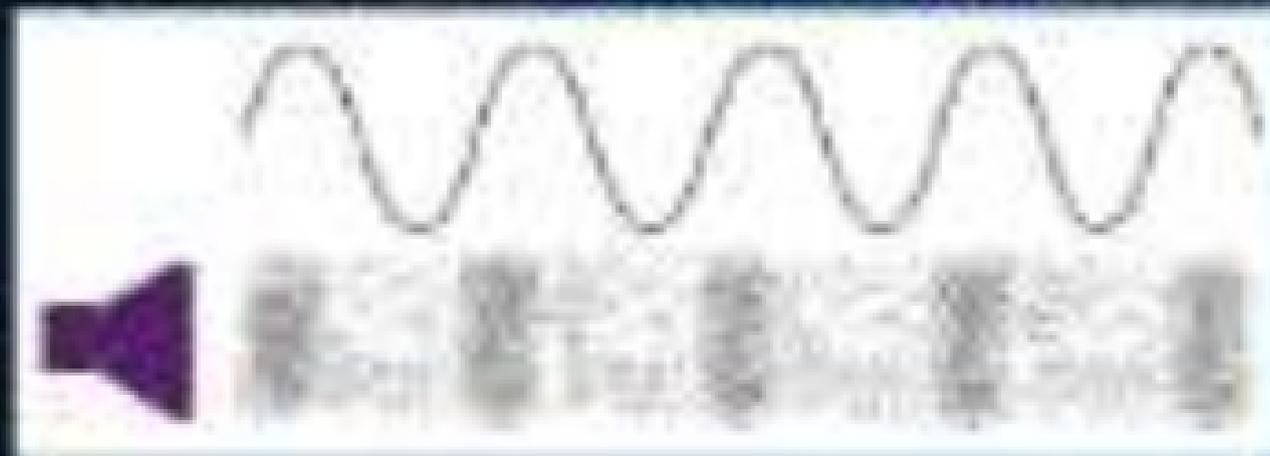
COMPRESSIBILITY: GASES VS LIQUIDS

	Compressibility (m ³ /N)
Water	5×10^{-10}
Air	10^{-5}

Implies the significant variation
in density with pressure in
gases

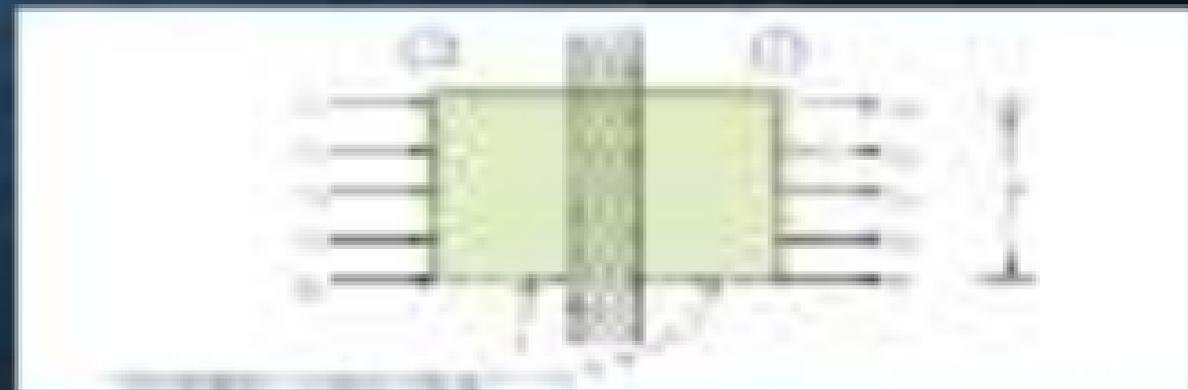
PROPAGATION OF SOUND

→ Sound travels as **SOUND WAVES** in air

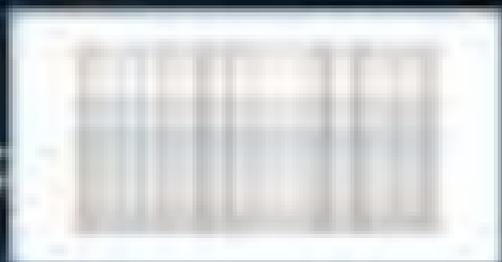


ONE-DIMENSIONAL PROPAGATION

- Variations across the coupled volume:



VARIATIONS ACROSS THE WAVE



- Charles I. Darwin's original propagation of a pressure wave
- **Separable, adiabatic propagation**
- The speed of sound depends on a reflecting a through the gas
- The pressure of the wave tends to change in proportion of the gas. This is because velocity $\propto \sqrt{p/\rho}$
- **Viewing from the relative speed of the wave,**
the gas moves with a velocity v and acceleration a that
 - Other gas properties are not necessarily



- Assuming constant mass, the continuity equation yields

$$\rho_0 \frac{dV}{dt} = \rho_0 \int_V \frac{dV}{dt} = \rho_0 \int_V \nabla \cdot \mathbf{u} dV$$

$$\rho_0 \frac{dV}{dt} = \rho_0 \int_V \nabla \cdot \mathbf{u} dV = \rho_0 \int_V \nabla \cdot \mathbf{u} dV$$

- Neglecting products of differentiated vectors

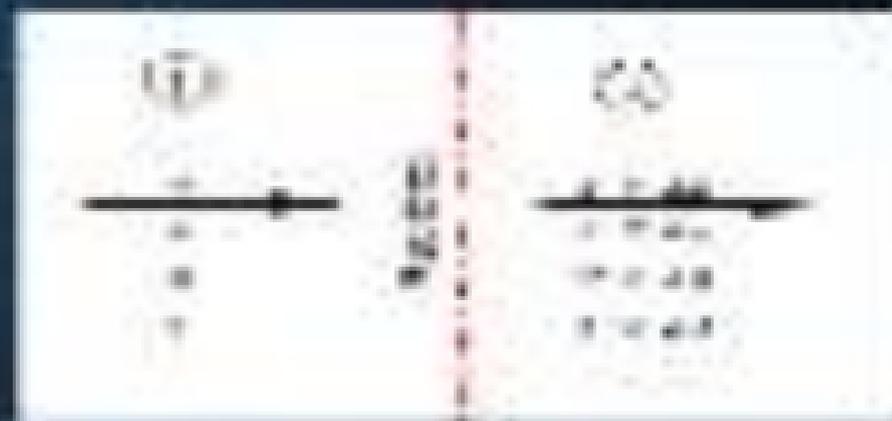
$$\rho_0 \frac{dV}{dt} = \rho_0 \int_V \nabla \cdot \mathbf{u} dV$$

- Measurement equation yields

$$\rho_0 \frac{dV}{dt} = \rho_0 \int_V \nabla \cdot \mathbf{u} dV = \rho_0 \int_V \nabla \cdot \mathbf{u} dV = \rho_0 \int_V \nabla \cdot \mathbf{u} dV$$

$$\rho_0 \frac{dV}{dt} = \rho_0 \int_V \nabla \cdot \mathbf{u} dV = \rho_0 \int_V \nabla \cdot \mathbf{u} dV$$

$$\rho_0 \frac{dV}{dt} = \rho_0 \int_V \nabla \cdot \mathbf{u} dV = \rho_0 \int_V \nabla \cdot \mathbf{u} dV$$



- Substitution

$$u = \sqrt{a^2 + b^2 - x^2}$$

- & solving for x^2

$$u^2 = a^2 + b^2 - x^2$$

- Since the whole process is **completely straightforward**

$$u^2 = \left(\frac{b^2}{a^2}\right) x^2$$

FOR ISENTROPIC FLOW OF A PERFECT GAS

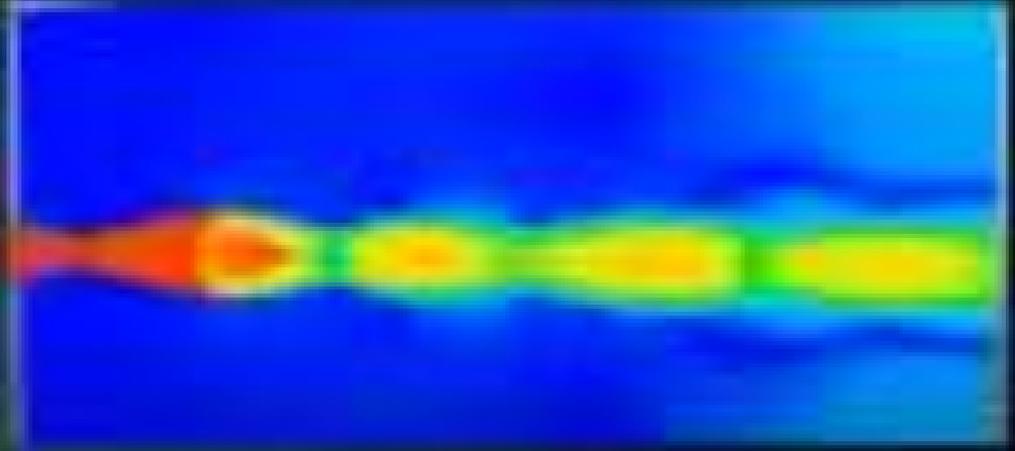
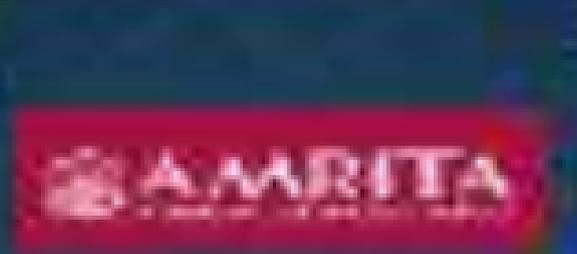
The isentropic process: $\frac{p}{\rho^\gamma} = \text{constant}, C \Rightarrow \frac{p}{\rho^\gamma} = C \Rightarrow \rho = C^{1/\gamma} p^{1/(\gamma-1)}$

$$a = \sqrt{\frac{\gamma p}{\rho}}$$

$$a = \sqrt{\gamma R T}$$

The
Velocity of
Sound in a
Perfect Gas

http://www.ck12.org



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Casecode-05

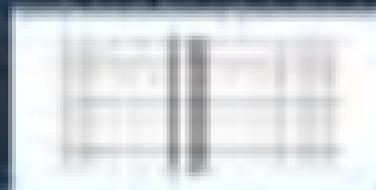
Mach Number

COMPRESSIBILITY & VELOCITY OF SOUND

- Remember the definition of compressibility:

$$\kappa = \frac{1}{\rho} \frac{d\rho}{dp}$$

- Remember the expression that we derived for the velocity of sound:



$$v = \left(\frac{\gamma P}{\rho} \right)^{1/2}$$

$$v^2 = \frac{\gamma P}{\rho}$$

Note the impact of compressibility

on the speed of sound through the medium.

MACH NUMBER

- The **MOST** important parameter in Compressible flow analysis
- Definition: **Velocity of the fluid / Velocity of sound through the fluid**

$$M = \frac{V}{a}$$

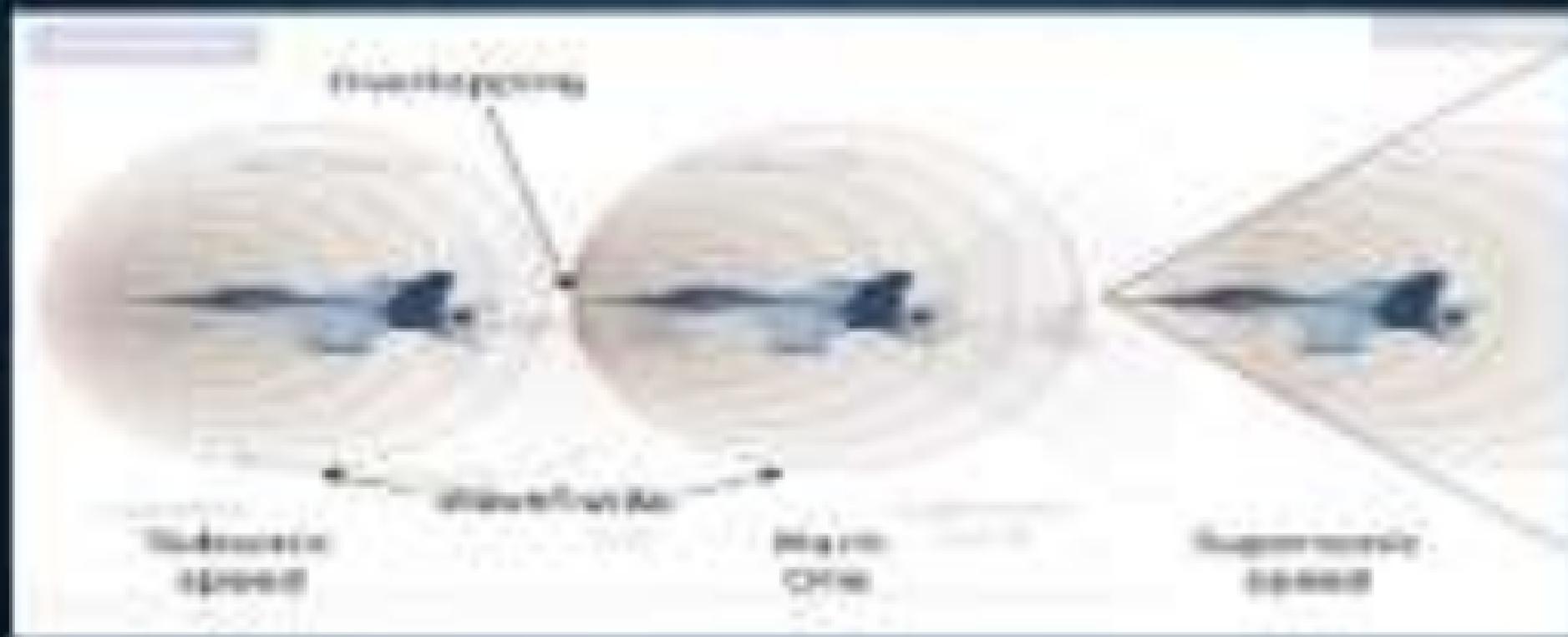
- The physical significance ? - A measure of the relative magnitude of kinetic energy and the internal energy

ERNST MACH

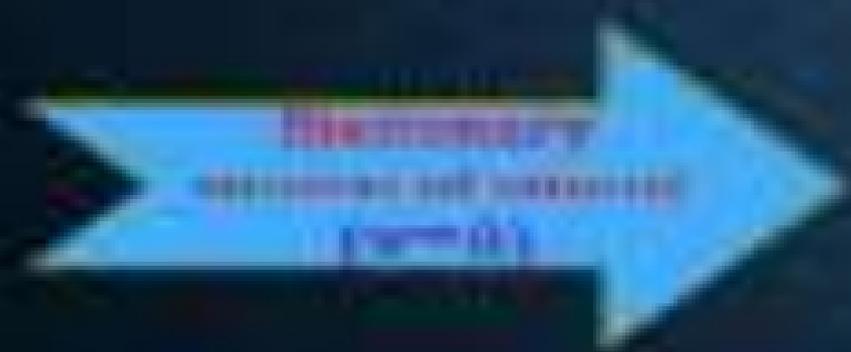
- Austrian Physicist, Philosopher
- Pioneering studies in compressible flows



SUBSONIC SONIC SUPERSONIC

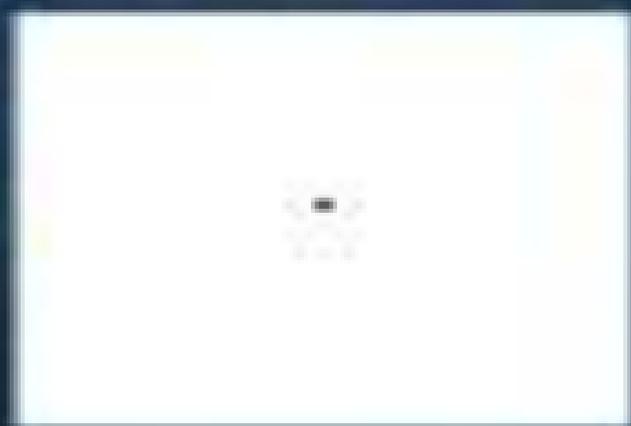


FLUID VELOCITY VS SOUND VELOCITY



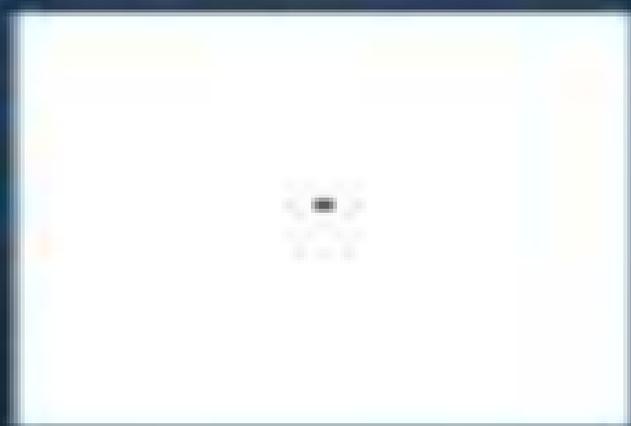
Flow is fully developed

FLUID VELOCITY VS SOUND VELOCITY



Wavefronts of sound waves

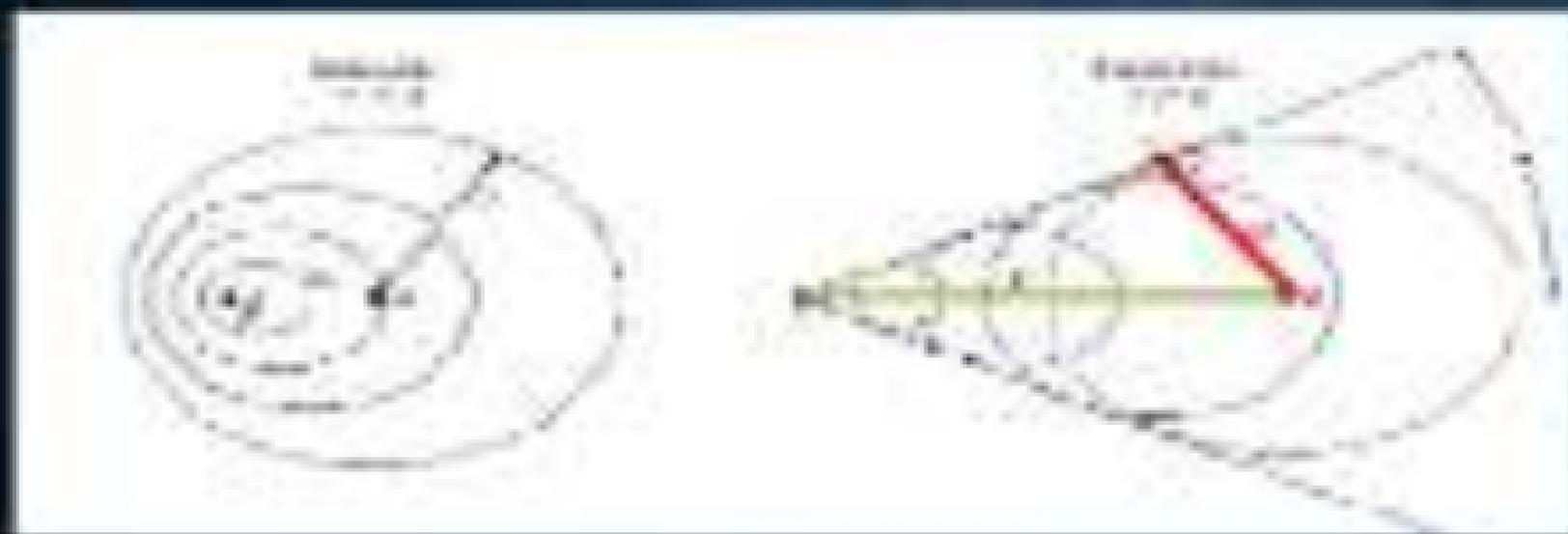
addition of a single
amino acid (methionine)
residues (e.g. Met-His)



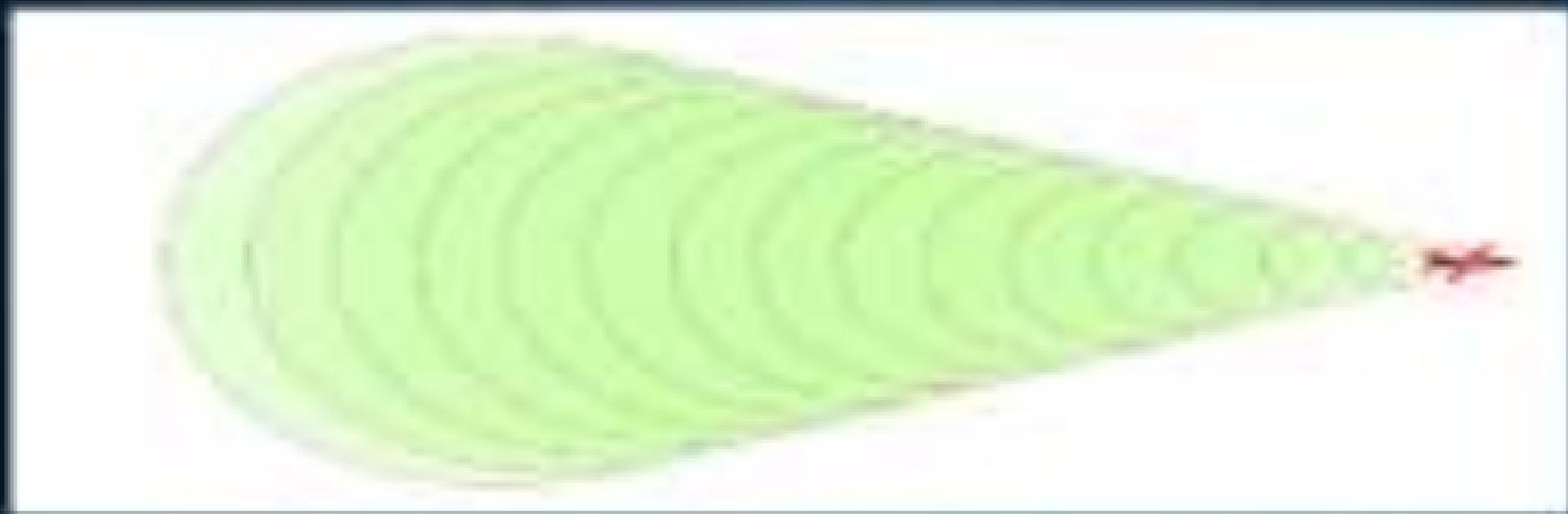
Met-His is a highly conserved
residue

MACH CONE & MACH ANGLE

- Consider the reach of waves created by a body moving at supersonic velocity.



$$\mu = \sin^{-1} \left(\frac{1}{M} \right)$$



TIDE RANGE: CAN YOU HEAR THE NOISE?

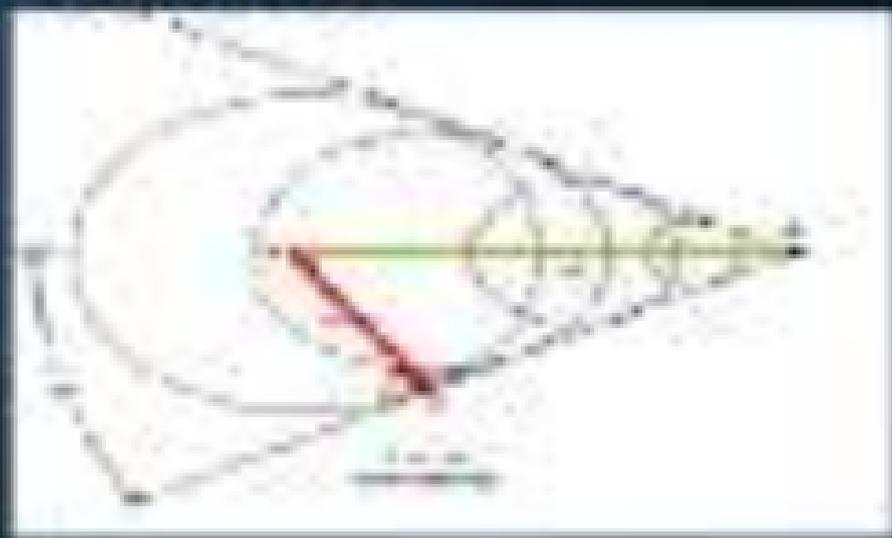


THE SONIC BARRIER & SONIC BOOM

→ <https://www.youtube.com/watch?v=U1DyKED34r0&list=PL0B319E3609599028>

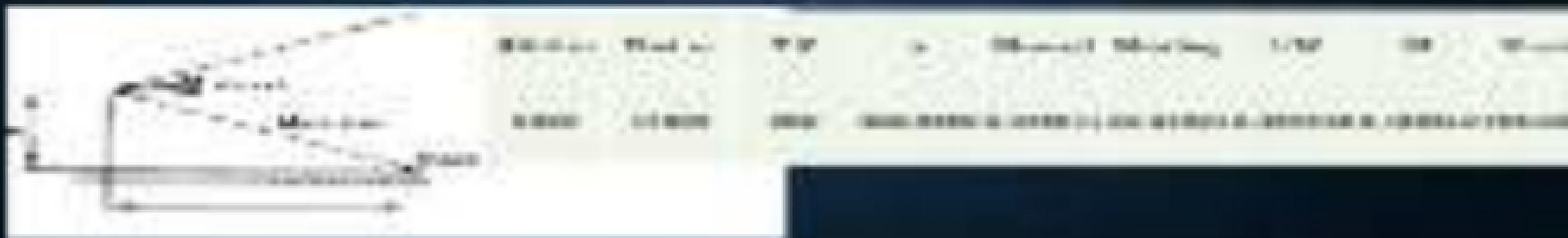
→ <https://www.youtube.com/watch?v=U1DyKED34r0&list=PL0B319E3609599028>

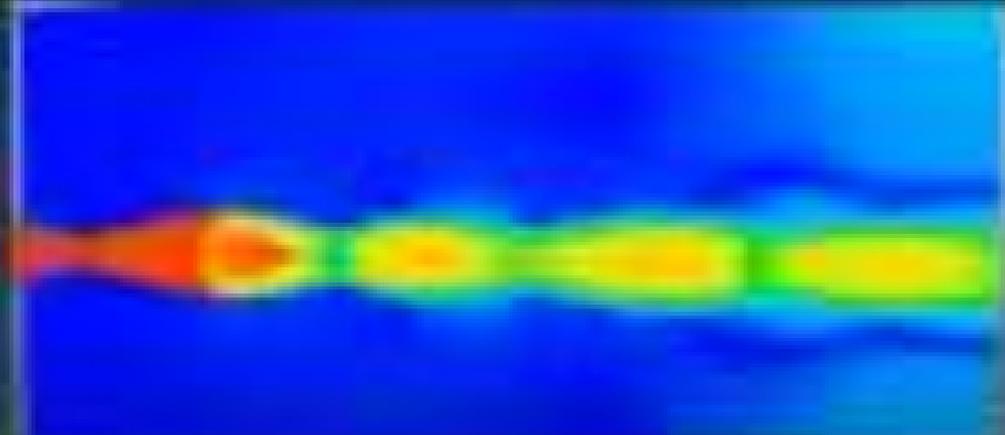
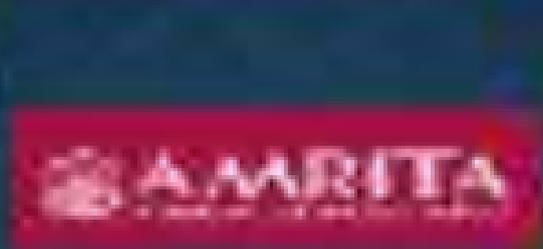
THE DISTANCE, THE ALTITUDE, THE MACH CONE & THE MACH NUMBER



NUMERICAL PROBLEM

- As shown in the given figure the total of an unsteady flow is maintained by the intake of MFR in which it has reached 3 L in the total this corrected position. Estimate the speed at which the cylinder is flying, assuming it is in a steady pressure condition. Use an average temperature of 20°C.





COMPRESSIBLE FLUID FLOW

Sections-31

Stagnation Properties &
Definition of COMPRESSIBLE FLOW

THE STAGNATION STATE

- Corresponds to the state when the flow is brought to zero velocity **without dissipation of energy**
 - Kinetic energy is completely converted into internal energy
- Stagnation properties are defined as those corresponding to the stagnation state of a fluid under the specified conditions
 - Take those of air in a storage tank

STAGNATION STATE & STORAGE CONDITIONS



STAGNATION PROPERTIES VS STATIC PROPERTIES

- **Static pressure:** Pressure (P) at a point in a flow where the velocity is 0
- **Stagnation pressure (P_0):** The pressure that would result **if** the flow is brought to rest in a **reversible, adiabatic process**
- **Static & Stagnation properties will be IDENTICALLY LINKED** in **high speed flows**

ENERGY EQUATION

- Steady, one-dimensional **control volume** **analysis** of mechanical work & heat transfer

$$\dot{m} \left(h_1 + \frac{V_1^2}{2} \right) = \dot{m} \left(h_2 + \frac{V_2^2}{2} \right) + \dot{Q}$$

- For **perfect gas**,

$$\dot{m} \left(c_p T_1 + \frac{V_1^2}{2} \right) = \dot{m} \left(c_p T_2 + \frac{V_2^2}{2} \right) + \dot{Q}$$

$$\frac{\dot{m} c_p T_1}{\dot{m} c_p T_2} = \frac{T_1}{T_2} = \frac{c_p T_1 + \frac{V_1^2}{2}}{c_p T_2 + \frac{V_2^2}{2}}$$

$$h_2 = h_1 + \frac{u_1^2}{2}$$

STAGNATION TEMPERATURE & MACH NUMBER

- If state 2 corresponds to stagnation (i.e. $u_2 = 0$ in the above equation) $\rightarrow T_2$ is stagnation temperature

$$h_2 = h_1 + \frac{u_1^2}{2}$$

$$\frac{h_2}{T} = 1 + \frac{u_1^2}{2c_p T} = 1 + \frac{u_1^2}{2\gamma R T (\gamma - 1)} = 1 + \frac{u_1^2}{2a^2 (\gamma - 1)}$$

$$\frac{T_2}{T} = 1 + \frac{\gamma - 1}{2} M^2$$

STAGNATION PRESSURE

- For an isentropic process, conditions at stagnation point are 0 velocity

$$\frac{p_0}{p} = \left(\frac{\rho_0}{\rho}\right)^{\gamma} = \left(\frac{T_0}{T}\right)^{\frac{\gamma}{\gamma-1}}$$

- 21 pages

$$\frac{p_0}{p} = \left(1 + \frac{\gamma-1}{2} M^2\right)^{\frac{\gamma}{\gamma-1}}$$

$$\frac{p_0}{p} = \left(1 + \frac{\gamma-1}{2} M^2\right)^{\frac{\gamma}{\gamma-1}}$$

STATIC VS STAGNATION: HOW DO THE VALUES COMPARE ?

- A car moving at 100km/h
- A low speed passenger aircraft
- A Cruise Ship (Traveling 20k)
- A supersonic jet - **MACH 2**



FEEL THE DIFFERENCE !

Calculations for Mach 10 Temperature $T = 200 \text{ K}$,
 Static Pressure $P = 1 \text{ atm}$

V_{∞} km/h	u_{∞} m/s	ρ_{∞}	Dynamic Pressure q_{∞}	Stagnation Pressure p_{01}
300	82	0.001225	3.42	1.004
1000	278	0.001225	38.1	1.009
3000	838	0.001225	347	1.127
10000	2778	0.001225	3810	1.520
25000	6944	0.001225	15000	2.414
50000	13889	0.001225	60000	3.140
100000	27778	0.001225	240000	3.791

TITLE DENSITY

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$$\frac{\rho}{\rho_0} = \left(1 - \frac{\gamma - 1}{2} M^2\right)^{\frac{\gamma}{\gamma - 1}}$$

INCOMPRESSIBLE FLOW

Conventionally

calculated as **$M \rightarrow 0.3$**

M	% Variation
0.0634	0.20
0.0864	0.37
0.3024	4.64
0.46011	10.98
0.8641	41.62
1.8722	277.37
3.0243	1246.45
5.7606	15016.80

NOTES ON COMPRESSIBLE FLOW

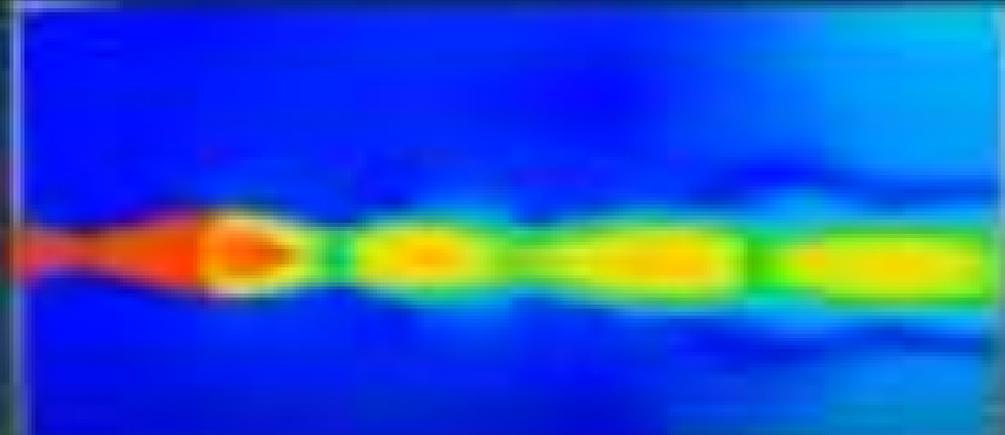
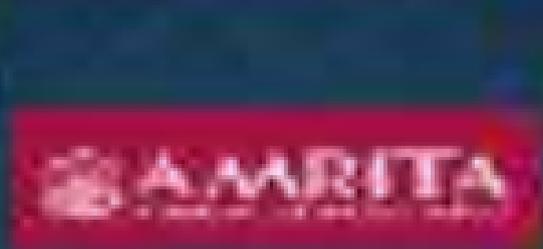
- The effect of compressibility gradually increases with Mach number
- Compressible flow as $M > 0.3$ is conventionally followed for convenience
 - Nothing happens dramatically at $M=0.3$, it's just a conventional limit
- Note the difference in the implications of subsonic vs transonic flow

CHARACTERISTIC TEMPERATURE

- Consider a flow at some velocity V , pressure P , Temperature T etc.
- What if the flow is accelerated/decelerated, **adiabatically**, to sonic velocity ($V = a$)
 - The Mach number becomes unity ($M = 1$)
- The resulting static temperature is called **characteristic temperature T^***
- The corresponding velocity of sound is a^*
- That is $a^* = \sqrt{\gamma R T^*}$
- A **NEW** Mach number is defined as $M^* = V/a^*$
- Note the difference between the definition of **M** (local Mach number) and **M^*** (characteristic Mach number)

CALCULATION OF M^* : EXAMPLE

- Determine the Mach number (M) and the critical Mach number (M^*) for the flow of air at a velocity of 850 m/s and static temperature = 300 K.
 - $M = 850/\text{SQRT}(1.4 \times 287 \times 300)$
 - $M^* = 850/\text{SQRT}(1.4 \times 287 \times T^*)$
 - $T^*/T = (1 + (\text{gamma} - 1) \times 0.5 \times M^2) / (1 + (\text{gamma} - 1) \times 0.5)$
 - Calculate T^* and then $M^* = V/\text{SQRT}(\text{gamma} \times R \times T^*)$



COMPRESSIBLE FLUID FLOW

Sections-31

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STAGNATION STATE & STORAGE CONDITIONS



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- **Static pressure:** Pressure (P) at a point in a flow where the velocity is 0
- **Stagnation pressure (P_0):** The pressure that would result **if** the flow is brought to rest in a **reversible, adiabatic process**
- **Ratio of stagnation properties will be SIGNIFICANTLY DIFFERENT** in high speed flows

ENERGY EQUATION

- Steady, one-dimensional **control volume** **analysis** of mechanical work & heat transfer

$$\dot{m} \left(h_1 + \frac{V_1^2}{2} \right) = \dot{m} \left(h_2 + \frac{V_2^2}{2} \right)$$

- For **perfect gas**,

$$\dot{m} \left(c_p T_1 + \frac{V_1^2}{2} \right) = \dot{m} \left(c_p T_2 + \frac{V_2^2}{2} \right)$$

$$\frac{c_p T_1}{R} + \frac{V_1^2}{2} = \frac{c_p T_2}{R} + \frac{V_2^2}{2}$$

$$h_2 - h_1 = c_p(T_2 - T_1)$$

STAGNATION TEMPERATURE & MACH NUMBER

- If state 2 corresponds to stagnation (i.e. $u_2 = 0$ in the above equation) $\rightarrow T_2$ is stagnation temperature

$$c_p T_2 = \frac{c_p T_1}{1} + \frac{c_p u_1^2}{2}$$

$$\frac{T_2}{T_1} = 1 + \frac{u_1^2}{2c_p T_1} = 1 + \frac{u_1^2}{2\gamma R T_1 (\gamma - 1)} = 1 + \frac{u_1^2}{2a_1^2 (\gamma - 1)}$$

$$\frac{T_2}{T_1} = 1 + \frac{\gamma - 1}{2} M_1^2$$

STAGNATION PRESSURE

- For an isentropic process, conditions at any point are related to stagnation conditions by

$$\frac{p_0}{p} = \left(\frac{T_0}{T}\right)^{\frac{\gamma}{\gamma-1}} = \left(\frac{\rho_0}{\rho}\right)^{\frac{\gamma}{\gamma-1}}$$

- or

$$\frac{p_0}{p} = \left(1 + \frac{\gamma-1}{2} M^2\right)^{\frac{\gamma}{\gamma-1}}$$

$$\frac{p_0}{p} = \left(1 + \frac{\gamma-1}{2} M^2\right)^{\frac{\gamma}{\gamma-1}}$$

STATIC VS STAGNATION: HOW DO THE VALUES COMPARE ?

- A car moving at 100km/h
- A low speed propeller aircraft
- A Cruise Ship (around 30k)
- A supersonic jet - **MACH**



FEEL THE DIFFERENCE !

Calculations for Mach 10 Temperature $T = 200 \text{ K}$,
 Static Pressure $P = 1 \text{ atm}$

V_{∞} km/h	u_{∞} m/s	ρ_{∞}	Temperature Reynolds number Re	Stagnation Temperature T_0
300	83	0.00012	100.2	1.000
1000	278	0.00012	100.4	1.000
3000	833	0.00012	111	1.127
10000	2778	0.00012	141	1.520
25000	6944	0.00012	350	6.414
50000	13889	0.00012	700	26.140
100000	27778	0.00012	1400	104.000

TITLE DENSITY

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$$\frac{\rho}{\rho_0} = \left(1 - \frac{\gamma - 1}{2} M^2\right)^{\frac{\gamma}{\gamma - 1}}$$

INCOMPRESSIBLE FLOW

Conventionally

calculated as **$M \rightarrow 0.3$**

M	% Variation
0.0634	0.20
0.0864	0.37
0.3024	4.64
0.46011	10.98
0.8641	41.62
1.8722	277.37
3.0243	1246.45
5.7606	15016.80

NOTES ON COMPRESSIBLE FLOW

- The effect of compressibility gradually increases with Mach number
- Compressible flow as $M > 0.3$ is conventionally followed for convenience
 - Nothing happens dramatically at $M=0.3$, it's just a convenient reference point
- Note the difference in the implications of subsonic vs transonic flow

CHARACTERISTIC TEMPERATURE

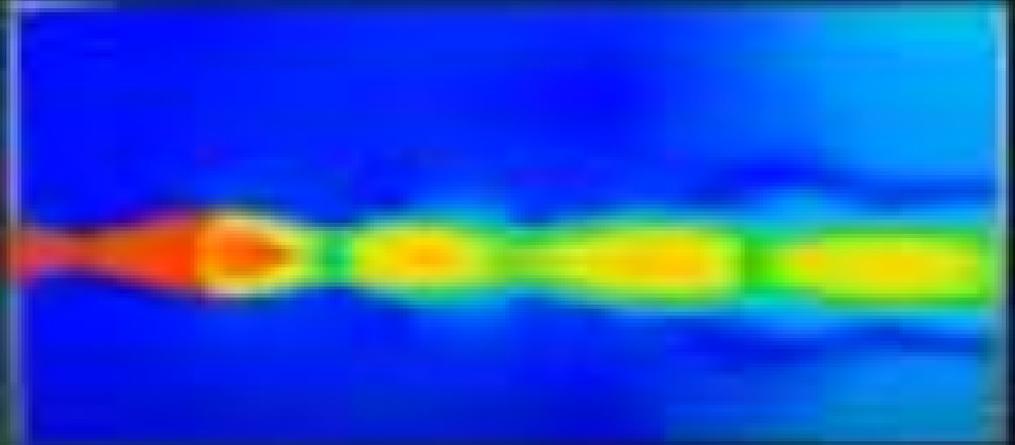
- Consider a flow at some velocity V , pressure P , Temperature T etc.
- What if the flow is accelerated/decelerated, **adiabatically**, to sonic velocity ($V = a$)
 - The Mach number becomes unity ($M = 1$)
- The resulting static temperature is called **characteristic temperature T^***
- The corresponding velocity of sound is a^*
- That is $a^* = \sqrt{\gamma R T^*}$
- A **NEW** Mach number is defined as $M^* = V/a^*$
- Note the difference between the definition of **M** (local Mach number) and **M^*** (characteristic Mach number)

CALCULATION OF M^* : EXAMPLE

- Determine the Mach number (M) and the critical Mach number (M^*) for the flow of air at a velocity of 850 m/s and static temperature = 300 K.
 - $M = 850/\sqrt{0.025(1.4 \times 287 \times 300)}$
 - $M^* = 850/\sqrt{0.025(1.4 \times 287 \times T^*)}$
 - $T^*/T = (1 + (\gamma - 1)M^2)/2$
 - Calculate T^* and then $M^* = V/\sqrt{\gamma R T^*}$



AARITA
AARITA

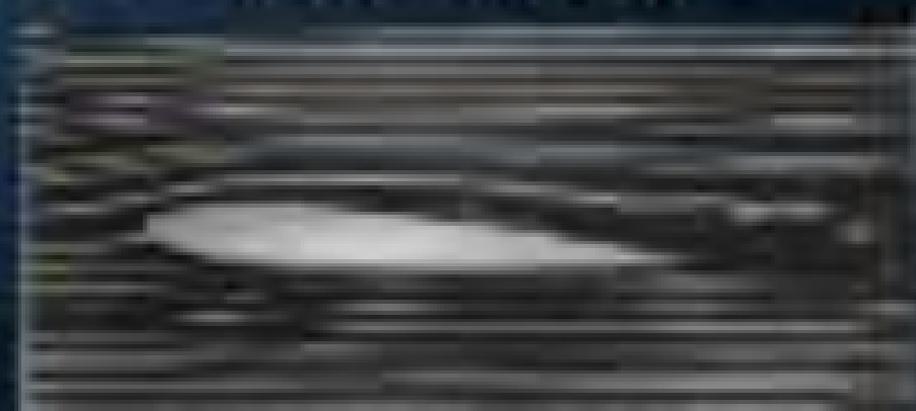


**Q. IS THERE TURBULENCE IN
THIS CASE?**

Normal ENDOCRIN

WHAT HAPPENS WHEN THE FLOW PROCEEDS FASTER THAN SOUND ($M > 1$) ?

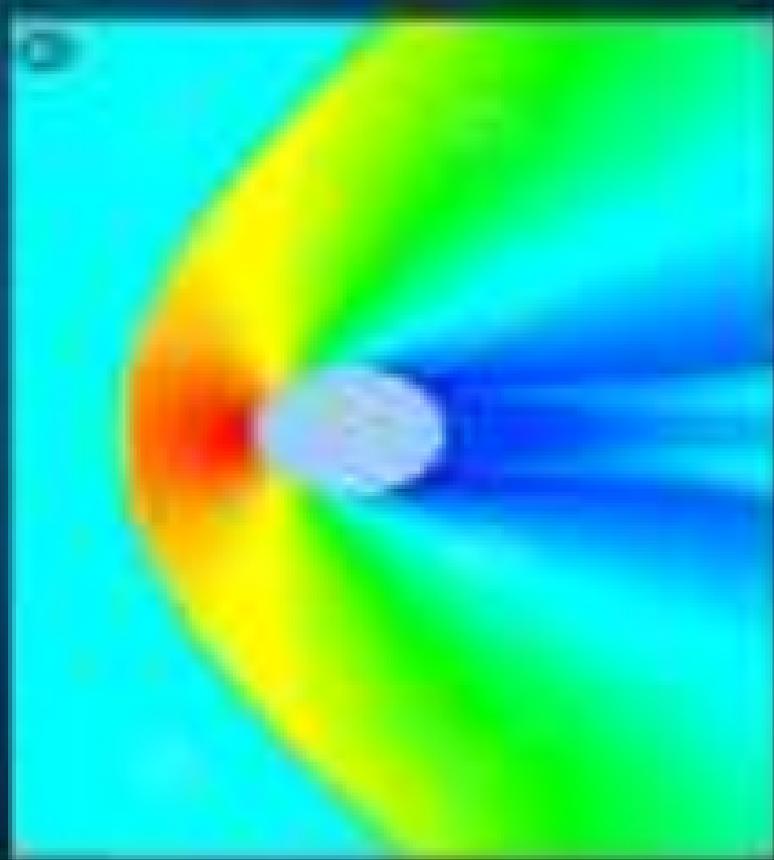
- The pressure disturbances cannot travel upstream!
- Do they travel upstream in other way?
 - Yes, they do - ρ waves from the streamlines bend away before reaching the obstruction.
 - Why do they bend before?
 - The pressure of the flow is a **PROPAGATING** wave, the pressure disturbance travels faster than the flow.





CAN THIS HAPPEN IN **SUPERSONIC FLOW** ?

- The flow travels faster than the pressure disturbance ($M > 1$)
- So the pressure waves cannot travel upstream
- The Consequence: **THE FLOW IS UNSTEADY !**



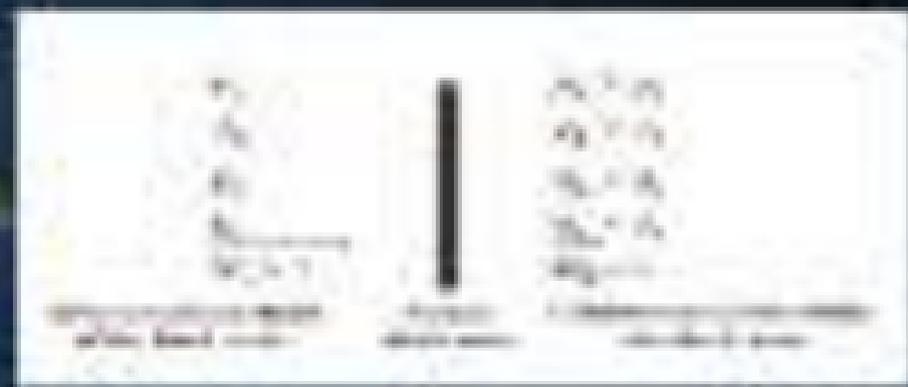


SUBSONIC VS SUPERSONIC



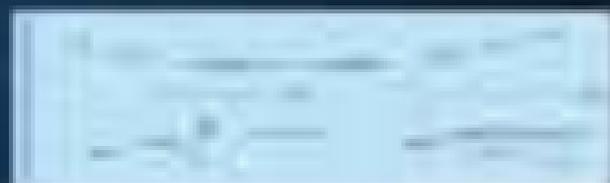
NORMAL SHOCK RELATIONS

- What happens upstream the shock?
- If we know properties upstream of the shock can we calculate those downstream?
- How do we design anticipating the impact of shock formation?



VARIATIONS ACROSS...

- Typically in all applications the systems/judges are drawn



THE GOVERNING EQUATIONS ACROSS A NORMAL SHOCK

- Steady, 1-dimensional adiabatic flow in constant area duct - for a perfect gas

$$\rho_1 V_1 = \rho_2 V_2$$

$$p_1 + \rho_1 V_1^2 = p_2 + \rho_2 V_2^2$$

$$\Delta_1 = \frac{p_1}{\rho_1} = \frac{p_2}{\rho_2} + \frac{V_2^2 - V_1^2}{2}$$

$$u = c_1 \rho^{\gamma}$$

$$h = c_2 \rho^{\gamma}$$

Stagnation properties

$$h_0 = c_p T_0 = \frac{\gamma}{\gamma - 1} \frac{p_0}{\rho_0}$$

$$\frac{p_1}{\rho_1^{\gamma}} = \frac{p_2}{\rho_2^{\gamma}} = \frac{p_0}{\rho_0^{\gamma}}$$

$$\frac{p_1}{\rho_1} = \frac{p_2}{\rho_2} = \frac{p_0}{\rho_0} = \frac{\gamma - 1}{\gamma} h_0$$

ENERGY EQUATION IN TERMS OF SOUND VELOCITY

$$c_2^2 = \frac{c_1^2}{\gamma} - c_1^2 + \frac{u^2}{\gamma} \quad \rightarrow \quad \frac{\gamma c_1^2}{\gamma} + \frac{u^2}{\gamma} = \frac{\gamma c_2^2}{\gamma} + \frac{u^2}{\gamma} \quad \rightarrow \quad \frac{c_1^2}{\gamma} + \frac{u^2}{\gamma} = \frac{c_2^2}{\gamma} + \frac{u^2}{\gamma}$$



$$\frac{c_1^2}{\gamma} + \frac{u^2}{\gamma} = \frac{c_2^2}{\gamma} + \frac{u^2}{\gamma}$$

$$\frac{c_1^2}{\gamma} - \frac{c_2^2}{\gamma} = \frac{u^2}{\gamma} - \frac{u^2}{\gamma}$$

$$\frac{c_1^2 - c_2^2}{\gamma} = \frac{u^2 - u^2}{\gamma}$$

$$c_1^2 - c_2^2 = u^2 - u^2$$

$$c_1^2 - c_2^2 = 0$$



$$c_1^2 - c_2^2 = 0$$

$$\begin{aligned}
 \frac{d^2x}{dt^2} &= \frac{d}{dt} \left(\frac{dx}{dt} \right) = \frac{d}{dt} \left(\frac{dx}{dr} \frac{dr}{dt} \right) \\
 &= \frac{d}{dr} \left(\frac{dx}{dt} \right) \frac{dr}{dt} \\
 &= \frac{d}{dr} \left(\frac{dx}{dr} \frac{dr}{dt} \right) \frac{dr}{dt} \\
 &= \frac{d}{dr} \left(\frac{dx}{dr} \right) \left(\frac{dr}{dt} \right)^2 + \frac{dx}{dr} \frac{d^2r}{dt^2}
 \end{aligned}$$

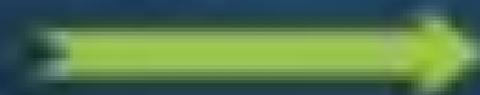
$$\frac{d^2x}{dt^2} = \frac{d}{dr} \left(\frac{dx}{dr} \right) \left(\frac{dr}{dt} \right)^2 + \frac{dx}{dr} \frac{d^2r}{dt^2}$$

$$\begin{aligned}
 \frac{d^2x}{dt^2} &= \frac{d}{dr} \left(\frac{dx}{dr} \right) \left(\frac{dr}{dt} \right)^2 + \frac{dx}{dr} \frac{d^2r}{dt^2} \\
 &= \frac{d}{dr} \left(\frac{dx}{dr} \right) \left(\frac{dr}{dt} \right)^2 + \frac{dx}{dr} \frac{d^2r}{dt^2} \\
 &= \frac{d}{dr} \left(\frac{dx}{dr} \right) \left(\frac{dr}{dt} \right)^2 + \frac{dx}{dr} \frac{d^2r}{dt^2}
 \end{aligned}$$

PRANDTL'S RELATION

- Simplifying

$$\frac{p + \frac{1}{2} \rho v^2}{\rho} + \frac{v^2}{2} = 1$$



$$\Delta p = \frac{1}{2} \rho v^2$$



Time Energy

- Time Energy equation:

$$\begin{aligned}
 \gamma m_0 c^2 &= \frac{1}{\sqrt{1-\beta^2}} = \frac{1}{\sqrt{1-\frac{v^2}{c^2}}} \\
 \gamma &= \frac{1}{\sqrt{1-\beta^2}} = \frac{1}{\sqrt{1-\frac{v^2}{c^2}}} \\
 \gamma m_0 c^2 &= \frac{m_0 c^2}{\sqrt{1-\beta^2}} = \frac{m_0 c^2}{\sqrt{1-\frac{v^2}{c^2}}}
 \end{aligned}$$

$$E = \frac{m_0 c^2}{\sqrt{1-\beta^2}}$$



$$E = \frac{m_0 c^2}{\sqrt{1-\beta^2}} = \frac{m_0 c^2}{\sqrt{1-\frac{v^2}{c^2}}}$$



$$\frac{dE}{dt} = \frac{d}{dt} \left(\frac{m_0 c^2}{\sqrt{1-\beta^2}} \right)$$

UPSTREAM TO DOWNSTREAM: THE MACH NUMBER RELATION

$$M_2^2 = \frac{1 + (\gamma - 1)/2 M_1^2}{\gamma M_1^2 - (\gamma - 1)/2}$$



DENSITY CHANGE ACROSS THE NORMAL SHOCK

$$\frac{\rho_2}{\rho_1} = \frac{\rho_2}{\rho_1} = \frac{\rho_1}{\rho_2} = \frac{M_1}{M_2} = \frac{M_1^2}{M_2^2}$$



$$\frac{\rho_2}{\rho_1} = \frac{\rho_1}{\rho_2} = \sqrt{\frac{1 + \gamma M_1^2}{1 + \gamma M_2^2}}$$

STATIC PRESSURE & STATIC TEMPERATURE

- An expression for the pressure coefficient can be derived using the momentum equation

$$P_2 - P_1 = \rho_1 U_1^2 - \rho_2 U_2^2$$

$$\frac{P_2}{P_1} = 1 + \frac{\rho_1}{\rho_2} \left[\frac{U_1^2}{U_2^2} - 1 \right]$$



- Applying ideal gas law:

$$\frac{\rho_1}{\rho_2} = \left(\frac{P_1}{P_2} \right) \left(\frac{T_2}{T_1} \right)$$

$$\frac{P_2}{P_1} = \frac{P_1}{P_2} \left[1 + \frac{\gamma}{\gamma - 1} \left(\frac{U_1^2}{U_2^2} - 1 \right) \right] \left[\frac{T_2 + (\gamma - 1) \frac{U_2^2}{2}}{T_1 + (\gamma - 1) \frac{U_1^2}{2}} \right]$$

LIMITING VALUES FOR PROPERTY VARIATION ACROSS NORMAL SHOCK AS $M \rightarrow \infty$

By substituting $M \rightarrow \infty$ in the respective relations

$$\lim_{M \rightarrow \infty} \frac{A_0}{A^*} = \frac{\sqrt{2} \sqrt{1 + \frac{\gamma - 1}{2} M^2}}{\frac{\gamma + 1}{2} M^2} = \frac{\sqrt{2}}{\sqrt{\gamma + 1}}$$

$$\lim_{M \rightarrow \infty} \frac{p_0}{p^*} = \frac{\sqrt{2} \sqrt{1 + \frac{\gamma - 1}{2} M^2}}{\frac{\gamma + 1}{2} M^2} = \frac{\sqrt{2}}{\sqrt{\gamma + 1}}$$

$$\lim_{M \rightarrow \infty} \frac{T_0}{T^*} = \frac{\gamma + 1}{2}$$

$$\lim_{M \rightarrow \infty} \frac{p_0}{p^*} = \frac{\sqrt{2}}{\sqrt{\gamma + 1}}$$

ACROSS THE NORMAL SHOCK:

INTEGRATING EULER'S EQUATION ACROSS SHOCKS YIELDS:

The static pressure increases: **COMPRESSION**

Density & static temperature increases:

Stagnation pressure drops

Stagnation Temperature **REMAINS CONSTANT**: Bernoulli energy equation

Entropy increases

$$\frac{p_2}{p_1} = \frac{\rho_2}{\rho_1} \frac{T_2}{T_1}$$

$$\frac{p_2}{p_1} = \frac{\rho_2}{\rho_1} \frac{T_2}{T_1} = \frac{\rho_2}{\rho_1} \frac{h_2}{h_1}$$

$$\frac{p_2}{p_1} = \frac{\rho_2}{\rho_1} \frac{T_2}{T_1} = \frac{\rho_2}{\rho_1} \frac{h_2}{h_1} = \frac{\rho_2}{\rho_1} \frac{h_1 + \frac{1}{2} u_1^2}{h_1 + \frac{1}{2} u_2^2}$$



SOME PRACTICAL ISSUES IN AEROSPACE APPLICATIONS

- Loss of stagnation pressure
- Increase in static temperature

Minimize these effects by
providing an efficient
Propulsion System...

Figure 10.10 Typical turbojet engine
Cross-sectional view of a turbojet engine, the most
widely used type of engine.



TABULAR PRESENTATION OF NORMAL SHOCK RELATIONS

- All the required parameters and ratios are indicated as functions of upstream Mach number
- **These relations are just the solutions of the relations that we have derived!**
- Values are listed in different tables for $\gamma = 1.4$, $\gamma = 1.67$ and $\gamma = 1.33$
- See the relation table above
- The tabulated ratios can be used with the **normal shock calculator** to calculate the downstream properties!



Normal Shock Table

Appendix B: A Table

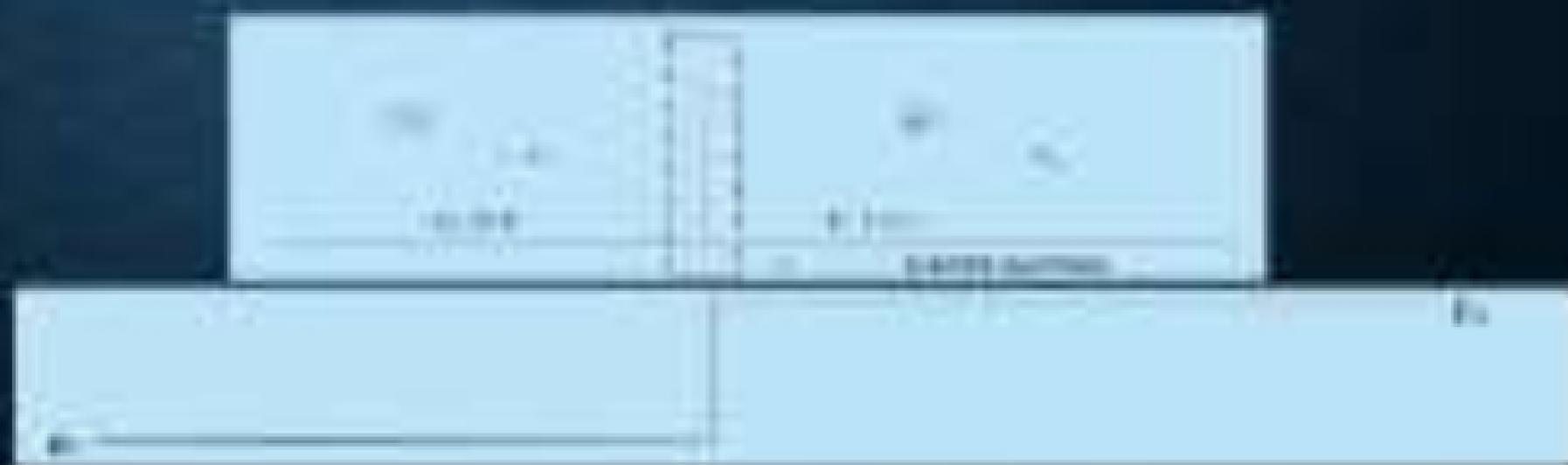
TABLE B.1. Standard Normal Table (p. 4, 14)

z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	0.5000	0.5040	0.5080	0.5120	0.5160	0.5199	0.5239	0.5279	0.5319	0.5359
0.1	0.5398	0.5438	0.5478	0.5518	0.5558	0.5598	0.5638	0.5677	0.5717	0.5757
0.2	0.5797	0.5837	0.5877	0.5917	0.5957	0.5997	0.6037	0.6076	0.6116	0.6156
0.3	0.6196	0.6235	0.6275	0.6315	0.6355	0.6395	0.6435	0.6474	0.6514	0.6554
0.4	0.6594	0.6634	0.6674	0.6714	0.6754	0.6794	0.6834	0.6874	0.6914	0.6954
0.5	0.6994	0.7034	0.7074	0.7114	0.7154	0.7194	0.7234	0.7274	0.7314	0.7354
0.6	0.7394	0.7434	0.7474	0.7514	0.7554	0.7594	0.7634	0.7674	0.7714	0.7754
0.7	0.7794	0.7834	0.7874	0.7914	0.7954	0.7994	0.8034	0.8074	0.8114	0.8154
0.8	0.8194	0.8234	0.8274	0.8314	0.8354	0.8394	0.8434	0.8474	0.8514	0.8554
0.9	0.8594	0.8634	0.8674	0.8714	0.8754	0.8794	0.8834	0.8874	0.8914	0.8954
1.0	0.8994	0.9034	0.9074	0.9114	0.9154	0.9194	0.9234	0.9274	0.9314	0.9354
1.1	0.9394	0.9434	0.9474	0.9514	0.9554	0.9594	0.9634	0.9674	0.9714	0.9754
1.2	0.9794	0.9834	0.9874	0.9914	0.9954	0.9994	1.0000	1.0000	1.0000	1.0000

NUMERICAL PROBLEM

- Consider a normal shock flowfield in a constant area duct in which air is flowing at Mach 3.0. If the static pressure & temperature before the shock are 3 atm & 3000 K, respectively, calculate the following after the shock.

- Static pressure
- Stagnation pressure
- Stagnation temperature
- Mach number



NUMERICAL PROBLEM

- Consider a normal shock in a duct. Upstream wind tunnel in which air is being supplied from a storage tank. The pressure inside the tank is 1.0 atm and the temperature inside the tank is 300 K. If a normal shock is formed at a constant area section within the tunnel where the Mach number is 2.0, calculate the static pressure & static temperature downstream of the shock.

- **Conditions inside the tank are**
 $P_0 = 1.01325 \times 10^5 \text{ Pa}$ & $T_0 = 300 \text{ K}$
- **P_0 & $T_0 \rightarrow P_1$ & T_1 & $M_1 \rightarrow T_1$**
- **The relationship for P_0/P_1 & T_0/T_1**
- **Calculate P_1 & T_1**



NUMERICAL PROBLEM GATE 2019

Ques 45 - 49 (MCQ): 2019 (2019) in category MCQ - 45/49

Q 47: The radii r_1 and r_2 of two circles, which are externally tangent to each other, is given by:

$$\frac{r_1}{r_2} = 1 + \frac{d}{r_2} \quad (d^2 = 1)$$

where d is the distance between centres. The distance between centres of circles is $d = 2 + \sqrt{2}$. The radii r_1 and r_2 of two circles which are externally tangent to each other, is given by $\frac{r_1}{r_2} = 1 + \frac{d}{r_2}$ where $d^2 = 1$. The value of r_1 is $\frac{1}{2}$ and the value of r_2 is $\frac{1}{2}$. The value of r_1 is $\frac{1}{2}$ and the value of r_2 is $\frac{1}{2}$.

(a)	(b)	(c)	(d)	(e)
1/2	1/2	1	1/2 + 1/2	1/2 + 1/2

NON-ISENTROPIC NATURE VS STAGNATION PRESSURE LOSS

- Flow can be considered isentropic up to the point where $M \approx 0.5$ (where δ is 1)
- The shock by itself is a very thin region - thicknesses of a few microns
- Shock profiles remain
 - Total enthalpy is conserved across the shock wave
 - Entropy gets increased from T_{01} to T_{02}
- Result for perfect gas:

$$P_0 = P_1 = P_2 \left(\frac{T_2}{T_1} \right)^{\frac{\gamma}{\gamma-1}} = P_1 \left(\frac{T_2}{T_1} \right)^{\frac{\gamma}{\gamma-1}}$$

- In terms of stagnation enthalpy:

$$T_{02} = T_2 = T_1 \left(\frac{P_2}{P_1} \right)^{\frac{\gamma-1}{\gamma}}$$

Remember T_{01} is T_{02} since $T_{01} = T_1 + \frac{V_1^2}{2}$ and $T_{02} = T_2 + \frac{V_2^2}{2}$
 The T_2 that comes out of the shock is proportional to $\frac{P_2}{P_1}$



HUGONIOT RELATIONS

- The relations for compression waves in terms of pressure and specific volume ($v > 1/\rho$)
 - Significant, because shock relations are in terms of Mass, energy, and entropy. They are three quantities
- If a given fluid property:
 - o Thermodynamic variables
 - o An appropriate description of shock as a compression process

TRANSFORMING THE EQUATIONS IN TERMS OF STATE VARIABLES (THERMODYNAMIC PROPERTIES)

• Energy equation in the form

$$\dot{Q} = \dot{m} \left(\frac{dH}{dt} \right)$$

• Using Maxwell's equations

$$\dot{Q} = \dot{m} \left(\frac{dH}{dt} \right) = \dot{m} \left(\frac{dU}{dt} + P \right)$$

$$\dot{Q} = \frac{dU}{dt} + \dot{m} P$$

• Energy equation in the form

$$\dot{Q} = \dot{m} \left(\frac{dU}{dt} + \frac{dW}{dt} \right)$$

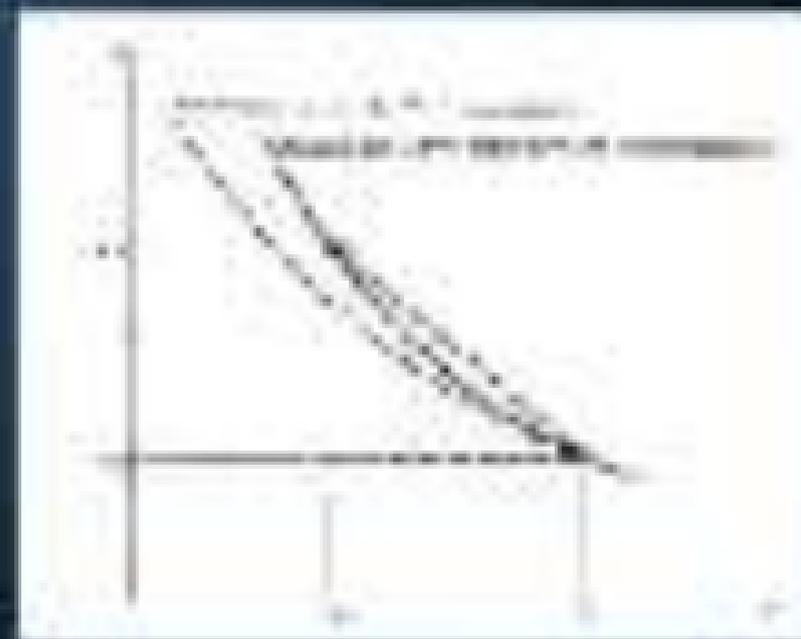
$$\dot{Q} + \frac{dW}{dt} = \dot{m} \left(\frac{dU}{dt} + \frac{dW}{dt} \right)$$

$$e_2 - e_1 = \frac{p_1 + p_2}{2} (v_2 - v_1)$$

- *the change in internal energy equals the mean pressure across the shock times the change in specific volume*

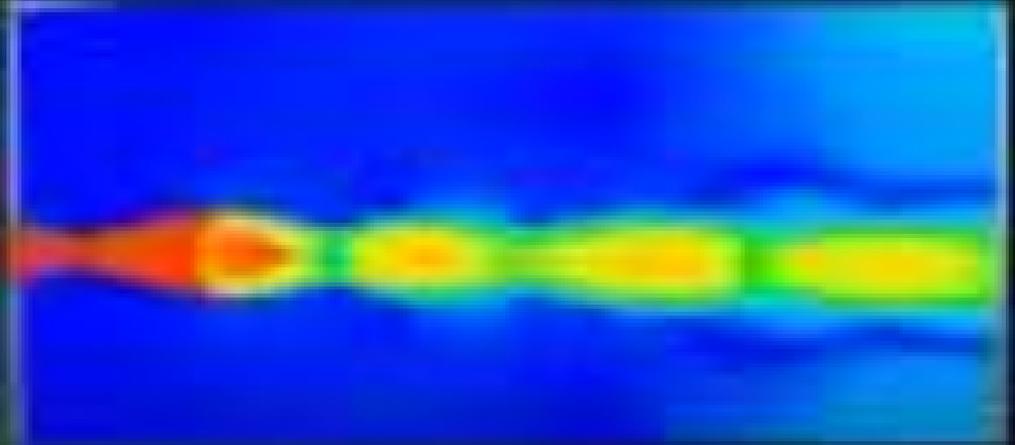
SHOCK ON A PV DIAGRAM

- A compression process which is more drastic than any other compression process.
 - Used when a large amount of work is needed.





LA ARITA



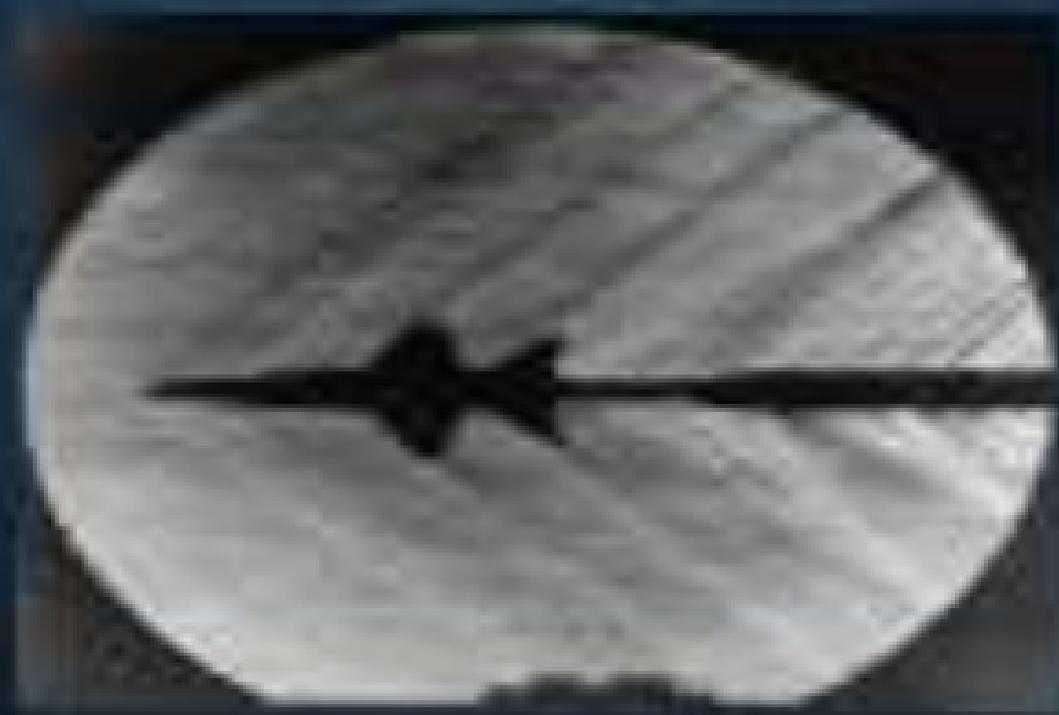
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LA ARITA

Obliqua in Oculis





OBLIQUE SHOCKS



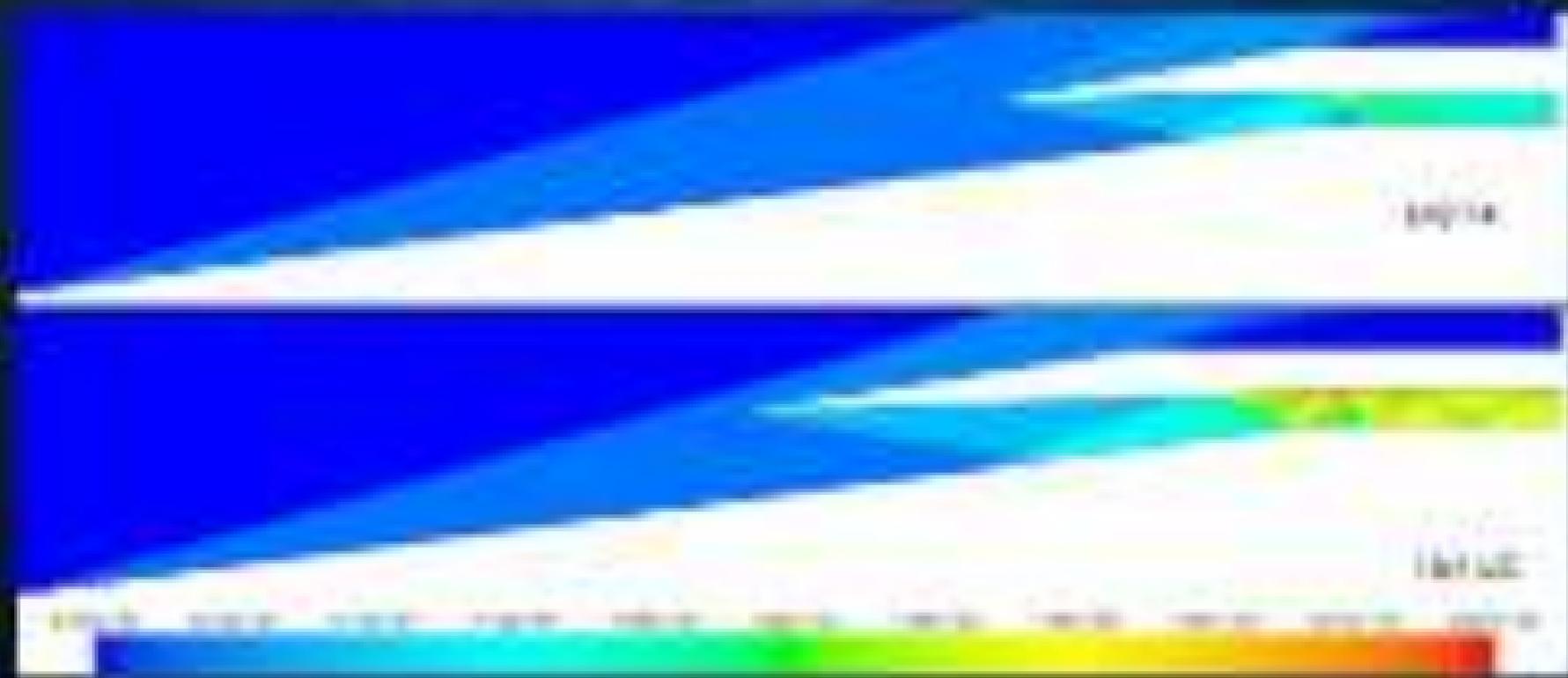
SCRAMJET CONFIGURATION

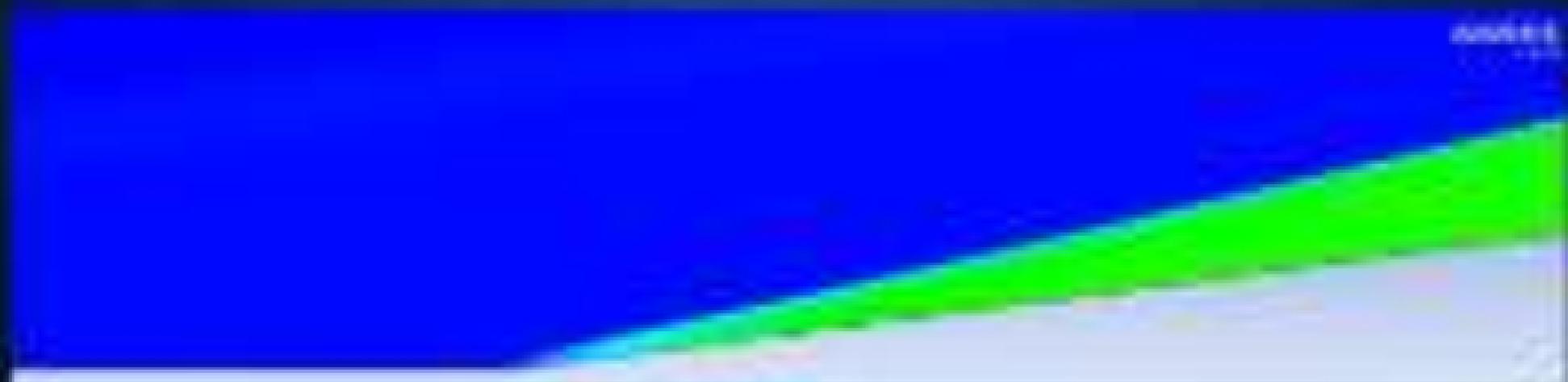


HOW DO STOCKS "LOOK LIKE"?

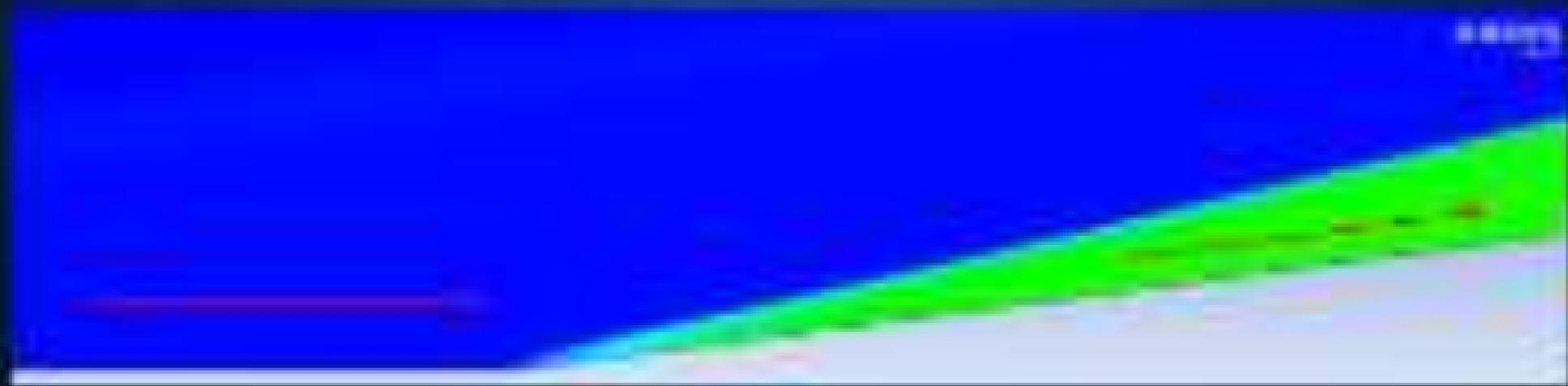






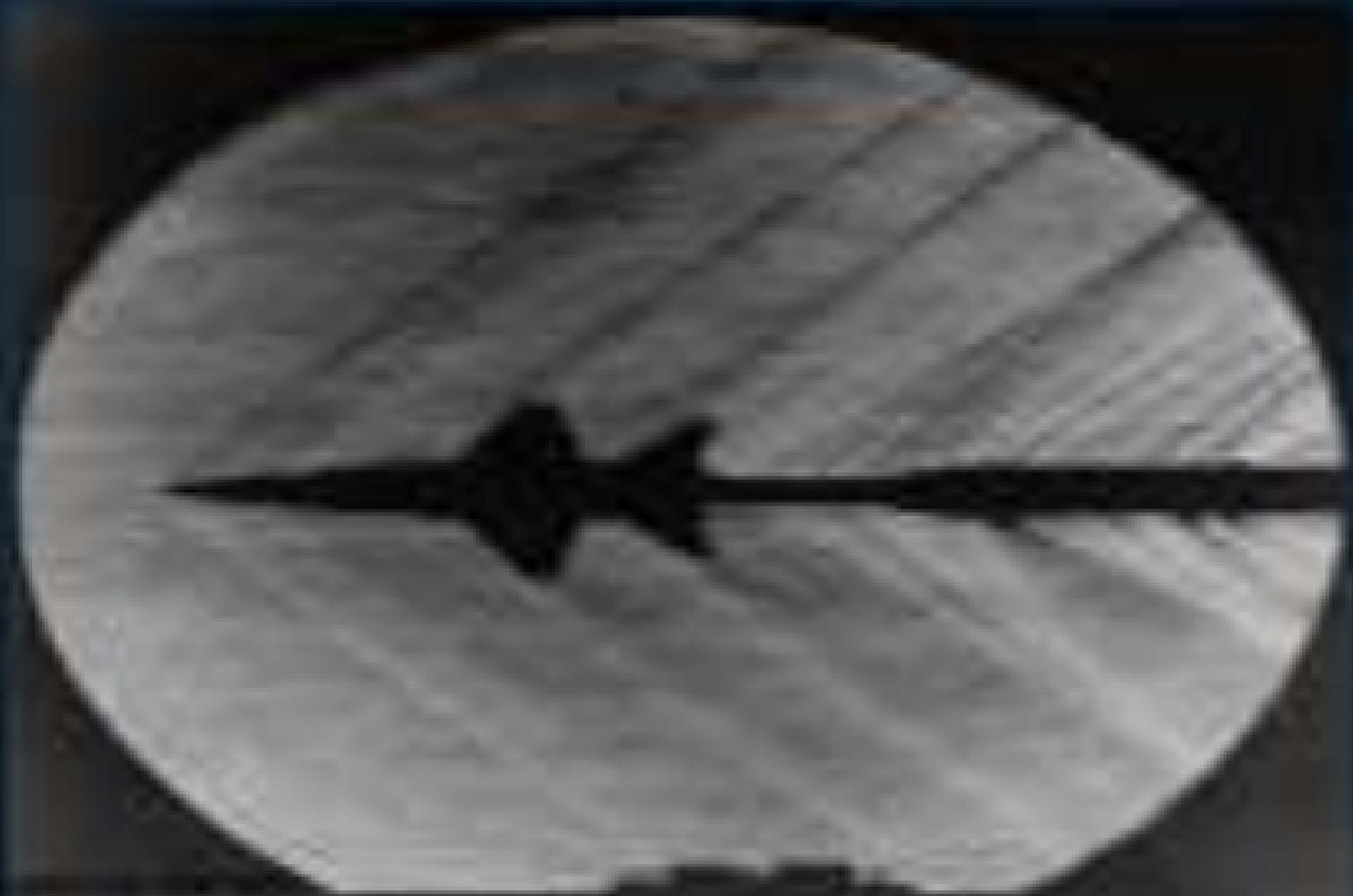


AFTER THE SHOCK, THE (INVISCID) FLOW
BECOMES PARALLEL TO THE DEFLECTING
SURFACE.



FLOW TURNING





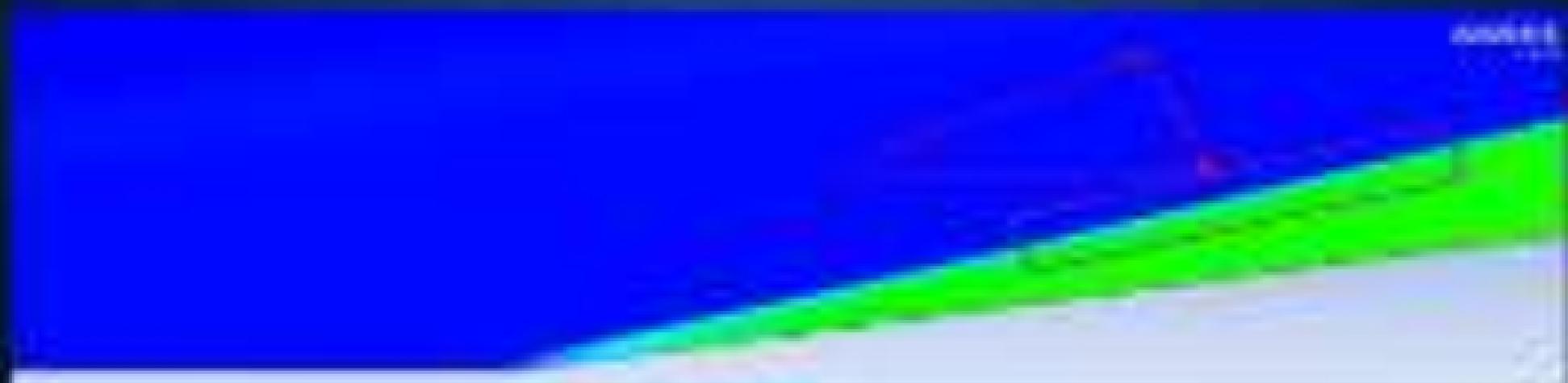
"WEDGE" AND "CORNER"

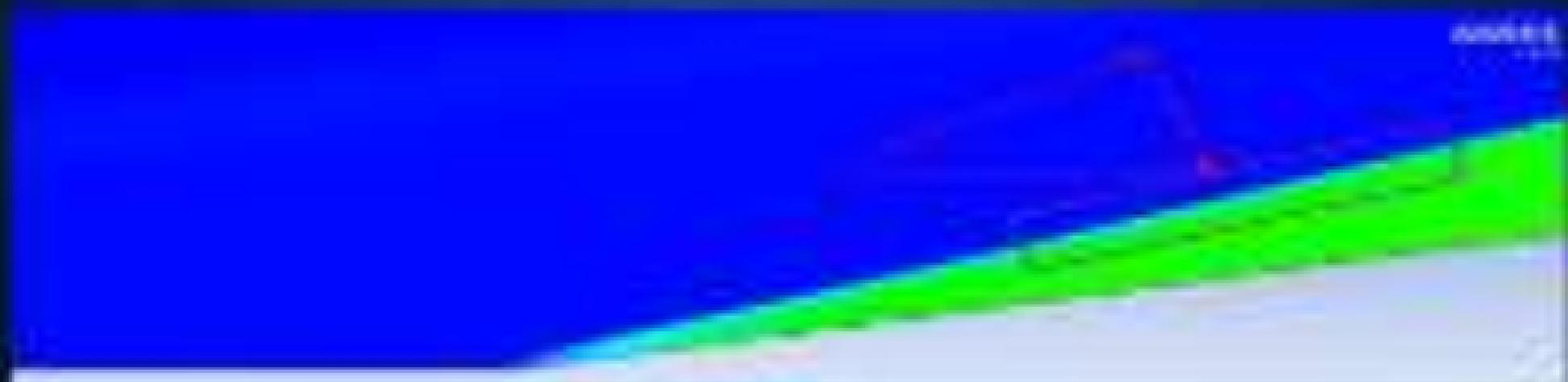
Note the difference:
Surface angle θ and
wedge angle β

"Wedge" and "Corner"



**THE TWO-DIMENSIONALITY:
INCORPORATED BY VELOCITY COMPONENTS**





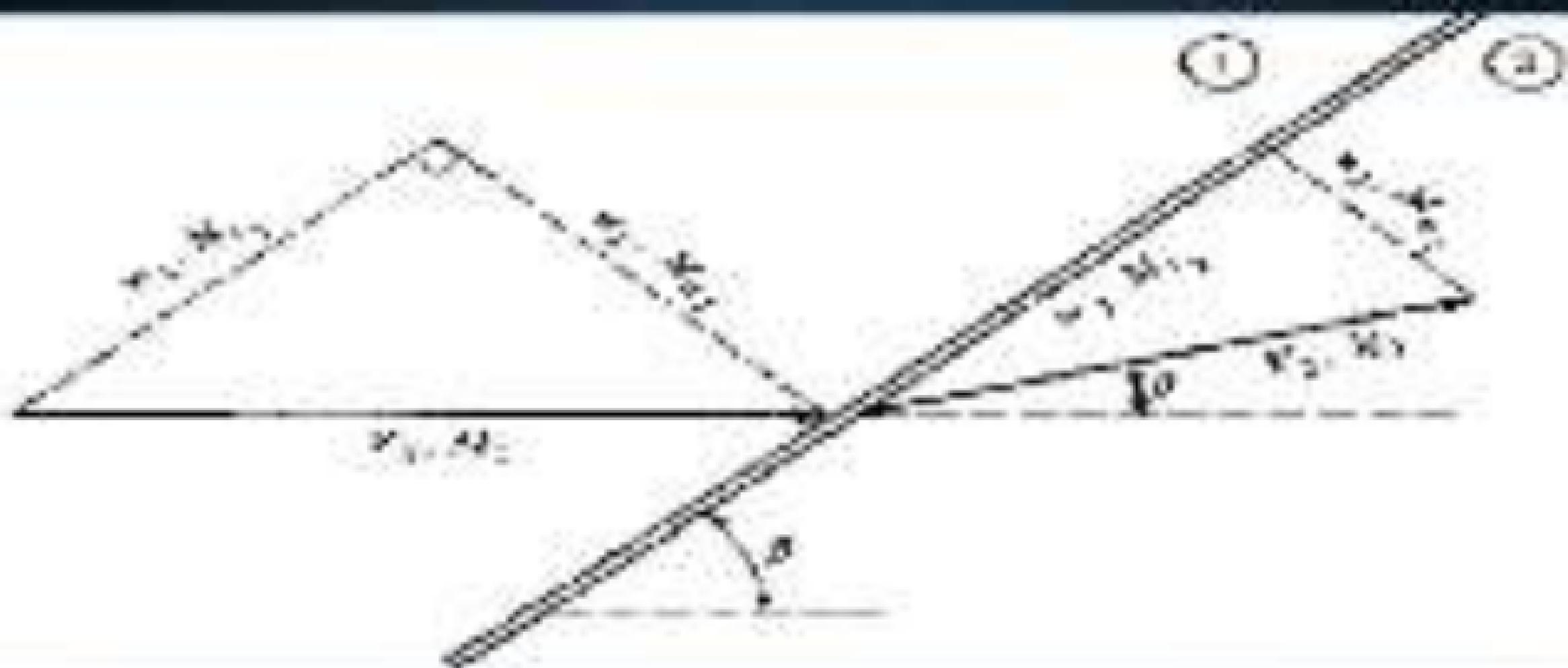
OBLIQUE SHOCK RELATIONS

- Analytical Relations from Governing Equations
 - same approach as for normal shocks
 - use conservation equations and state equations
- Conservation Equations:
 - mass, energy and momentum
 - Use same 2 conservation equations - ~~momentum~~ for a 2-d oblique shock
- Assumptions
 - steady flow (stationary shock), inviscid except inside shock, adiabatic, no work but flow work

OBLIQUE SHOCK GEOMETRY



- $u_{1n} = V_1 \sin \beta$; $u_{1t} = w_1 = V_1 \cos \beta$
- $u_{2n} = V_2 \sin(\beta - \theta)$; $u_{2t} = w_2 = V_2 \cos(\beta - \theta)$
- $M_{1n} = M_1 \sin \beta$; $M_{1t} = M_1 \cos \beta$
- $M_{2n} = M_2 \sin(\beta - \theta)$; $M_{2t} = M_2 \cos(\beta - \theta)$



CONTINUITY EQUATION ACROSS OS

→ Consider flow across the control volume

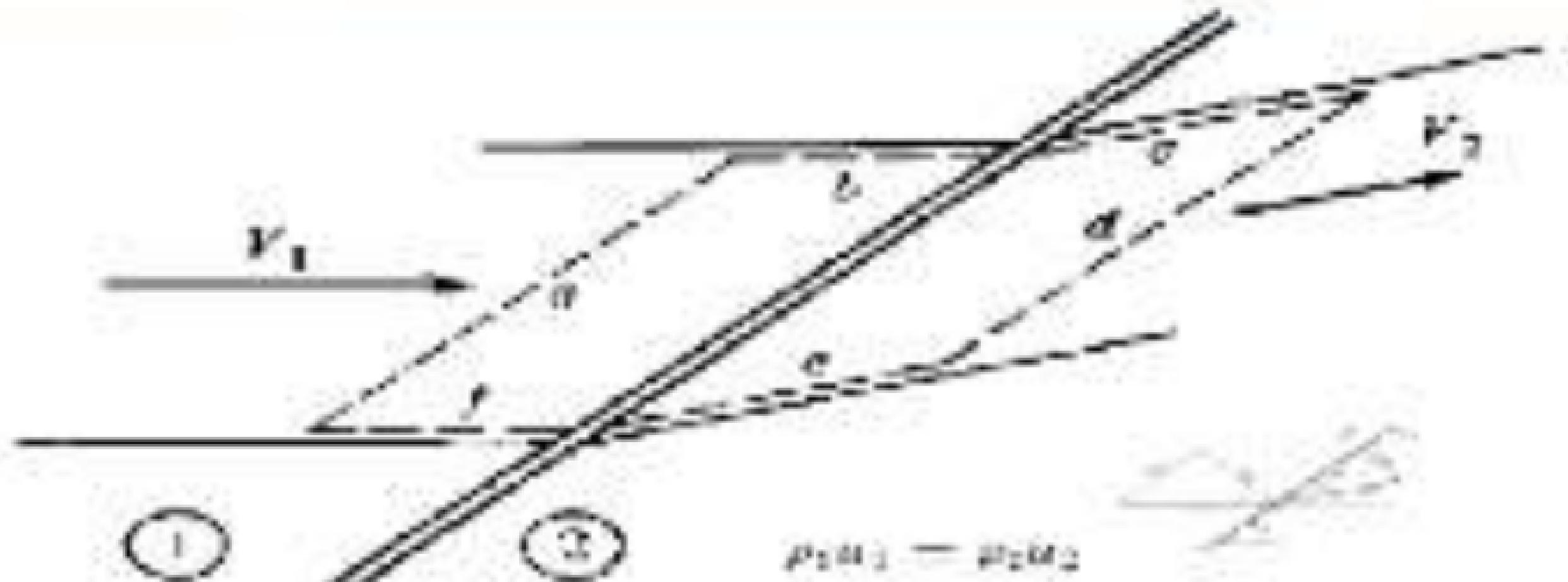
→ Note that the horizontal component of velocity does not change across the control volume. The upstream & the downstream flows

→ The side flows ($\rho u_1 \delta y$ & $\rho u_2 \delta y$) are parallel to P in the respective regions (1 & 2) and hence have no component of velocity perpendicular to the plane AB .

→ Hence the continuity equation holds across the mass flow balance due to the normal velocity u

$$\rho_1 u_1 = \rho_2 u_2$$





MOMENTUM EQUATION – **TANGENTIAL DIRECTION**

- Shear stress equation: Rate of change of momentum = 0
Assume velocity in x
(Neglecting body forces & surface forces)



- Consider tangential component: **Important!** component of the surface force **τdy**
 - They cancel on $L dy$
 - They cancel on $a \Delta x$
 - They are **equal** on $a \Delta x$
- Assume rate of change of momentum due to longitudinal velocity = 0

$$(-\rho_1 u_1) u_1 + (\rho_2 u_2) u_2 = 0$$

$$\zeta = \rho_1 K_1 \Delta W_1 + (\rho_2 K_2) W_2 = 0$$

→ $\rho_1 K_1 \Delta W_1 = -(\rho_2 K_2) W_2$

$$\Delta W_1 = -\frac{\rho_2 K_2}{\rho_1 K_1} W_2$$

→ $W_2 = \frac{\rho_1 K_1}{\rho_2 K_2} \Delta W_1$

The tangential component of velocity remains unchanged across the oblique shock

Therefore, all the oblique shocks across the oblique shock are considered to be normal shock for oblique shock. The normal component of velocity

NORMAL DIRECTION

- Considering Forces in the NORMAL direction

$$(-p_1 u_1) u_1 + (p_2 u_2) u_2 = -(p_1 + p_2)$$



$$p_1 + p_2 u_1^2 = p_2 + p_2 u_2^2$$

ENERGY EQUATION

$$\frac{d}{dt} \left(\frac{1}{2} m \dot{x}^2 + \frac{1}{2} k x^2 \right) = 0$$

• Example: **Energy** is conserved in a closed system with no external forces.

• The energy equation:

$$\frac{d}{dt} \left(\frac{1}{2} m \dot{x}^2 + \frac{1}{2} k x^2 \right) = 0$$
$$\left(m \dot{x} \ddot{x} + k x \dot{x} \right) = 0$$

$$\dot{x} \left(m \ddot{x} + k x \right) = 0$$

$$\dot{x} = 0 \quad \text{or} \quad \ddot{x} + \frac{k}{m} x = 0$$

$$\dot{x} = 0 \quad \text{or} \quad \ddot{x} = -\omega^2 x$$

RECALL THE CONTEXT OF ANALYSIS/APPLICATION



OVERVIEW OF THE 3 EQUATIONS

$$\Delta P_i = \rho A v_i$$

$$\Delta P_i + \Delta P_{VP} = \rho v_i \Delta v_i$$

$$\rho v_i \frac{dv_i}{dt} = \rho v_i \frac{dv_i}{dt}$$



- They are ALL identical with the 2nd one! should include **NON-STEADY STATE FLOW**
- **COMPONENTS OF VELOCITY FOR COLLISION IMPACT**
- **For impaction of air?**

NORMAL VS OBLIQUE

- The an oblique shock with upstream velocity \vec{V}_1 has normal component in u_{1n}
- Hence the wave will correspond to a normal shock with upstream velocity u_{1n}
- That is with upstream Mach number M_{1n}

- Recall

$$M_{1n} = M_1 \sin \theta$$



OS RELATIONS FOLLOW FROM THE RESPECTIVE NS RELATIONS...

- The various properties which concern shapes can be understood using the formal correspondences

$$\begin{aligned} \rho_1 &= \frac{1}{2} (1 + \sqrt{5}) \\ \rho_2 &= \frac{1}{2} (1 - \sqrt{5}) \end{aligned}$$

$$\begin{aligned} \frac{F_n}{F_{n-1}} &= \frac{F_n}{F_{n-1}} \left(\frac{\rho_1^n}{\rho_1^{n-1}} + \frac{\rho_2^n}{\rho_2^{n-1}} \right) \\ &= \frac{F_n}{F_{n-1}} \left(\rho_1 + \rho_2 \right) \end{aligned}$$

$$\frac{F_{n+1}}{F_n} = \frac{F_n + F_{n-1}}{F_n} = \frac{F_n}{F_n} + \frac{F_{n-1}}{F_n} = 1 + \frac{F_{n-1}}{F_n}$$

$$\begin{aligned} \rho_1 &= \frac{1}{2} (1 + \sqrt{5}) \\ \rho_2 &= \frac{1}{2} (1 - \sqrt{5}) \end{aligned}$$

THE

θ-β-M

RELATION

→ The Mach number can be used to calculate the shock angle as well.

$$M_2 = \frac{M_1}{\cos(\beta - \theta)}$$

$$\cos \beta = \frac{M_1}{M_2}$$

$$\frac{\cos(\beta - \theta)}{\cos \beta} = \frac{1 + \gamma}{2} + \frac{(\gamma - 1) M_1^2 \sin^2 \beta}{(\gamma + 1) M_2^2 \cos^2 \beta}$$

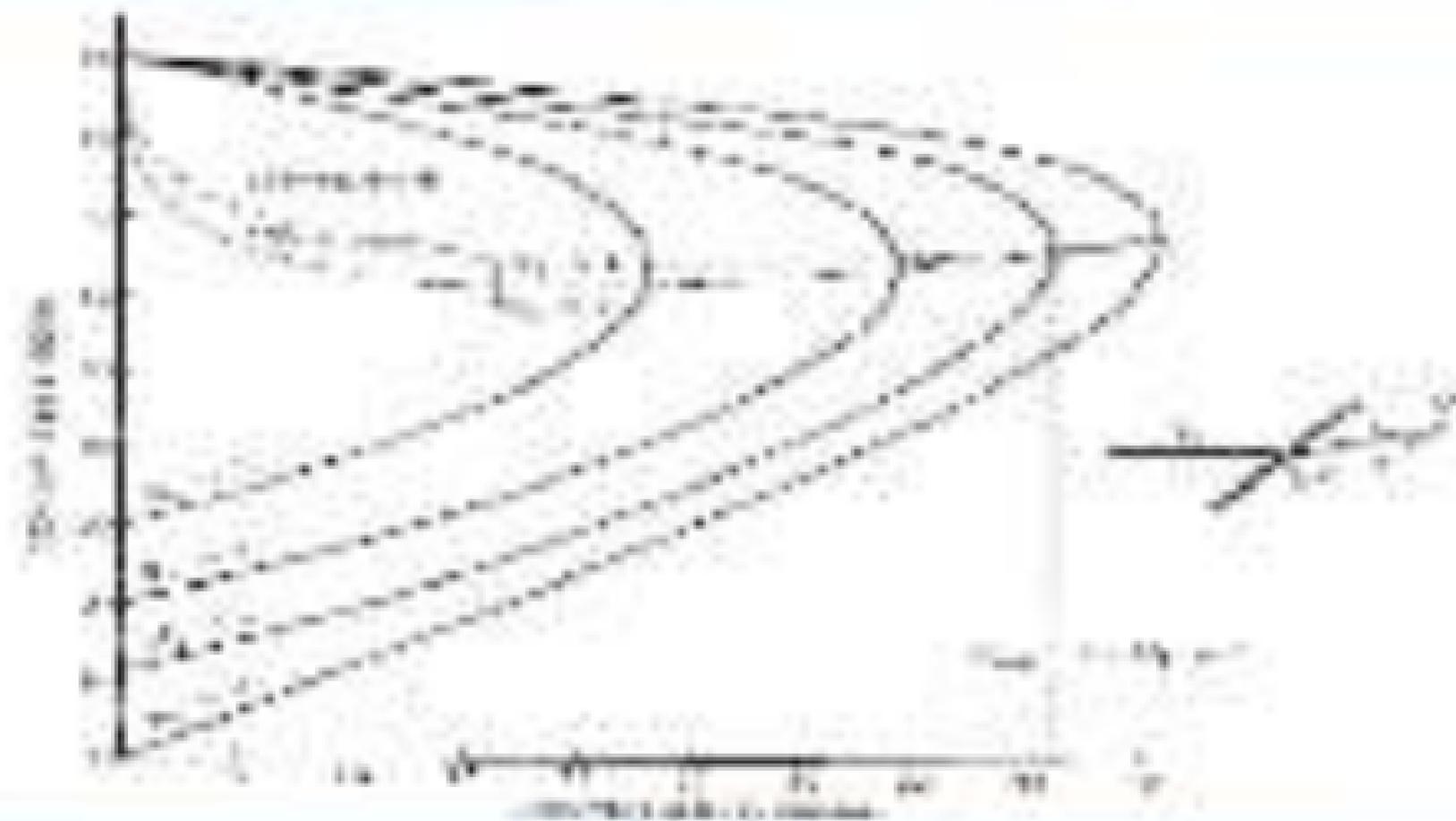
$$\cos(\beta - \theta) = \frac{M_1}{M_2} \left[\frac{1 + \gamma}{2} + \frac{(\gamma - 1) M_1^2 \sin^2 \beta}{(\gamma + 1) M_2^2 \cos^2 \beta} \right]$$

→ The Mach number can be used to calculate the shock angle as well.

→ The Mach number can be used to calculate the shock angle as well.

$$\tan \theta = 2 \cot \beta \left[\frac{M_1^2 \sin^2 \beta - 1}{M_1^2 (\gamma + \cos 2\beta) + 2} \right]$$

θ - β - M GRAPH

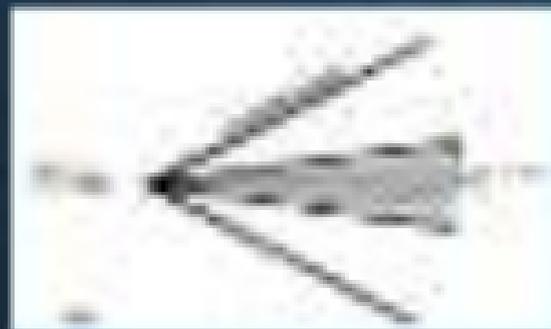


THE STRONG SHOCK & THE WEAK SHOCK

- For a given M_1 & β there can be **TWO** solutions for various angles β .
- Only one **weak shock** will exist.
- The lower value of β results in the formation of a **WEAK SHOCK**.
- The higher value of β \Rightarrow **STRONG shock**.
- Most practical applications involve **weak shocks** for various reasons.



WHAT HAPPENS WITH INCREASE IN DEFLECTION ANGLE θ ??



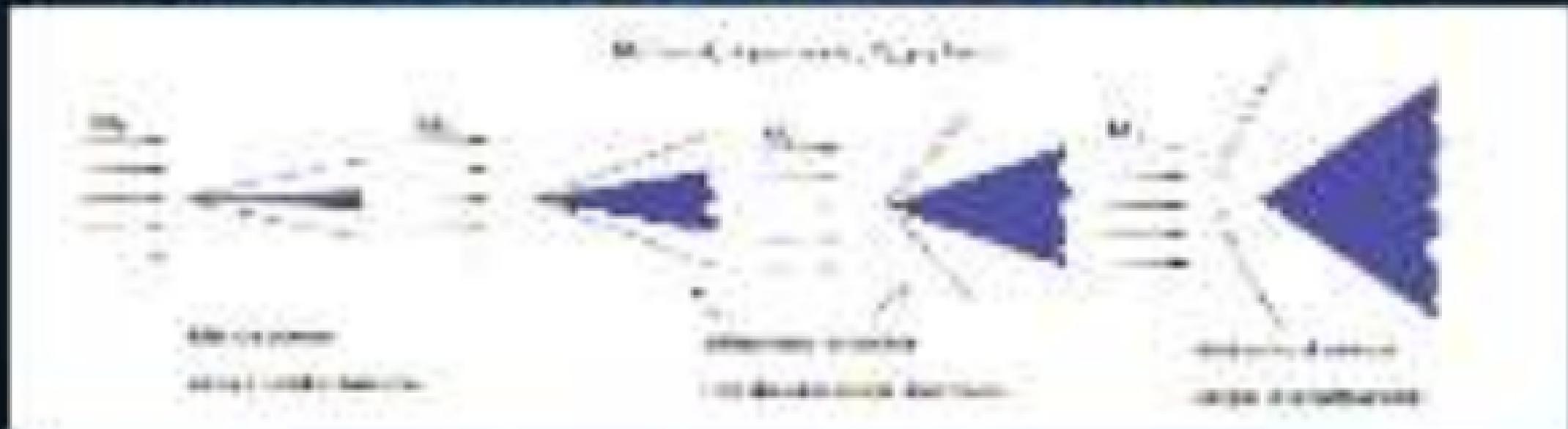
As the deflection angle increases, the maximum stress increases. The rate of increase in stress decreases as the deflection angle increases.

DETACHED / BOW SHOCK

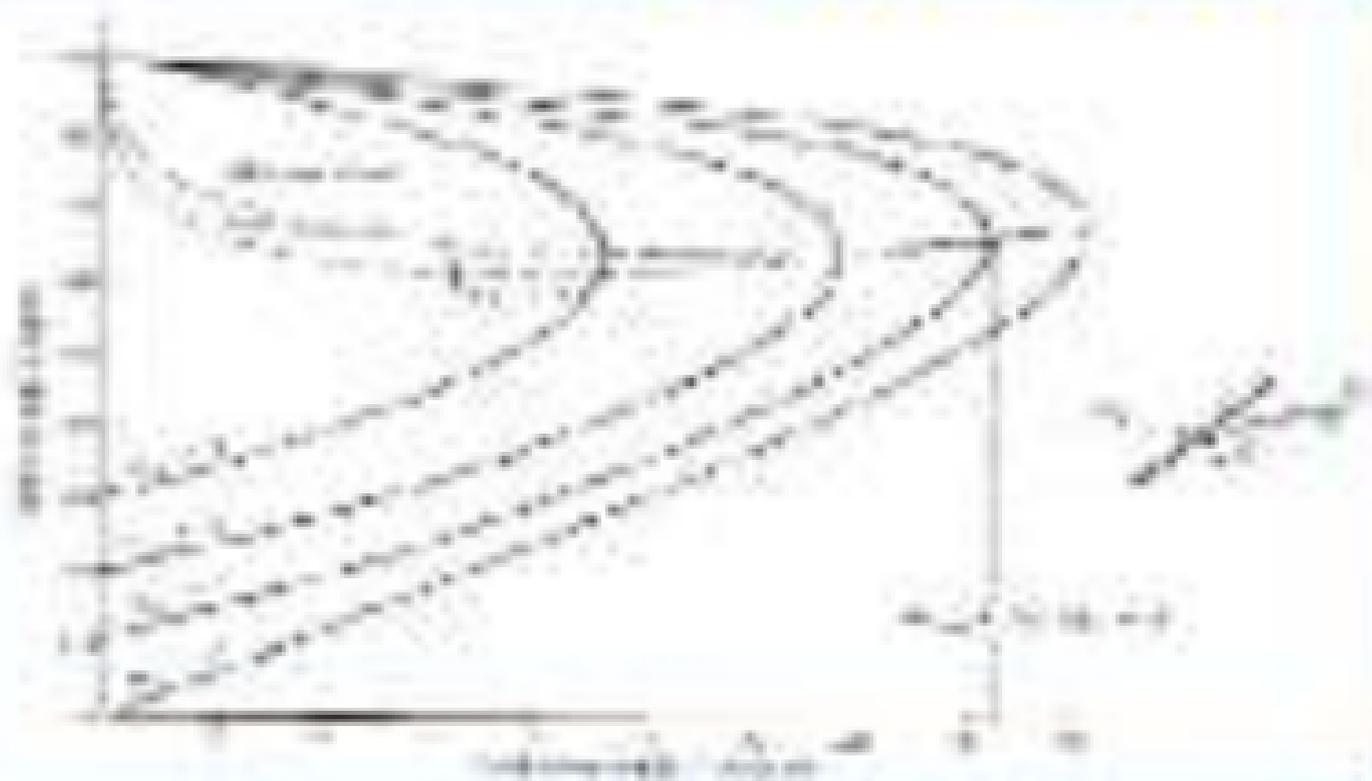
- For supersonic Mach numbers $Ma > 1$, there is a maximum value of deflection angle θ_{max} which an attached oblique shock can be formed
- Beyond that value of θ_{max} , an oblique shock detaches from the surface
 - The shock structure near the corner
- Critical nature of the detached shock will be in the form of a curved shock
- As you move away, the shock relaxes to straight, laminar oblique
- Expansion processes from a corner is called Prandtl-Göddert expansion shock region
- A detached shock is to be attached to blunt bodies that is where a hypothetical expansion of flow is not allowed



MACH WAVE TO DETACHED SHOCK – THE IMPACT OF DEFLECTION ANGLE...



SOME IMPORTANT OBSERVATIONS



As ML increases, there is a positive correlation.

As ML increases, there is a positive correlation between the number of pages and the number of lines.

As ML increases, there is a positive correlation between the number of pages and the number of lines.

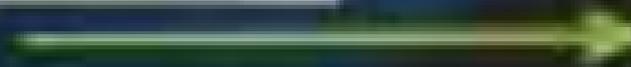
As ML increases, there is a positive correlation between the number of pages and the number of lines.

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APPLICATION: AN INSTANCE OF REAL OBLIQUE SHOCK FORMATION (PLANET INTAKE)



The supersonic flow is a complex phenomenon that is characterized by the formation of shock waves and the resulting changes in the flow properties. The shock waves are formed when the flow is deflected by a curved surface, and they are characterized by a sudden change in the flow properties. The flow lines are shown as dashed lines, and the shock wave is shown as a solid line. The flow is shown as a series of lines that are deflected by the surface.

NUMERICAL PROBLEM

- Consider air at 300 K flowing over a wedge-shaped airfoil with a half angle of 20° . The upstream velocity of the flow is 100 m/s and the static temperature is 300 K and 100 kPa respectively.
 1. Calculate the static pressure, total pressure, and stagnation pressure downstream of the shock formed assuming to be a weak shock.
 2. Calculate the shock parameters as shown if a STRONG shock is formed.Answer all to 4 decimal places with unit given. (4)

SOLUTION FOR WEAK SHOCK

$$\rightarrow P_2/P_1 = 2.721 \rightarrow P_2 = 272.1 \text{ kPa}$$

$$\rightarrow T_2/T_1 = 1.708 \rightarrow T_2 = 407.3 \text{ K}$$

$$\rightarrow M_2 = 0.5775$$

$$\rightarrow T_{02} = 463.8 \text{ kPa}$$



STRONG SHOCK

Strong Shock Table 1 - 1

M ₁	M ₂	Shock Wave Properties				Flow Properties			
		ρ ₂ /ρ ₁	u ₂ /u ₁	T ₂ /T ₁	p ₂ /p ₁	ρ ₂ /ρ ₁	u ₂ /u ₁	T ₂ /T ₁	p ₂ /p ₁
1.0	1.0	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
1.1	0.5774	1.0412	0.7209	1.0540	1.2333	0.7209	1.0540	1.2333	1.7080
1.2	0.5413	1.0843	0.6281	1.1071	1.5777	0.6281	1.1071	1.5777	2.2833
1.3	0.5111	1.1297	0.5584	1.1711	2.0461	0.5584	1.1711	2.0461	3.0433
1.4	0.4849	1.1774	0.5061	1.2461	2.6611	0.5061	1.2461	2.6611	4.0433
1.5	0.4617	1.2271	0.4661	1.3311	3.4461	0.4661	1.3311	3.4461	5.3433
1.6	0.4407	1.2787	0.4351	1.4261	4.4461	0.4351	1.4261	4.4461	7.0033
1.7	0.4215	1.3321	0.4101	1.5311	5.7461	0.4101	1.5311	5.7461	9.1033
1.8	0.4039	1.3871	0.3891	1.6461	7.4461	0.3891	1.6461	7.4461	11.7033
1.9	0.3877	1.4437	0.3721	1.7711	9.6461	0.3721	1.7711	9.6461	14.9033
2.0	0.3727	1.5017	0.3581	1.9061	12.4461	0.3581	1.9061	12.4461	18.8033
2.1	0.3587	1.5607	0.3461	2.0511	15.9461	0.3461	2.0511	15.9461	23.5033
2.2	0.3457	1.6207	0.3361	2.2061	20.2461	0.3361	2.2061	20.2461	29.1033
2.3	0.3335	1.6817	0.3281	2.3711	25.4461	0.3281	2.3711	25.4461	35.7033
2.4	0.3221	1.7437	0.3211	2.5461	31.6461	0.3211	2.5461	31.6461	43.5033
2.5	0.3113	1.8067	0.3151	2.7311	38.9461	0.3151	2.7311	38.9461	52.7033
2.6	0.3011	1.8707	0.3091	2.9261	47.4461	0.3091	2.9261	47.4461	63.5033
2.7	0.2915	1.9357	0.3041	3.1311	57.2461	0.3041	3.1311	57.2461	76.1033
2.8	0.2823	2.0017	0.3001	3.3461	68.4461	0.3001	3.3461	68.4461	90.7033
2.9	0.2735	2.0687	0.2961	3.5711	81.0461	0.2961	3.5711	81.0461	107.7033
3.0	0.2651	2.1367	0.2921	3.8061	95.1461	0.2921	3.8061	95.1461	127.5033
3.1	0.2571	2.2057	0.2881	4.0511	110.8461	0.2881	4.0511	110.8461	150.5033
3.2	0.2495	2.2757	0.2841	4.3061	128.2461	0.2841	4.3061	128.2461	176.3033
3.3	0.2423	2.3467	0.2801	4.5711	147.4461	0.2801	4.5711	147.4461	205.5033
3.4	0.2355	2.4187	0.2761	4.8461	168.5461	0.2761	4.8461	168.5461	238.7033
3.5	0.2291	2.4917	0.2721	5.1311	191.6461	0.2721	5.1311	191.6461	275.7033
3.6	0.2231	2.5657	0.2681	5.4261	216.8461	0.2681	5.4261	216.8461	317.3033
3.7	0.2175	2.6407	0.2641	5.7311	244.2461	0.2641	5.7311	244.2461	363.5033
3.8	0.2123	2.7167	0.2601	6.0461	273.9461	0.2601	6.0461	273.9461	415.3033
3.9	0.2075	2.7937	0.2561	6.3711	306.0461	0.2561	6.3711	306.0461	473.7033
4.0	0.2031	2.8717	0.2521	6.7061	340.6461	0.2521	6.7061	340.6461	539.7033
4.1	0.1991	2.9507	0.2481	7.0511	377.8461	0.2481	7.0511	377.8461	614.5033
4.2	0.1955	3.0307	0.2441	7.4061	417.7461	0.2441	7.4061	417.7461	699.3033
4.3	0.1923	3.1117	0.2401	7.7711	460.4461	0.2401	7.7711	460.4461	795.3033
4.4	0.1895	3.1937	0.2361	8.1461	506.0461	0.2361	8.1461	506.0461	903.7033
4.5	0.1871	3.2767	0.2321	8.5311	554.6461	0.2321	8.5311	554.6461	1025.7033
4.6	0.1851	3.3607	0.2281	8.9261	606.4461	0.2281	8.9261	606.4461	1161.7033
4.7	0.1835	3.4457	0.2241	9.3311	661.6461	0.2241	9.3311	661.6461	1313.3033
4.8	0.1823	3.5317	0.2201	9.7461	720.4461	0.2201	9.7461	720.4461	1481.3033
4.9	0.1815	3.6187	0.2161	10.1711	783.0461	0.2161	10.1711	783.0461	1667.3033
5.0	0.1811	3.7067	0.2121	10.6061	849.6461	0.2121	10.6061	849.6461	1873.3033
5.1	0.1811	3.7957	0.2081	11.0511	920.4461	0.2081	11.0511	920.4461	2099.7033
5.2	0.1815	3.8857	0.2041	11.5061	995.6461	0.2041	11.5061	995.6461	2347.7033
5.3	0.1823	3.9767	0.2001	11.9711	1075.4461	0.2001	11.9711	1075.4461	2618.7033
5.4	0.1835	4.0687	0.1961	12.4461	1160.0461	0.1961	12.4461	1160.0461	2913.7033
5.5	0.1851	4.1617	0.1921	12.9311	1249.6461	0.1921	12.9311	1249.6461	3234.7033
5.6	0.1871	4.2557	0.1881	13.4261	1344.4461	0.1881	13.4261	1344.4461	3582.7033
5.7	0.1895	4.3507	0.1841	13.9311	1444.6461	0.1841	13.9311	1444.6461	3958.7033
5.8	0.1923	4.4467	0.1801	14.4461	1550.4461	0.1801	14.4461	1550.4461	4364.7033
5.9	0.1955	4.5437	0.1761	14.9711	1662.0461	0.1761	14.9711	1662.0461	4802.7033
6.0	0.1991	4.6417	0.1721	15.5061	1779.6461	0.1721	15.5061	1779.6461	5274.7033
6.1	0.2031	4.7407	0.1681	16.0511	1903.4461	0.1681	16.0511	1903.4461	5782.7033
6.2	0.2075	4.8407	0.1641	16.6061	2033.6461	0.1641	16.6061	2033.6461	6328.7033
6.3	0.2123	4.9417	0.1601	17.1711	2170.4461	0.1601	17.1711	2170.4461	6914.7033
6.4	0.2175	5.0437	0.1561	17.7461	2314.0461	0.1561	17.7461	2314.0461	7542.7033
6.5	0.2231	5.1467	0.1521	18.3311	2464.6461	0.1521	18.3311	2464.6461	8214.7033
6.6	0.2291	5.2507	0.1481	18.9261	2622.4461	0.1481	18.9261	2622.4461	8932.7033
6.7	0.2355	5.3557	0.1441	19.5311	2787.6461	0.1441	19.5311	2787.6461	9698.7033
6.8	0.2423	5.4617	0.1401	20.1461	2960.4461	0.1401	20.1461	2960.4461	10514.7033
6.9	0.2495	5.5687	0.1361	20.7711	3141.0461	0.1361	20.7711	3141.0461	11382.7033
7.0	0.2571	5.6767	0.1321	21.4061	3329.6461	0.1321	21.4061	3329.6461	12304.7033
7.1	0.2651	5.7857	0.1281	22.0511	3526.4461	0.1281	22.0511	3526.4461	13282.7033
7.2	0.2735	5.8957	0.1241	22.7061	3731.6461	0.1241	22.7061	3731.6461	14318.7033
7.3	0.2823	6.0067	0.1201	23.3711	3945.4461	0.1201	23.3711	3945.4461	15414.7033
7.4	0.2915	6.1187	0.1161	24.0461	4168.0461	0.1161	24.0461	4168.0461	16572.7033
7.5	0.3011	6.2317	0.1121	24.7311	4399.6461	0.1121	24.7311	4399.6461	17794.7033
7.6	0.3113	6.3457	0.1081	25.4261	4640.4461	0.1081	25.4261	4640.4461	19082.7033
7.7	0.3221	6.4607	0.1041	26.1311	4890.6461	0.1041	26.1311	4890.6461	20438.7033
7.8	0.3335	6.5767	0.1001	26.8461	5150.4461	0.1001	26.8461	5150.4461	21864.7033
7.9	0.3457	6.6937	0.0961	27.5711	5420.0461	0.0961	27.5711	5420.0461	23362.7033
8.0	0.3587	6.8117	0.0921	28.3061	5700.6461	0.0921	28.3061	5700.6461	24934.7033
8.1	0.3727	6.9307	0.0881	29.0511	5992.4461	0.0881	29.0511	5992.4461	26582.7033
8.2	0.3877	7.0507	0.0841	29.8061	6295.6461	0.0841	29.8061	6295.6461	28308.7033
8.3	0.4039	7.1717	0.0801	30.5711	6610.4461	0.0801	30.5711	6610.4461	30114.7033
8.4	0.4215	7.2937	0.0761	31.3461	6937.0461	0.0761	31.3461	6937.0461	32002.7033
8.5	0.4407	7.4167	0.0721	32.1311	7275.6461	0.0721	32.1311	7275.6461	34074.7033
8.6	0.4617	7.5407	0.0681	32.9261	7626.4461	0.0681	32.9261	7626.4461	36332.7033
8.7	0.4849	7.6657	0.0641	33.7311	7989.6461	0.0641	33.7311	7989.6461	38778.7033
8.8	0.5111	7.7917	0.0601	34.5461	8365.4461	0.0601	34.5461	8365.4461	41414.7033
8.9	0.5413	7.9187	0.0561	35.3711	8754.0461	0.0561	35.3711	8754.0461	44242.7033
9.0	0.5774	8.0467	0.0521	36.2061	9155.6461	0.0521	36.2061	9155.6461	47264.7033

STRONG SHOCK SOLN.

$\rightarrow R_2 = 32.52 \text{ mm}$

$\rightarrow \rho_2/\rho_1 = 11.517 \rightarrow \rho_2 = 1.1517 \text{ g/cm}^3$

$\rightarrow T_2/T_1 = 2.74 \rightarrow T_2 = 274 \text{ K}$

$\rightarrow M_2 = 0.58860$



NUMERICAL PROBLEM - 2

- Consider a pressure flow over a sharp leading corner with a sharp angle of 30° . It is required to a Mach number of 0.5 calculate the free surface and static pressure along the surface. Assuming the upstream static pressure is 1 bar (assume the shock to be weak).
- What is the maximum ramp angle which you have an assumed shock system than considered?

SHOCK POLAR

- Graphical representation of oblique shock
- Provides information of various flow parameters upstream and downstream of shock
- Useful for analyzing, particularly, when compressible shock structures are possible under the given flow/geometric conditions
- Coordinate system aligned with the flow direction (x -axis) and the shock is z (y -axis)
 - As opposed to the normal and tangential components that we considered for the oblique shock.
- This relation is called **Mach-Zugler's eqn**

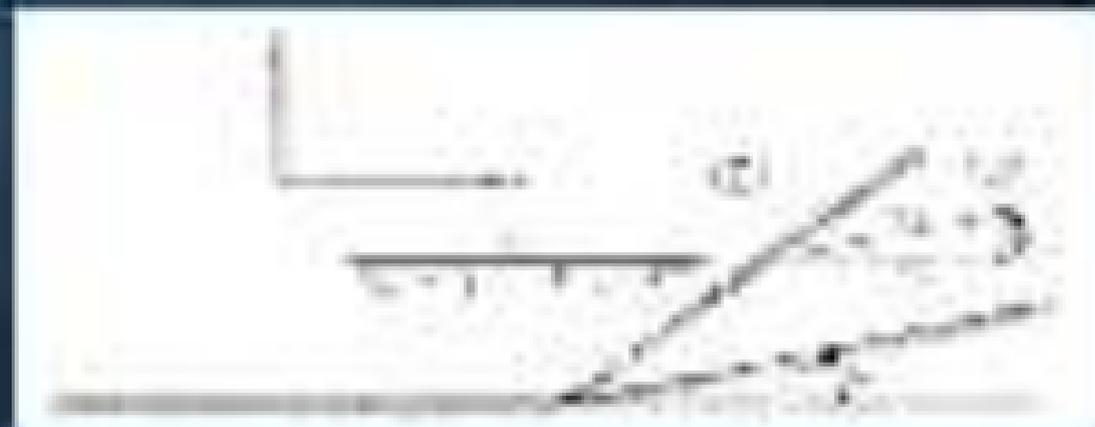
THE COMPONENTS



WEAK SHOCK & STRONG SHOCK SOLUTIONS IN HODOGRAPH PLANE



WEAK SHOCK & STRONG SHOCK SOLUTIONS IN HODOGRAPH PLANE





- Blood vessels are also affected by mechanical compression during injury.



WHAT HAPPENS WHEN AN OBLIQUE SHOCK HITS A SURFACE ?

- What does it depend on ?



SHOCK REFLECTION

- When an oblique shock impinges on a wall, the flow across it acquires a new angle and a new Mach number in the upper half-space.
- This takes place by a way of a new oblique shock that originates from the surface being hit.
- The nature of the new shock depends on the geometry and flow conditions.
- This phenomenon is called **SHOCK-KINKING**.

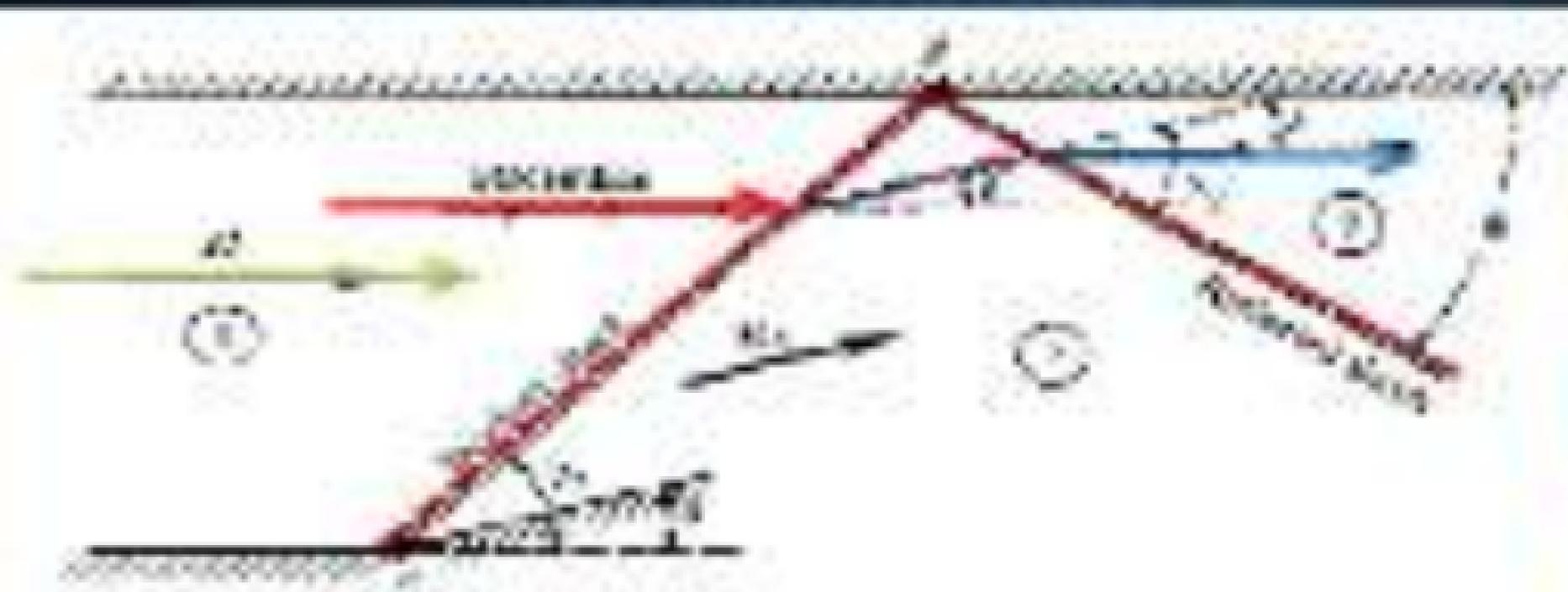


INCIDENT & REFLECTED STROKES



SHOCK REFLECTION - THE SEQUENCE

and the flow velocity u is constant

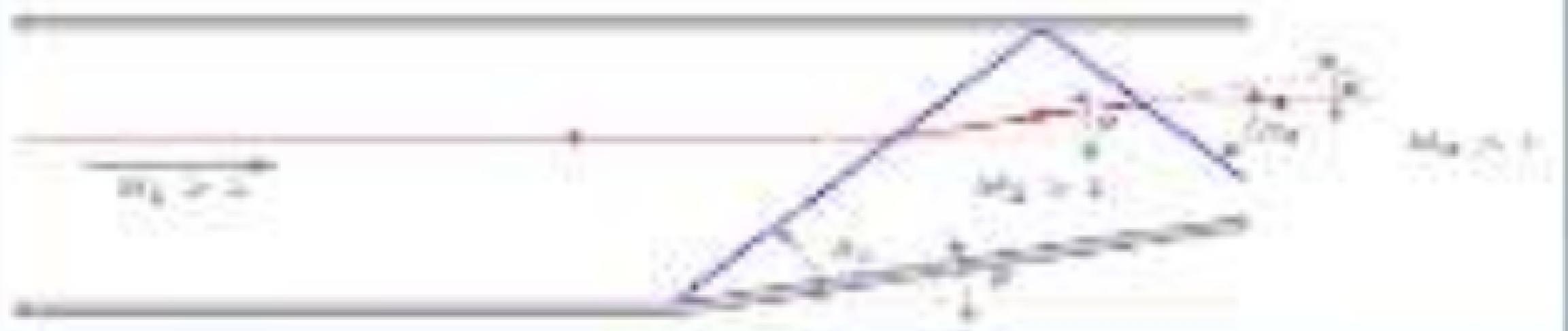


As pressure p and density ρ increase, the shock wave moves towards the wall.

As the shock wave moves towards the wall, the flow velocity u decreases.

As the shock wave moves towards the wall, the flow velocity u increases.

As the shock wave moves towards the wall, the flow velocity u decreases.



SHOCK REFLECTION

- **State the terminology: INCIDENT shock & REFLECTED shock**
- When that $M_1 > M_2$ occurs, the reflected shock is formed at a lower Mach number. Hence the reflected shock is **WEAKER** than the incident shock.
 - The velocity of flow is **INCREASED** across reflection.
 - The **total pressure decreases** across shock although gradually increases.



SHOCK INTERSECTION

- Freedom of the shock. The collision of **Left** moving & **Right** moving shocks



- What happens if in a supersonic medium, two shocks of opposite families **intersect**?

SHOCK INTERSECTION

- Consider the intersection of shock E1 and E2
- Regions I & II will have different properties
- Mach no., pressure etc. in E1 & E2 are not of same strength
- As we move through E1 & E2 we'll be crossed
- Regions I & II actually do have same flow properties: **Throat's Dy**
 - 1. For this situation you can't differentiate
 - 2. A slip line can be identified across which flow still constant and constant mass flux



NUMERICAL PROBLEM

- In a rectangular flow passage, the flow velocity given through a horizontal channel with velocity and flow is represented inside on the surface wall of the passage. The velocity is an U^* with the approximating flow diameter. The velocity starts toward the corner and ends at the top wall so that the flow is directed to be in the horizontal direction. If the approximating flow area $M = 2.8$ m², the passage U^* is, calculate the flow diameter and the area given a flow rate influence. Answer: 0.4 m, 0.4 m² respectively.



SOLUTION



- Shock 1: $M_1 = 3$, $\theta_1 = 10$ deg.

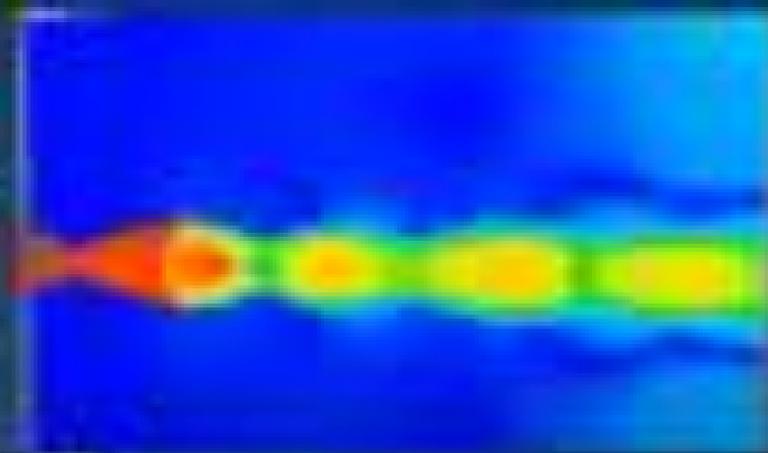
- CD tables for weak shock $\Rightarrow M_2 = 2.505$
- $\theta_2 = 81.30$ deg
- $P_2/P_1 = 2.004$

- Shock 2: $M_2 = 2.505$, $\theta_2 = 10$ deg. (we know that we require θ_2 as at shock 1 and therefore parallel to the top horizontal surface 20 deg inclined)

- $\theta_1 = 81.30$ deg, $M_1 = 3.88$, $P_1/P_2 = 1.800$
- $P_3 = p_1 \cdot (P_2/P_1) \cdot (P_1/P_2) = 100 \cdot 2.004 \cdot 1.800 = 360.72 Pa$



ANURITA

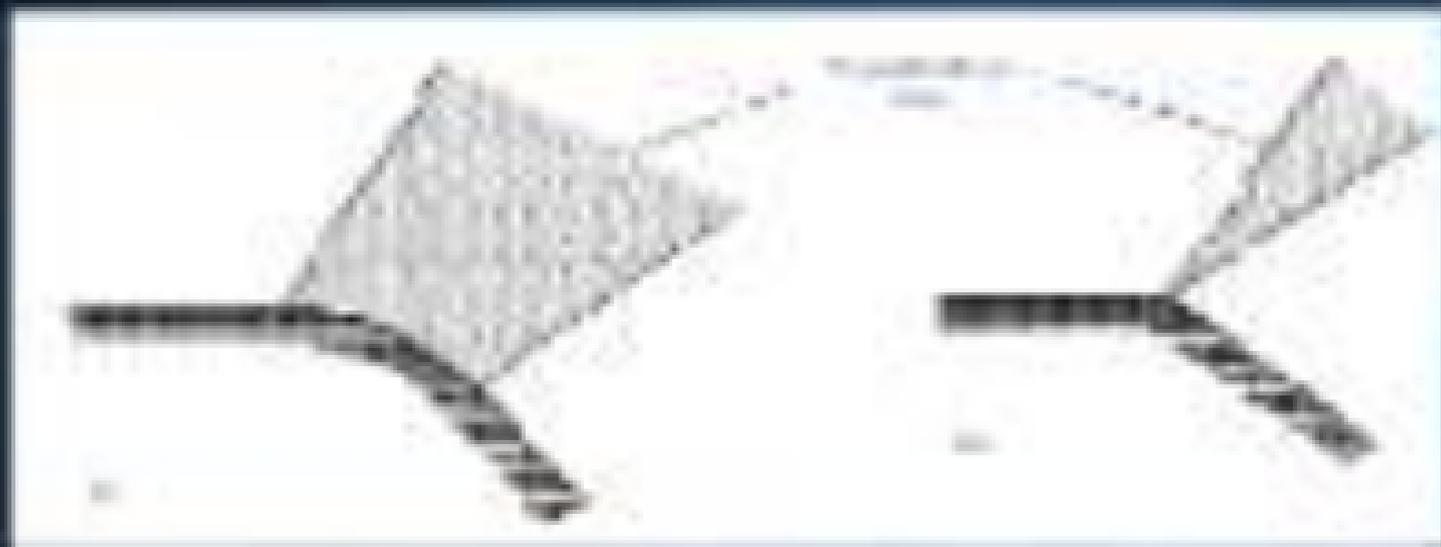


23AES212
COMPRESSIBLE
FLUID FLOW

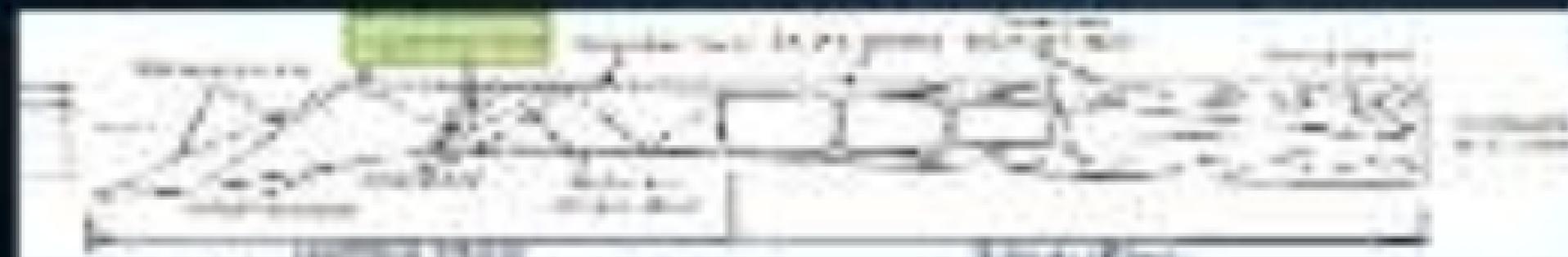
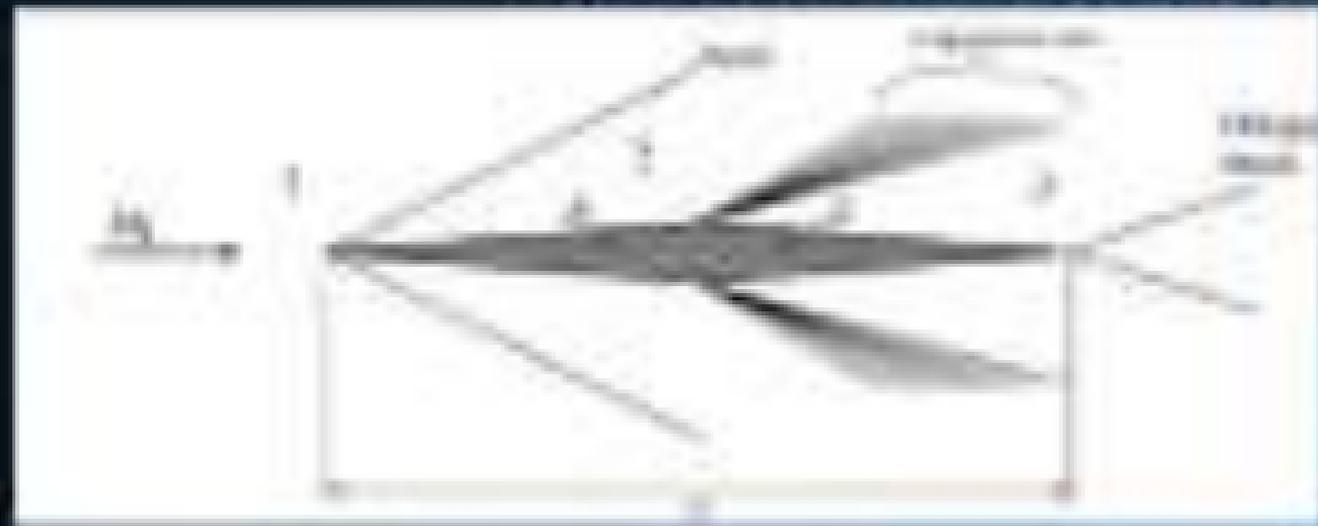
Expansion Waves

EXPANSION WAVES

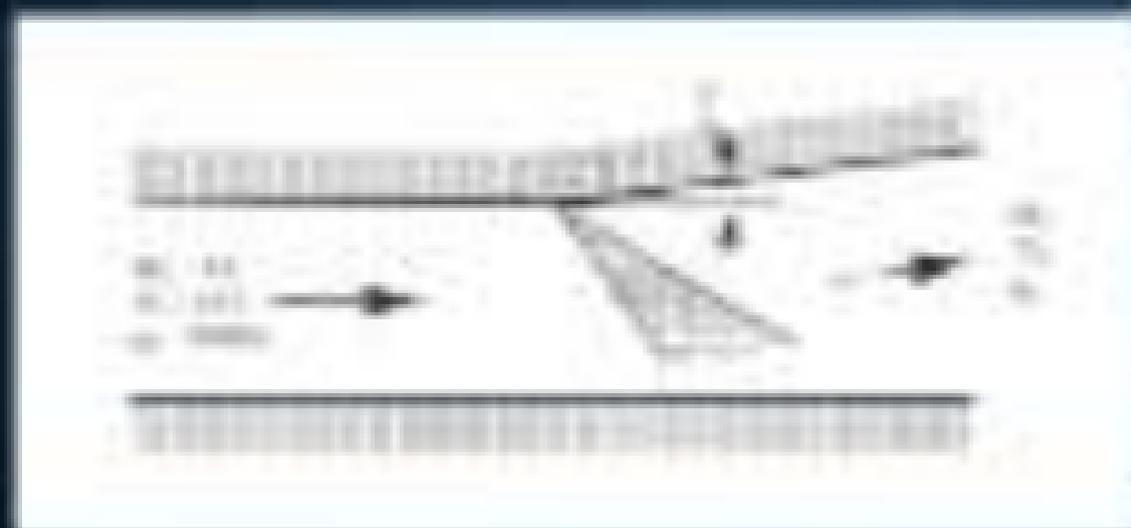




APPLICATIONS..



A TYPICAL EXPANSION..



EXPANSION WAVES

- Flow phenomena that occur in the closed flow where a portion of flow is treated as free surface conditions → **transient open channel flow**
- A one dimensional unsteady flow regime which can represent flow systems composed of an infinite number of MCs across a channel (Practical) **Mayer expansion waves**
- The flow does not have a sharp transition, it shock, but gradually with increasing Mach wave from flow by an infinitesimal amount.
 - flow with increased magnitude, there is small backflow **the flow nature of water expansion fan or flow energy expansion**
- The Mach number characteristics of the expansion waves are $(M_2 = M_1 + 1)$, while pressure, density, and temperature decrease.

DURING EXPANSION...

- The static pressure drops & the flow accelerates in a continuous process
- The expansion proceeds through a fan of Mach lines
- Consider expansion from M_1 to M_2
 - The leading Mach wave has
 - $\mu_1 = \sin^{-1}(1/M_1)$
 - And the trailing Mach wave has
 - $\mu_2 = \sin^{-1}(1/M_2)$





PRANDTL

PRANDTL-MEYER EXPANSION



MEYER

- Application of supersonic flow **compressible**
- An isentropic process of gradual expansion over an infinite number of Mach waves
- Streamline pressure remains constant
- This was to solve a specific question, as it flows around a sharp nose and 1-D
- Analysis: determine M_2 , θ_2 , T_2 etc. if M_1 and there are specific θ given, may determine the deflection required as follows a lot of properties
- Goody flow, also later with

Theodor Meyer

1882-1972



Theodor Meyer





CHANGES ACROSS A WEAR WAVE...



Applying the **level set** to the margin: **2008**

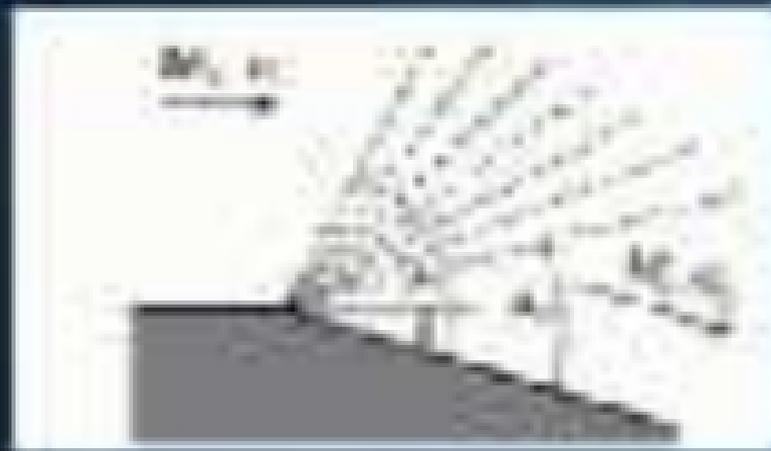
$$\frac{\partial \phi}{\partial t} + \mathbf{v} \cdot \nabla \phi = 0$$

$$\frac{\partial \phi}{\partial t} + \mathbf{v} \cdot \nabla \phi = 0$$

$$\frac{\partial \phi}{\partial t} + \mathbf{v} \cdot \nabla \phi = 0$$



THE EXPANSION FAN



INFINITESIMAL DEFLECTION

$\rightarrow \text{The } \delta u \rightarrow 0$

$$\rightarrow \text{Stress} = \frac{E}{L} \delta u$$

$\rightarrow \text{The } \delta u \rightarrow 0$

$$\rightarrow \text{Strain} = \frac{\delta u}{L}$$

Displacement higher order $\rightarrow \frac{1}{2} \delta u \delta u$

$$\frac{1}{2} \delta u \delta u$$

$$\frac{1}{2} \delta u \delta u$$

INTEGRATION ACROSS THE FINITE DEFLECTION ANGLE..

$$\frac{d\theta}{d\alpha} = \frac{1}{\cos^2 \alpha}$$

• Can be represented as

$$\frac{d\theta}{d\alpha} = \frac{1}{1 - \sin^2 \alpha}$$

$$\theta = \frac{\alpha}{\cos^2 \alpha}$$

PRANDTL-MEYER FUNCTION

The integral

$$\int \frac{d\theta}{\sqrt{1 - \frac{5}{4} \theta^2}}$$

is evaluated as:

$$\frac{1}{\sqrt{1 - \frac{5}{4} \theta^2}} = \frac{1}{\sqrt{1 - \frac{5}{4} \theta^2}} \cdot \frac{\sqrt{1 - \frac{5}{4} \theta^2}}{\sqrt{1 - \frac{5}{4} \theta^2}} = \frac{\sqrt{1 - \frac{5}{4} \theta^2}}{1 - \frac{5}{4} \theta^2}$$

Since that based on the inverse trig. the functions and the angles are related as:

$$1 - \frac{5}{4} \theta^2 = \cos^2 \theta$$

Therefore, $\theta = \cos^{-1} \sqrt{1 - \frac{5}{4} \theta^2}$



MAXIMUM EXPANSION ANGLE

$$\sin \theta_{\max} = \frac{\sqrt{1 - \frac{1}{M^2}}}{\sqrt{1 - \frac{1}{M^2} + \frac{1}{M^2}}} = \sqrt{1 - \frac{1}{M^2}}$$

• When hypersonic, where $M \gg 1$

$$\sin \theta_{\max} \approx \frac{1}{M}$$

• The expansion angle is $\propto 1/M$ (small angle)

$$\theta_{\max} \approx \frac{1}{M}$$



ANGLES OF THE LEADING & THE TRAILING MACH WAVES



$$\mu = \frac{1}{M} \quad \text{or} \quad \sin \mu = \frac{1}{M} \quad \text{or} \quad \mu = \sin^{-1} \left(\frac{1}{M} \right)$$

NUMERICAL PROBLEM



• Example

- $f(x, y) = 100(x^2 - y)^2 + (x - 1)^2$ (Rosenbrock)
- $f(x, y) = 1.5x^2 - 0.5y^2$ (Saddle)
- $f(x, y) = 1.5x^2 - 0.5y^2$ (Saddle)

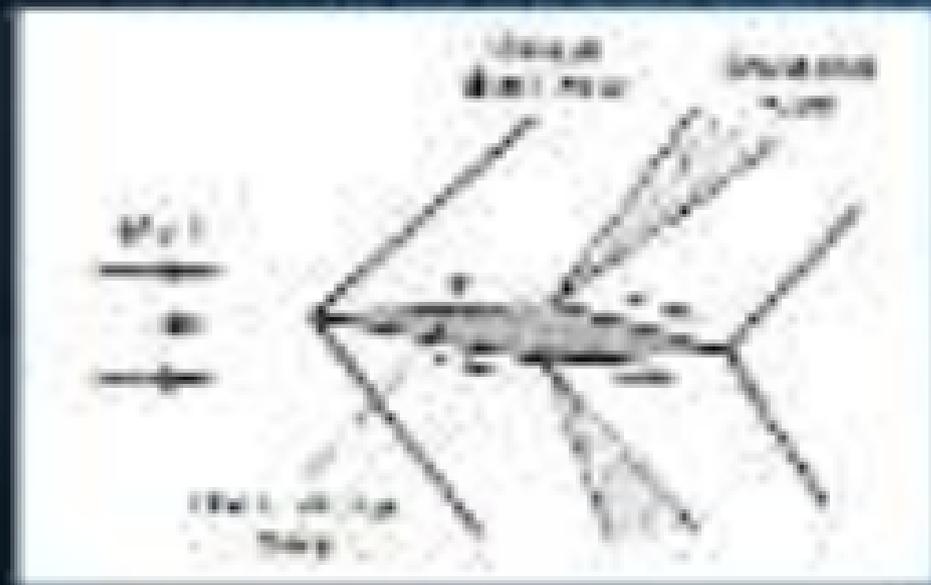
INTRUSIVE MEASUREMENTS IN SUPERSONIC FLOWS



Probe with multiple ports for total pressure measurement

An expensive device that measures total pressure of the probe as it takes a direct hit; resulting in the formation of a curved shock wave. This is important since for the conventional rectangular probe, shock formation. Hence the stagnation pressure measured will be $P_{0,2}$. Can be related to M_0 .

FLOWS WITH SHOCKS AND EXPANSIONS



SHOCK-EXPANSION METHOD

- + Several supersonic flow cases can be analyzed (without) by **patching together appropriate relationships** of the oblique shock wave, and the Prandtl-Meyer expansion fan.
- + TWO distinct aspects of the flow field:
 - The structure ahead of the leading edge is **non-coupled**, **streamlines do not influence**
 - The flow streams over the upper and lower surfaces are **completely independent of one another**



SHOCK EXPANSION **44** **COMPLEX** **77**

[METHOD]

- An approach for calculation of aerodynamic forces on two-dimensional airfoils using the **Prandtl-Glauert singularity method** and the approximation of a **flowfield by vortex sheets**.
 - Flat plate, or airfoil, at α .
 - Discrete vortices.
- **Using the oblique shock relations and the Prandtl-Glauert function**.
 - shock pressure on all surfaces of the diamond wedge and flat plate and their lift and drag forces are calculated.
- **Applicable to sharp-nosed bodies with an attached shock wave**.



A FLAT PLATE AT AN ANGLE OF ATTACK

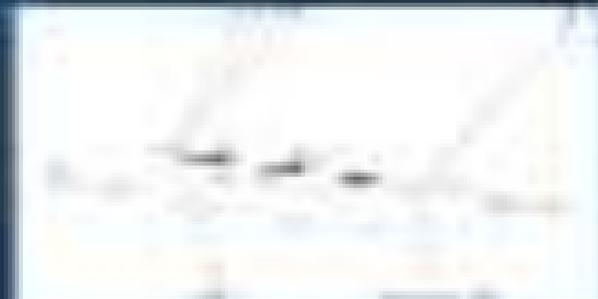
- The flow in the upper surface is curved through an expansion angle by means of a Prandtl-Glauert singularity line situated in the leading edge of the airfoil, whereas the flow in the lower side is turned through a compression angle by means of an oblique shock.
- The flow in the upper surface is recompressed to the upstream pressure by means of an oblique shock wave situated in the trailing edge of the airfoil. Likewise, the flow in the lower profile is recompressed to the upstream pressure by means of an oblique shock.



DRAG & LIFT

- Drag and lift forces give rise to lift and drag

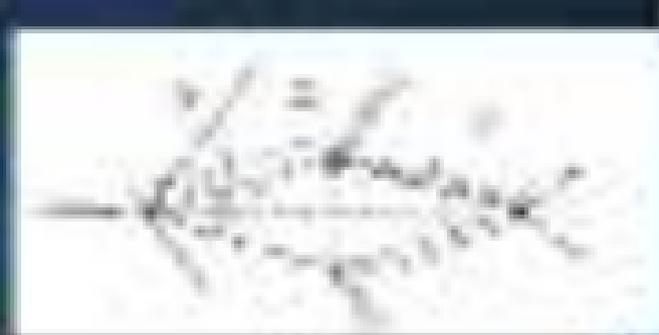
$$L = (\rho V^2 - p_0) c_l \frac{1}{2} S$$
$$D = (\rho V^2 - p_0) c_d \frac{1}{2} S$$



- What is the reason for this drag? Note that we have a permanent velocity loss.....



WAVE DRAG



→ 2D wave pattern transverse waves form good a body waves (2D) drag coefficient C_D $\propto U^2$

→ Mod. for 3D approximations, then

→ 3D wave pattern form transverse waves and longitudinal waves due to the 3D structure approximations

The drag caused by transverse & longitudinal waves is called **WAVE DRAG**

→ Drag per unit area

$$D = \frac{1}{2} \rho U^2 C_D = \frac{1}{2} \rho U^2 C_D \frac{A}{L} = \frac{1}{2} \rho U^2 C_D \frac{A}{L}$$

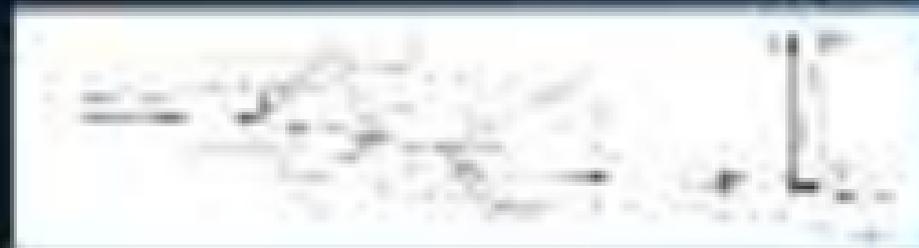
$$D = \frac{1}{2} \rho U^2 C_D$$

Wave drag coefficient C_D

WAVE DRAG – TERM FROM SHIP
HYDRODYNAMICS ?



WAVE DRAG – ON FLAT PLATE



Wave drag is a component of the total drag force that arises due to the presence of shock waves in the flow field.

$$\begin{aligned} D &= \int_0^L \rho U^2 \sin^2 \theta dx \\ D^* &= \int_0^L \rho U^2 \sin^2 \theta \frac{dx}{L} \\ D^* &= \int_0^1 \rho U^2 \sin^2 \theta dx^* \end{aligned}$$

ACCURACY OF SHOCK-EXPANSION MODEL.



Shocks from a
real world data
from CFD
model

NUMERICAL PROBLEM

PROBLEM PROPOSED BY **MAHESHKANTH K. S**

- A thermocouple placed in a supersonic stream of air records a stagnation pressure of 100 kPa and a static pressure of 100 kPa. If the stagnation temperature is 400 K, determine the Mach number & velocity of the main stream flow.

- Given: The value of P_{02} is 100 kPa
 - Calculate Mach
- Give M_1 from table
- To find M_1 given T_1 , T_0 & M_1 give V_1



Velocity = 1000 m/s and 300 K

Stagnation pressure = 200 kPa

→ $M_1 = 1.5$

→ $V_1 = 1000$ m/s

NUMERICAL PROBLEM

- Consider a symmetric, diamond-shaped object, see figure at $Mach = 0.8$ past the initial surface, with inlet angle of attack. The freestream static pressure is 1 atm. The included angle for the leading and the rear faces is $(10/10)^\circ$ and $(20/20)^\circ$ respectively. Determine the pressure distribution on the object surface.



SOLUTION

Steps

- Solve for the unknown stock at including output MC & demand curve
- Calculate MR/P
- Share has the representative firm, firm MC & MR
- Calculate MR
- $P_{MR} = P_{MC}$ (Equilibrium firm)
- Then MR & P_{MC} to calculate Q

Calculate output for MR by 3 firms at 1/3
of total output from each plant
Price = demand curve = $280 - 0.0001Q$
 $MR = P_{MR} = 2P$

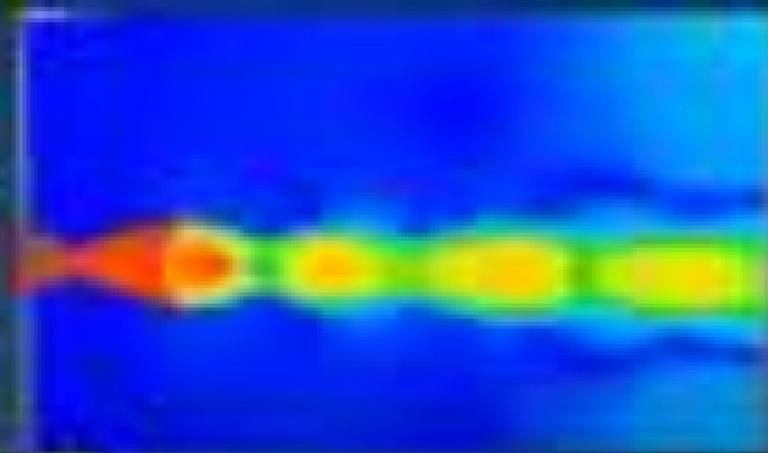
$$\text{Revenue } P_{MR} = 420000$$

Representative firm at 1/3 = 22.5
Share = $120000 - 0.0001Q$ (Long Run
price)

$MR = P_{MR} = 420000$ (Long Run price 200)
= Revenue for each firm for 1/3 share
Revenue = $MR = 120000 - 0.0001Q$
 $120000 - 0.0001Q = 420000$



ANURITA



23AES212

COMPUTATIONAL FLUID DYNAMICS
PROF. DR. P. S. RAO

Nozzle Flow



FLOW WITH **AFRICA CHANGE**

- **What is the impact of rising and not responsible flow?**
- Most of the politicians that we considered are not concerned for any change.
- How does the same change impact the flow position where the flow is **COMFORTABLE**?
- **For more perspective:**
 - How do the same politicians view on national progress in the last years?
 - How do we measure the flow just made today?



QUASI 1-DIMENSIONAL FLOW

- Velocity is only a function of the streamwise coordinate



CONTINUITY EQUATION

= Steady, 3-dimensional flow

$$\frac{\partial}{\partial x}(\rho u) + \frac{\partial}{\partial y}(\rho v) + \frac{\partial}{\partial z}(\rho w) = 0$$



$$\rho_1 V_1 A_1 = \rho_2 V_2 A_2$$

MOMENTUM EQUATION

$$\frac{d}{dt} \int_{CV} \rho \mathbf{u} dV + \int_{CS} \rho \mathbf{u} (\mathbf{u} \cdot \mathbf{n}) dA = \sum \mathbf{F}_{ext}$$

- Steady flow without body forces & mass no. forces

$$\rho_1 A_1 \mathbf{u}_1 + \int_{CS} \rho \mathbf{u} (\mathbf{u} \cdot \mathbf{n}) dA = \rho_2 A_2 \mathbf{u}_2 + \rho \mathbf{u} \mathbf{u} \cdot \mathbf{n} dA$$

ENERGY EQUATION

$$\begin{aligned} \frac{d}{dt} \int_{\Omega} \rho \mathbf{v} \cdot \mathbf{v} &= \int_{\Omega} \rho \mathbf{v} \cdot \mathbf{a} + \int_{\partial \Omega} \rho \mathbf{v} \cdot \mathbf{v} \mathbf{n} \\ &= \int_{\Omega} \rho \mathbf{v} \cdot \mathbf{a} + \int_{\partial \Omega} \rho \mathbf{v} \cdot \mathbf{v} \mathbf{n} \end{aligned}$$

- In the absence of external forces, we'll consider such free boundary, but steady flow of incompressible fluid in the Ω domain

$$\frac{d}{dt} \int_{\Omega} \rho \mathbf{v} \cdot \mathbf{v} = \int_{\Omega} \rho \mathbf{v} \cdot \mathbf{a}$$

$$\rho \mathbf{v} \cdot \mathbf{a} = \rho \mathbf{v} \cdot \nabla \mathbf{v} \cdot \mathbf{v}$$

Handwritten text on a page, likely bleed-through from the reverse side. The text is extremely faint and illegible due to the image quality. It appears to be a list or a series of notes, possibly containing names and dates, but the specific content cannot be discerned.

$$\begin{aligned} L &= \frac{1}{2} \dot{\theta}^2 - \frac{1}{2} \omega^2 \theta^2 \\ P &= \dot{\theta} \end{aligned}$$

$$\frac{dP}{dt} = -\frac{\partial H}{\partial \theta} = -\omega^2 \theta$$

→ Lösung des Ansatzes in Form von $\theta(t)$

$$\theta = \theta_0 \cos(\omega t)$$

$$\begin{aligned} \frac{d\theta}{dt} &= -\dot{\theta}_0 \sin(\omega t) \\ \frac{d^2\theta}{dt^2} &= -\omega \dot{\theta}_0 \cos(\omega t) \\ \frac{d^2\theta}{dt^2} &= -\omega^2 \theta \end{aligned}$$

$$\left[\frac{d\theta}{dt} = (\dot{\theta}_0^2 - \omega^2) \frac{d\theta}{dt} \right]$$

VARIATION OF VELOCITY WITH AREA

$$\frac{dV}{V} = -\frac{dA}{A} \quad \text{--- (1)}$$

• **Continuity equation (Case 1) $\rho = \text{const}$**

→ How does A vary with V ?

→ When ρ is constant, eq. (1) can be integrated as follows by eq. (1)

• **Continuity equation (Case 2) $\rho \neq \text{const}$**

→ How does A vary with V ?

→ When ρ is not constant, eq. (1) can be integrated as follows by eq. (1)

Initial position	Initial velocity	Final velocity
------------------	------------------	----------------

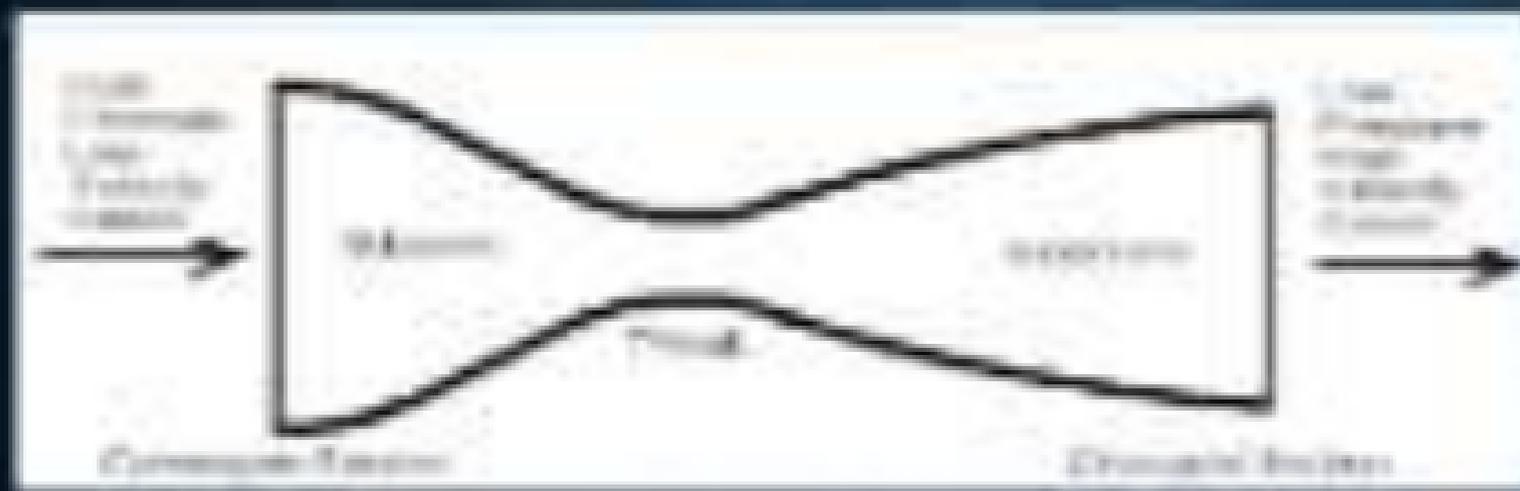


HOW DO WE ACCELERATE THE FLOW TO SUPERSONIC ?

- In the subsonic regime, a **convergent** duct is required to accelerate the flow to Mach 1.0.
- At $M = 1$ the flow requires a **change** in section or curvature to then become supersonic.
- Hence, a **CONVERGENT-DIVERGENT** duct is required for accelerating the flow from subsonic to supersonic Mach numbers.

C-D NOZZLE

[DE LAVAL NOZZLE]



DESIGN OPERATION OF A CD NOZZLE: HOW DO THE FLOW PROPERTIES VARY ACROSS ?

- Velocity: Increases continuously
- Mach number: Increases continuously
- Static pressure: Decreases to Mach 1 then increases (Expansion)
- Density: \downarrow
- Static temperature: \downarrow
- Stagnation temperature: Remains constant (adiabatic flow with no heat)
- Stagnation pressure: Decreases (due to friction, losses, etc.)



CD NOZZLE IN ROCKETS





NOZZLE DESIGN: HOW TO DESIGN FOR A SPECIFIED MACH NUMBER

- What should be the exit area to achieve a desired Mach number?
- How should we determine the pressure ratio of some section? (The THRUST area) ?

AREA VARIATION IN THE DIVERGENT SECTION

Applying continuity between a
 upstream section A and the throat section B

$$\frac{A_1}{A_2} = \frac{V_2}{V_1} = \frac{C_2}{C_1} = \frac{C_2}{C_1} \frac{V_2}{V_1}$$

$$= \frac{C_2}{C_1} \frac{C_1}{C_2} \frac{V_2}{V_1}$$

$$\frac{A_1}{A_2} = \frac{C_2}{C_1} \frac{C_1}{C_2} \frac{V_2}{V_1} = \frac{C_2}{C_1} \frac{C_1}{C_2} \frac{V_2}{V_1}$$

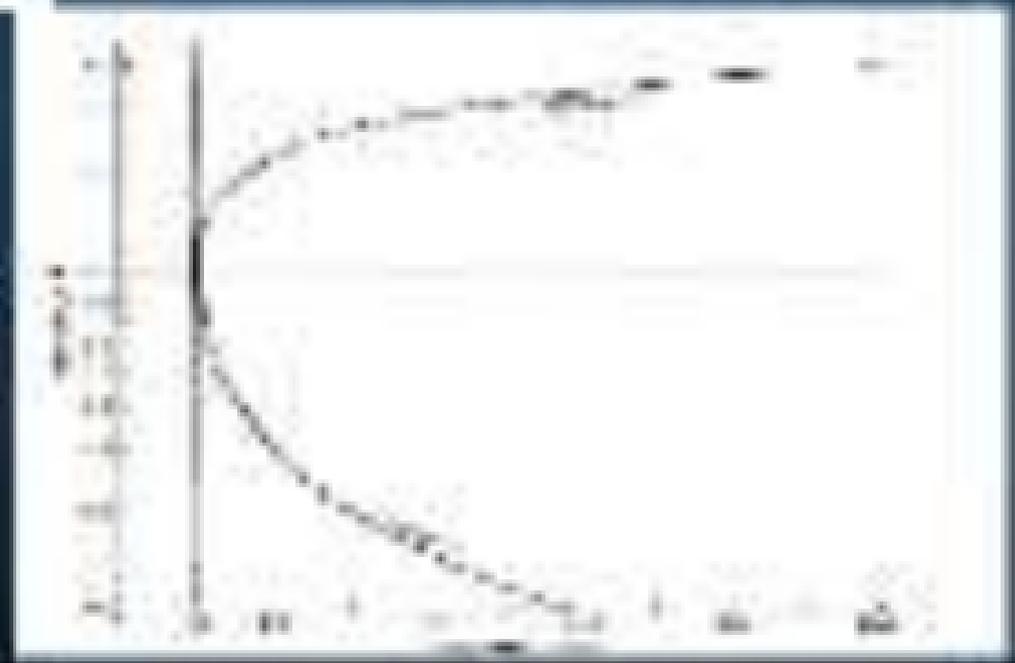
$$\frac{A_1}{A_2} = \frac{C_2}{C_1} \frac{C_1}{C_2} \frac{V_2}{V_1} = \frac{C_2}{C_1} \frac{C_1}{C_2} \frac{V_2}{V_1}$$

$$\frac{A_1}{A_2} = \frac{C_2}{C_1} \frac{C_1}{C_2} \frac{V_2}{V_1}$$



AREA-MACH NUMBER RELATION

$$\left(\frac{A}{A^*}\right)^2 = \frac{1}{M^2} \left[\frac{2 + \gamma}{2} + \frac{\gamma - 1}{2} M^2 \right]^{\frac{\gamma + 1}{\gamma - 1}}$$



STAGNATION VS STATIC CONDITIONS



MACH NUMBER VARIATION ALONG THE NOZZLE

- Design operation: A combustion chamber is designed to Mach 0.5, and the nozzle is designed to Mach 2.5.

- Design: $A_c = 0.5 A^*$



ISENTROPIC FLOW TABLES

M	$\frac{r_2}{r_1}$	$\frac{T_2}{T_1}$	$\frac{P_2}{P_1}$	$\frac{\rho_2}{\rho_1}$	$\frac{A_2}{A_1}$	$\frac{V_2}{V_1}$	$\frac{M_2}{M_1}$
1.00	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
1.05	1.0474	0.9948	0.9896	1.0036	1.0000	0.9952	1.0500
1.10	1.1070	0.9896	0.9794	1.0072	1.0000	0.9904	1.1000
1.15	1.1788	0.9848	0.9692	1.0108	1.0000	0.9856	1.1500
1.20	1.2628	0.9804	0.9590	1.0144	1.0000	0.9808	1.2000
1.25	1.3600	0.9764	0.9488	1.0180	1.0000	0.9760	1.2500
1.30	1.4714	0.9728	0.9386	1.0216	1.0000	0.9716	1.3000
1.35	1.6000	0.9696	0.9284	1.0252	1.0000	0.9672	1.3500
1.40	1.7480	0.9668	0.9182	1.0288	1.0000	0.9628	1.4000
1.45	1.9180	0.9644	0.9080	1.0324	1.0000	0.9584	1.4500
1.50	2.1130	0.9624	0.8978	1.0360	1.0000	0.9540	1.5000
1.55	2.3380	0.9608	0.8876	1.0396	1.0000	0.9496	1.5500
1.60	2.5980	0.9596	0.8774	1.0432	1.0000	0.9452	1.6000
1.65	2.8980	0.9588	0.8672	1.0468	1.0000	0.9408	1.6500
1.70	3.2440	0.9584	0.8570	1.0504	1.0000	0.9364	1.7000
1.75	3.6440	0.9584	0.8468	1.0540	1.0000	0.9320	1.7500
1.80	4.1040	0.9588	0.8366	1.0576	1.0000	0.9276	1.8000
1.85	4.6320	0.9596	0.8264	1.0612	1.0000	0.9232	1.8500
1.90	5.2360	0.9608	0.8162	1.0648	1.0000	0.9188	1.9000
1.95	5.9240	0.9624	0.8060	1.0684	1.0000	0.9144	1.9500
2.00	6.7040	0.9644	0.7958	1.0720	1.0000	0.9100	2.0000
2.05	7.5840	0.9668	0.7856	1.0756	1.0000	0.9056	2.0500
2.10	8.5720	0.9696	0.7754	1.0792	1.0000	0.9012	2.1000
2.15	9.6760	0.9728	0.7652	1.0828	1.0000	0.8968	2.1500
2.20	10.9040	0.9764	0.7550	1.0864	1.0000	0.8924	2.2000
2.25	12.2640	0.9804	0.7448	1.0900	1.0000	0.8880	2.2500
2.30	13.7640	0.9848	0.7346	1.0936	1.0000	0.8836	2.3000
2.35	15.4120	0.9896	0.7244	1.0972	1.0000	0.8792	2.3500
2.40	17.2160	0.9948	0.7142	1.1008	1.0000	0.8748	2.4000
2.45	19.1840	0.9996	0.7040	1.1044	1.0000	0.8704	2.4500
2.50	21.3240	1.0048	0.6938	1.1080	1.0000	0.8660	2.5000
2.55	23.6440	1.0096	0.6836	1.1116	1.0000	0.8616	2.5500
2.60	26.1520	1.0148	0.6734	1.1152	1.0000	0.8572	2.6000
2.65	28.8560	1.0204	0.6632	1.1188	1.0000	0.8528	2.6500
2.70	31.7640	1.0264	0.6530	1.1224	1.0000	0.8484	2.7000
2.75	34.8840	1.0328	0.6428	1.1260	1.0000	0.8440	2.7500
2.80	38.2240	1.0396	0.6326	1.1296	1.0000	0.8396	2.8000
2.85	41.7920	1.0468	0.6224	1.1332	1.0000	0.8352	2.8500
2.90	45.5960	1.0544	0.6122	1.1368	1.0000	0.8308	2.9000
2.95	49.6440	1.0624	0.6020	1.1404	1.0000	0.8264	2.9500
3.00	53.9440	1.0708	0.5918	1.1440	1.0000	0.8220	3.0000
3.05	58.4960	1.0796	0.5816	1.1476	1.0000	0.8176	3.0500
3.10	63.3040	1.0888	0.5714	1.1512	1.0000	0.8132	3.1000
3.15	68.3720	1.0984	0.5612	1.1548	1.0000	0.8088	3.1500
3.20	73.7040	1.1084	0.5510	1.1584	1.0000	0.8044	3.2000
3.25	79.3040	1.1188	0.5408	1.1620	1.0000	0.8000	3.2500
3.30	85.1720	1.1296	0.5306	1.1656	1.0000	0.7956	3.3000
3.35	91.3040	1.1408	0.5204	1.1692	1.0000	0.7912	3.3500
3.40	97.7040	1.1524	0.5102	1.1728	1.0000	0.7868	3.4000
3.45	104.3720	1.1644	0.5000	1.1764	1.0000	0.7824	3.4500
3.50	111.3040	1.1768	0.4900	1.1800	1.0000	0.7780	3.5000
3.55	118.5040	1.1896	0.4800	1.1836	1.0000	0.7736	3.5500
3.60	125.9720	1.2028	0.4700	1.1872	1.0000	0.7692	3.6000
3.65	133.7040	1.2164	0.4600	1.1908	1.0000	0.7648	3.6500
3.70	141.7040	1.2304	0.4500	1.1944	1.0000	0.7604	3.7000
3.75	150.0000	1.2448	0.4400	1.1980	1.0000	0.7560	3.7500
3.80	158.5960	1.2596	0.4300	1.2016	1.0000	0.7516	3.8000
3.85	167.4920	1.2748	0.4200	1.2052	1.0000	0.7472	3.8500
3.90	176.6960	1.2904	0.4100	1.2088	1.0000	0.7428	3.9000
3.95	186.2040	1.3064	0.4000	1.2124	1.0000	0.7384	3.9500
4.00	196.0000	1.3228	0.3900	1.2160	1.0000	0.7340	4.0000



THE IMPACT OF "BACK" PRESSURE

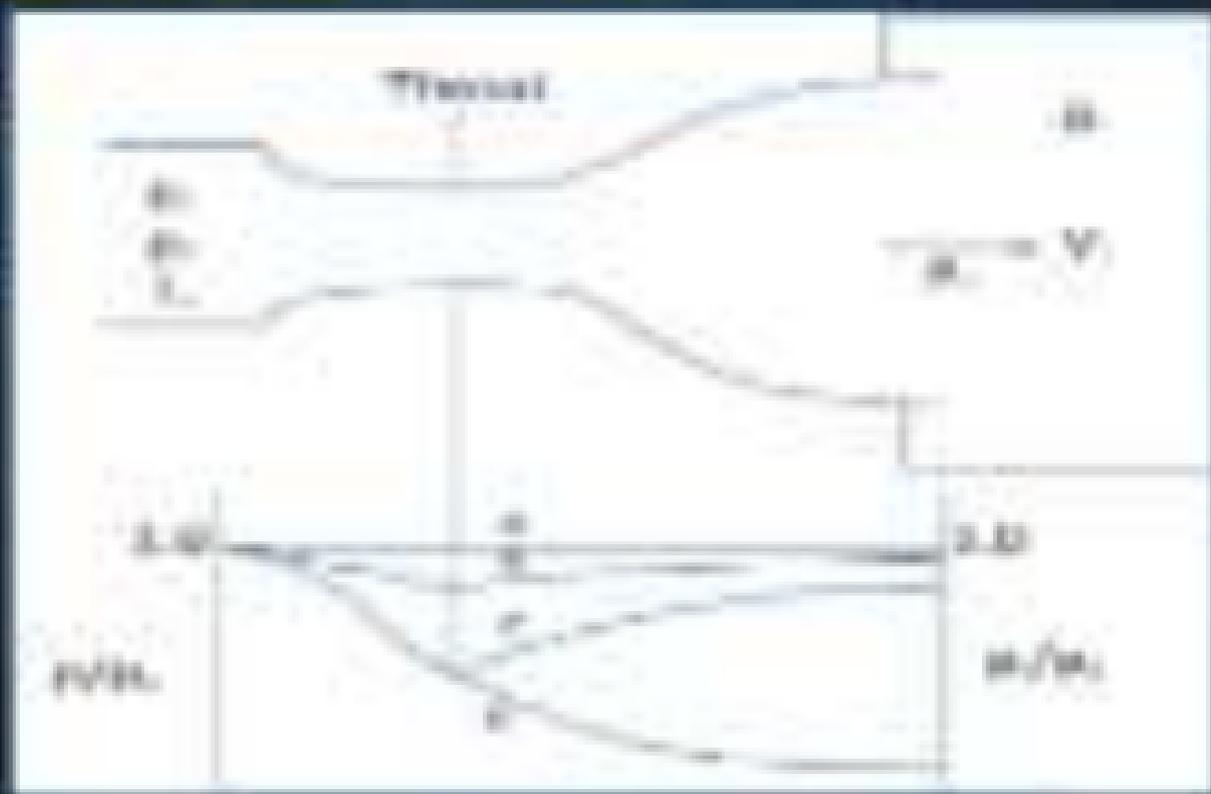
- Consider an arrangement where you can vary the P_b , the pressure to which the gas discharges from the nozzle
- The supply is from a tank maintained at P_0
- What if $P_b = P_0$ to begin with?
 - Dis flow!



When the P_b the static pressure of the gas at the nozzle exit P_b is the same as the pressure in the tank P_0 , the gas does not flow. **NO FLOW!**

THE IMPACT OF "BACK" PRESSURE - CTD.

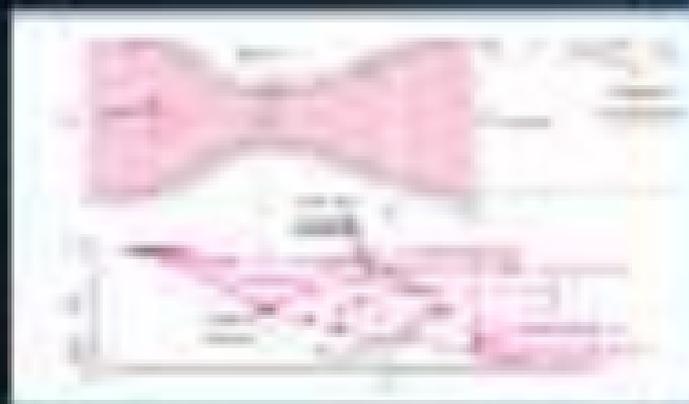
- Keep recording P, T, S, σ_t in flight
- Two main parts
- Recovery of the instrument & sampling methods
- Depth operation in the water - stopping position is a problem
- When the main flow slow down with delay on P



FORMATION OF SHOCK WAVES INSIDE...



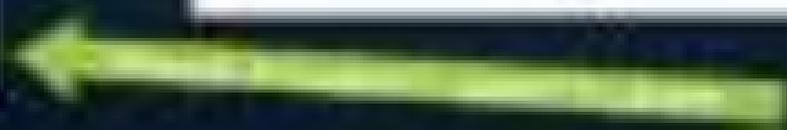
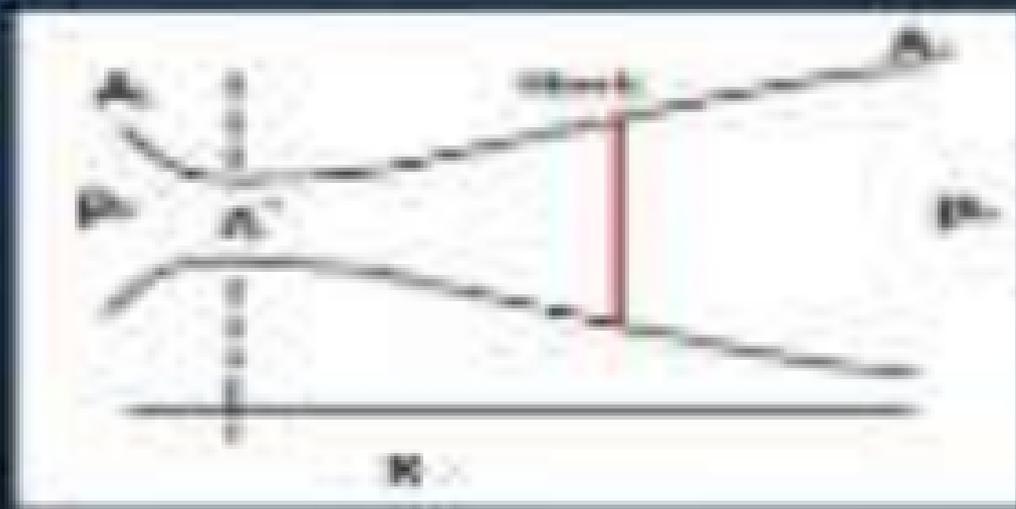
NOZZLE PRESSURE RATIO & MASS FLOW RATE - **CHOKING**



- The **critical** flow condition is reached when the pressure ratio is equal to the critical pressure ratio.
- The mass flow rate is maximum at this point and is called the **choking** condition.
- The mass flow rate is constant for all pressure ratios greater than the critical pressure ratio.
- The mass flow rate is independent of the upstream pressure for all pressure ratios greater than the critical pressure ratio.
- The mass flow rate is independent of the downstream pressure for all pressure ratios greater than the critical pressure ratio.
- The mass flow rate is independent of the nozzle geometry for all pressure ratios greater than the critical pressure ratio.

NORMAL SHOCK INSIDE THE NOZZLE

- **CU Omega Industries**
 - Main demand for typical applications
- They provide when $P_2 = P_{critical}$
- They provide when $P_2 > P_{critical}$ after the nozzle exit
- They provide when $P_2 < P_{critical}$ before the nozzle exit



UNDEREXPANSION vs OVEREXPANSION

- **Recessed:** P_2 is not immediately equal to P_1 but a significant jet issuing from a CD nozzle
 - Small P_2 - the static pressure of the gas in the jet is low
 - P_2 is not equal to P_1
 - It is also less specific than expansion in a normal shock wave and the jet velocity is less than P_1
- **The nozzle exit is the point of flow:**
 - (UNDEREXPANSION) when $P_2 < P_1$
 - (OVEREXPANSION) when $P_2 > P_1$
- **The Ideal/Characteristic Nozzle Expansion:** $P_2 = P_1$

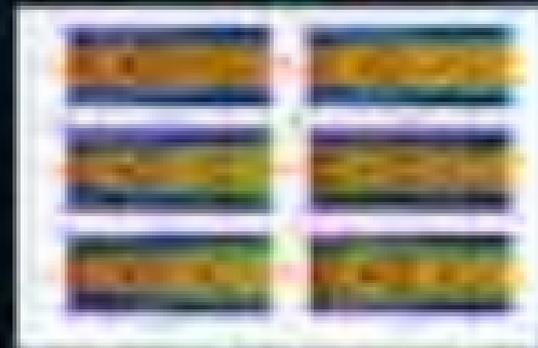


UNDER/OVEREXPANSION: THE CONSEQUENCES

- **Flare:** A series of expansion management moves (decisions) that consistently lead to an expansion strategy with the lowest **market share**
- **Under-expansion:** **underperformance** in market share over time for some years
Type A, three lines (in red, green, & blue)
 - **Flare:** see description on slide
 - **Over-expansion:** see slide

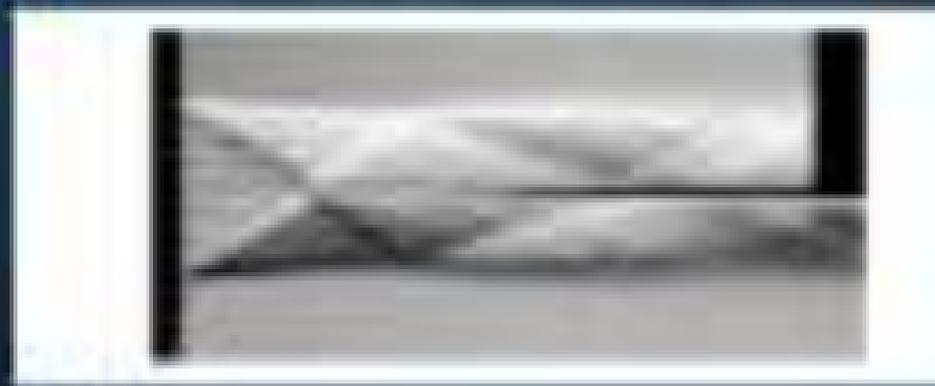
UNDEREXPANDED JET

- At the exit, the jet has a higher pressure than the ambient pressure.
- The jet expands and undergoes a series of shock & expansion waves subsonically.



OVEREXPANDED JET

- At the exit, the jet has a static pressure LOWER than the ambient pressure
- The jet undergoes overexpansion and must undergo multiple reflections to adjust to ambient pressure



ROCKET JET

- A rocket with fixed geometry results in a fixed throat area (Throat Pa) only at ONC altitude.
 - As the velocity increases pressure drops, changing with altitude.



Number of nodes in network



UNDER-EXPANDED

OPTIMUM (IDEAL)

SLIGHTLY OVER-EXPANDED

OVER-EXPANDED

Number of edges in network

SHOCK DIAMONDS



VIDEO

Image of human population growth from 1950 to 2050
by [BBC](#)

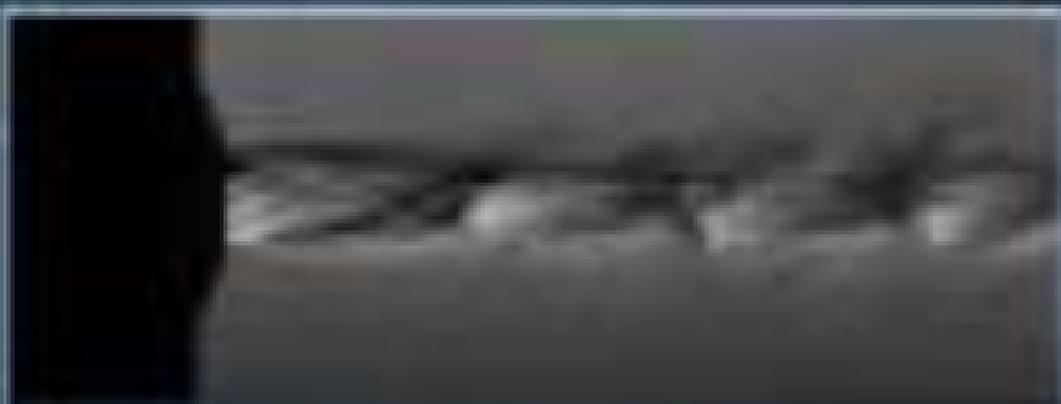
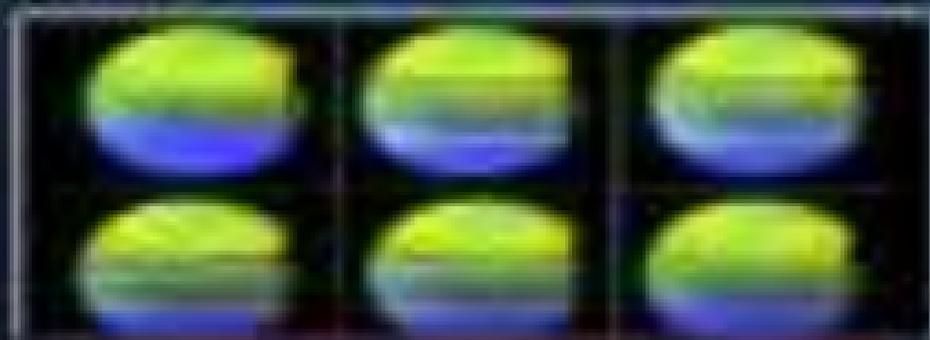


Image of human population growth from 1950 to 2050
by [BBC](#)



NUMERICAL PROBLEM - 1

- From the following Mach number values obtain an idea through how a shock wave Mach number of 2.0. Calculate the Area Ratio (A_0/A^*) for the normal expansion function in 1.29 for the respective gases.

$$\frac{A}{A^*} = \frac{1}{M} \left[\frac{2}{\gamma + 1} \left(1 + \frac{\gamma - 1}{2} M^2 \right) \right]^{1/(\gamma + 1)}$$



- The above eqn. can be used to compare the value of A_0/A^* for $\gamma = 1.4$ from the calculation of value for $\gamma = 1.29$.

GAS TABLES FOR GAMMA = 1.4

Table 6.1: Continues

M	β	$\frac{P}{P_0}$	$\frac{\rho}{\rho_0}$	$\frac{A}{A^*}$	$\frac{P}{P_0}$	$\frac{\rho}{\rho_0}$	$\frac{A}{A^*}$
0.000	0.0	1.000	1.000	1.000	0.720	1.000	1.000
0.005	0.1	0.995	0.995	1.000	0.718	0.995	1.000
0.010	0.2	0.990	0.990	1.000	0.716	0.990	1.000
0.015	0.3	0.985	0.985	1.000	0.714	0.985	1.000
0.020	0.4	0.980	0.980	1.000	0.712	0.980	1.000
0.025	0.5	0.975	0.975	1.000	0.710	0.975	1.000
0.030	0.6	0.970	0.970	1.000	0.708	0.970	1.000
0.035	0.7	0.965	0.965	1.000	0.706	0.965	1.000
0.040	0.8	0.960	0.960	1.000	0.704	0.960	1.000
0.045	0.9	0.955	0.955	1.000	0.702	0.955	1.000
0.050	1.0	0.950	0.950	1.000	0.700	0.950	1.000

NUMERICAL PROBLEM - 2

- A management development course is to be designed for 400 hours being applied for in 1991 with $\mu_0 = 100$ hours and $\sigma_0 = 100$ h. The data shows results shown in the following table. Determine the number and size and the design is based on the data if the class size is 100 hrs?
- What will be the main points and results shown in the table?



SOLUTION

1. Determine the flow table area for the inlet flow \dot{M}_1 from P. 1 (1)

2. Show that $\dot{M}_1 = \dot{M}_2 = \dot{M}_3 = 51.21 \text{ kg/s}$

3. Determine \dot{M}_4 and \dot{M}_5 in kg/s

4. Determine \dot{M}_6 and \dot{M}_7 in kg/s

5. T_2 from T_1 & P_2

6. T_3 from T_2 & P_3

7. Calculate Mass flow rate $\dot{M}_6 = \dot{M}_7$
 $= 0.525 \text{ kg/s}$

8. Calculate P_6 , from P_2 & P_3



GATE 2016

For a complete discussion see [www.gate.ac.in](#). **Admission to the course will be held on the following dates:**

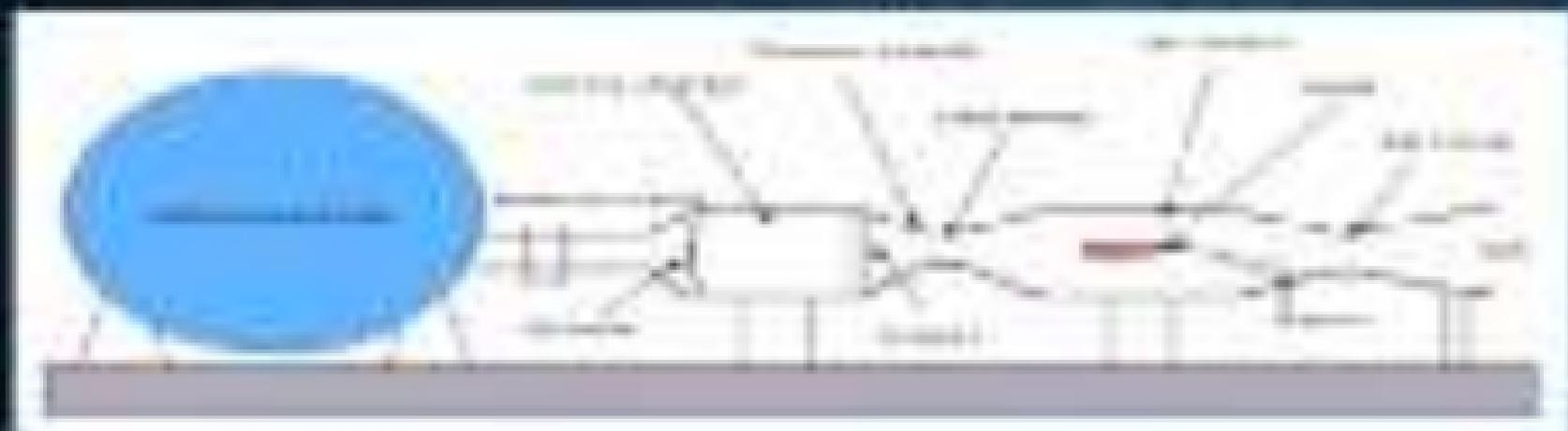
- 1st round: 15th to 19th August 2016
- 2nd round: 22nd to 26th August 2016
- 3rd round: 29th to 31st August 2016
- 4th round: 5th to 7th September 2016

SUPERSONIC WIND TUNNEL

- Experimental set up to provide a solution experiences than at the test section
- **Requirements**
 - A high pressure storage (tank)
 - Nozzle structure
 - A CD model to understand the flow of the supersonic impinging flow
 - Test section with the required pressure ratios



SUPERSONIC WIND TUNNEL



THE **SECOND** CD PASSAGE - WHY IS IT REQUIRED ? - I

- Consider a TFT with one arm on Mach number 3.8
- Assume the Dry Density of CD is atmospheric (1.225 kg/m³)
- What is the average regressive pressure required (Pa) ?
 - $P_0 = \rho C_0^2 \Rightarrow P_0 = 76.3 \text{ atm}$
- Checked that a normal shock is formed before the flow starts to reattach to atmospheric pressure



THE **SECOND** CD PASSAGE - WHY IS IT REQUIRED ? - II

- Consider that a normal cloud is formed by hydrogen gas. Over its discharge by electrolytic processes
 - Solar pressure inside normal (ion cloud) from 50 (bar) to 55 - 60
 - 14.155
 - $\rho = 1.4 \times 10^{-12} \text{ kg/m}^3$
 - $\rho = 1.4 \times 10^{-12} \text{ kg/m}^3$
 - $\rho = 1.4 \times 10^{-12} \text{ kg/m}^3$

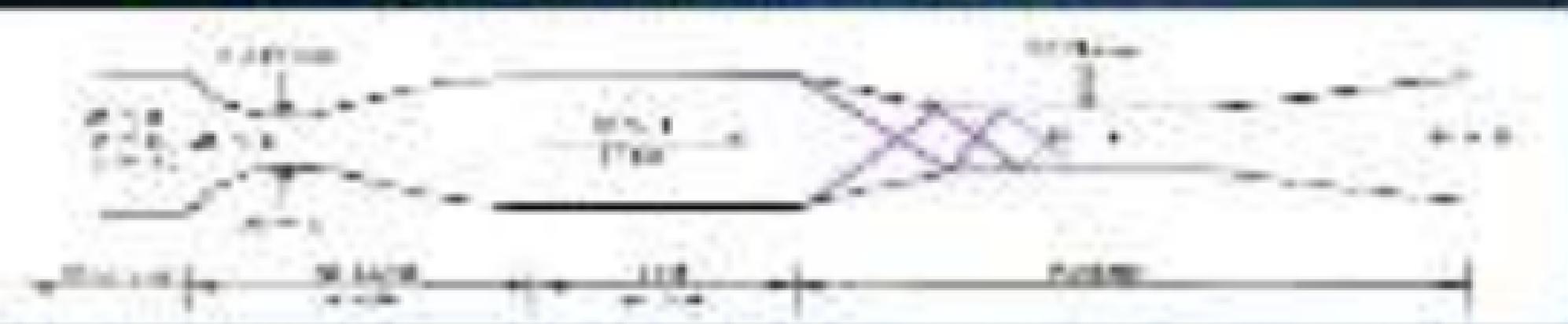


Consider that normal **CD** ions which are produced by action of solar pressure inside normal cloud.

A normal CD ion cloud always presents a normal composition, presents **CD** ions.

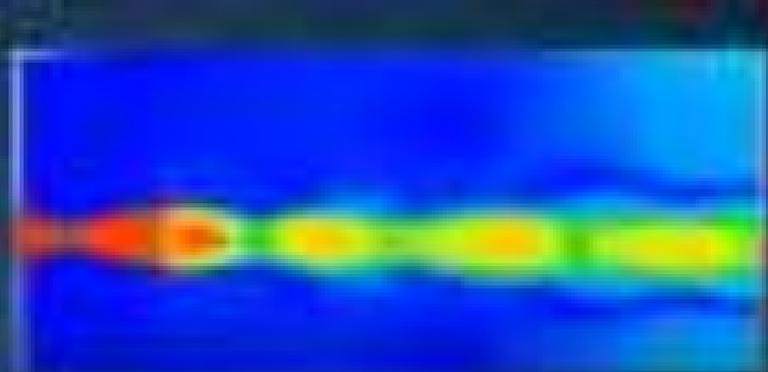
SUPERSONIC WIND TUNNEL COMPONENTS

- Initially supersonic flow is impeded by the nozzle by having a **CD Diffuser**
 - The diffuser decelerates the supersonic flow through a series of oblique shocks
- After the **nozzle** (all the oblique shocks in the nozzle are spaced such that the oblique shock waves do not intersect) flow is supersonic pressure is low through the nozzle





WASAFETA



23AES212

COMPTON SCATTER

PHOTO-PEAK

IDENTIFICATION

PLEASE VISIT AT www.iaea.org

ASSIGNMENT 2

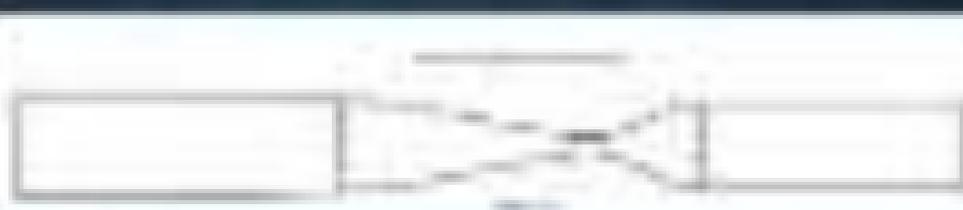
- The experimental setup includes an Air Temperature Logger connected to a USB reader, with an air flow controller of 0.5 L/min and a flowmeter with a resolution of 0.1. The storage tank provides the inlet flow with a total capacity of 10,000 kg of air and is assumed to be well-mixed. The inlet air pressure is 101,325 Pa. The melting chamber pressure is maintained at 0.1 MPa. The mass flow rate \dot{m} is the number measured by the last five digits of peak value number. Under these conditions, determine the time to melt 1 kg of ice in the hourly ice machine. Assume the ice is initially at 0°C and the melting chamber temperature is 0°C. The properties of air, as perfect gas. Assume perfect expansion in the nozzle.
 - [Lecture 10: Thermodynamics of Fluids in a Turbine](#)
 - [Lecture 11: Thermodynamics of Fluids in a Turbine](#)

**THE ISSUE OF NON-UNIFORMITY IN
MACH NUMBER DISTRIBUTION IN
SUPERSONIC NOZZLE DESIGN FOR
WIND TUNNELS - A CASE STUDY**

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TWO DIMENSIONAL CD NOZZLE FOR WIND TUNNEL

- The CD nozzle alignment nozzle (CD) has an experimental ability to work better.
- The nozzle block, it will support the nozzle and will be able to adjust the distance of the nozzle.

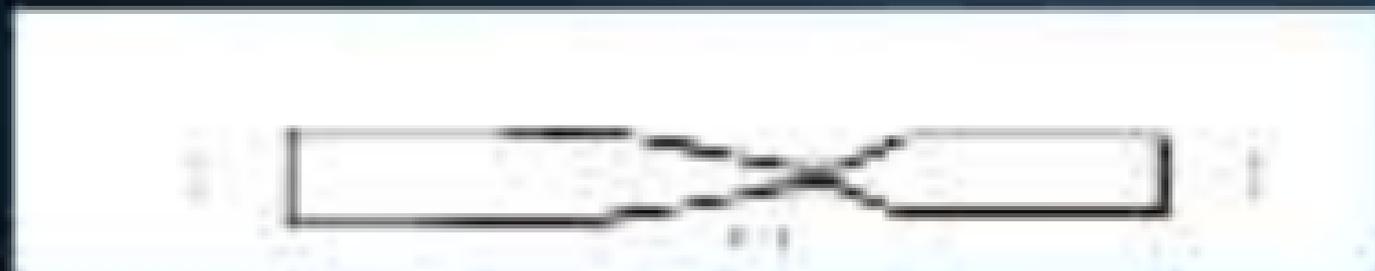


THE SLIDING BLOCK.



Diagram and photograph showing the sliding block mechanism.

- Waves that travel across the interface in the same direction as the wave speed are called **transmitted waves**, resulting in a **transmitted pulse** moving across the interface.



• The sinking itself results either "accidental deaths" or subject the survivors generally of the tragedy.

→ These deaths include the loss of lives of the passengers and crew. The sinking of the Titanic itself is a disaster caused by the structural failure of the hull and the loss of the ship's propulsion system.

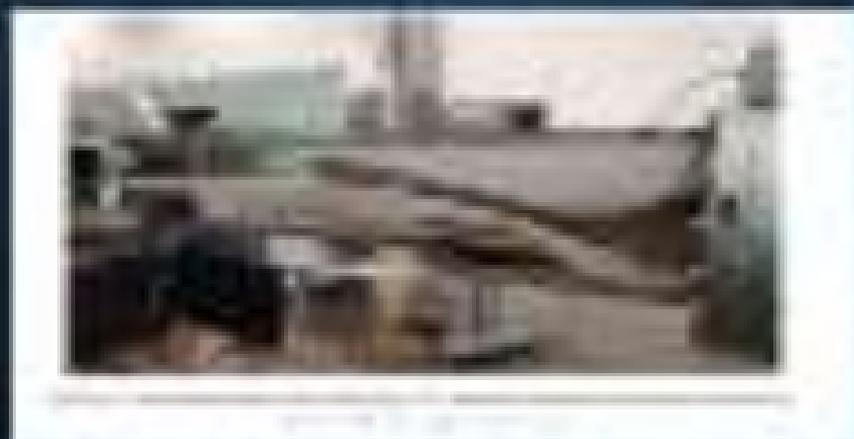
• The (IMO) International Maritime Organization (IMO) has its International Safety of Life at Sea Convention.

→ This convention, known as SOLAS, sets out the minimum standards for the construction, equipment, and operation of ships.

→ SOLAS also covers the requirements for the crew of the ship, the ship's equipment, and the ship's operation, including the use of lifeboats and life jackets.

THIS PROBLEM

- The water level in the river has been raised in heavy rain flow resulting in the boat motion, which is where the principal loads applied to the vessel.
- Moreover, much more to the point of vessel's hull displacement, more stresses in the divergent section of the vessel hull's member.



OBJECTIVE OF THE STUDY

- Analyze the flow within the ODU asymmetric nozzle used for jet
- Suggest a method for improving the flow uniformity using the MFC and regulated air flow
→ not used at the time of the creation of the ODU jet
- Establish a baseline for improvement of the Mach number uniformity in the test section.

THE MESH USED FOR CFD ANALYSIS



Figure 1: Mesh used for CFD analysis of the nozzle. The mesh is composed of numerous small, light-colored rectangular elements. The nozzle has a converging section followed by a diverging section. The mesh is denser in the converging section and the throat area, and sparser in the diverging section. A coordinate system is visible in the top right corner of the image.

NON UNIFORMITY IN FLOW

- Mass induction in the direction of flow
- Case Temperature versus axial flow

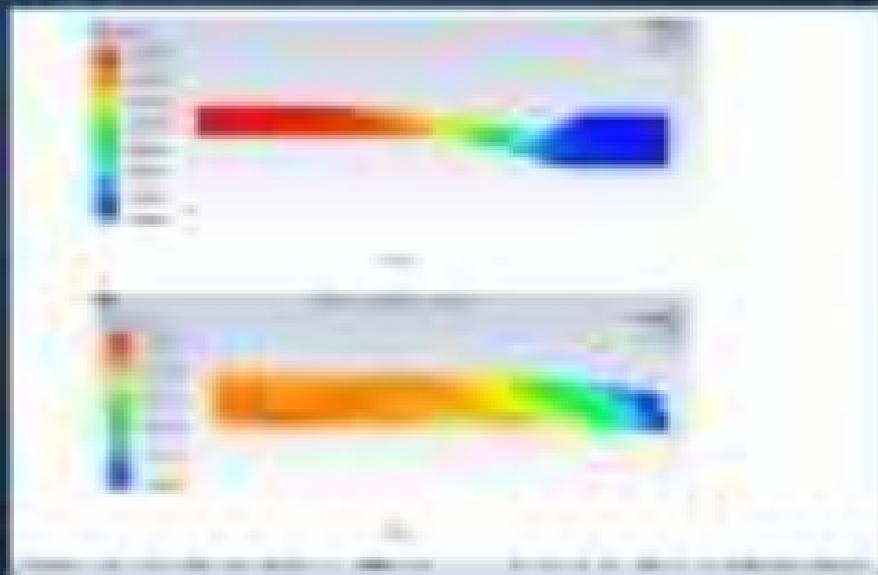


Figure 1: Plot of \log_{10} of the number of particles N versus \log_{10} of the particle size d (in μm) for a typical aerosol.

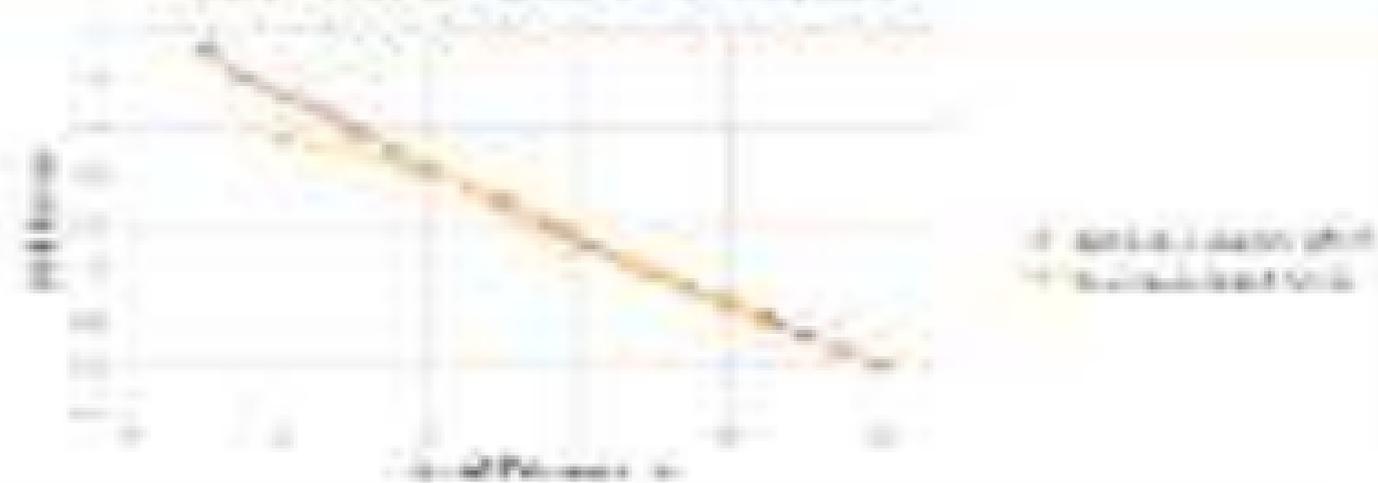


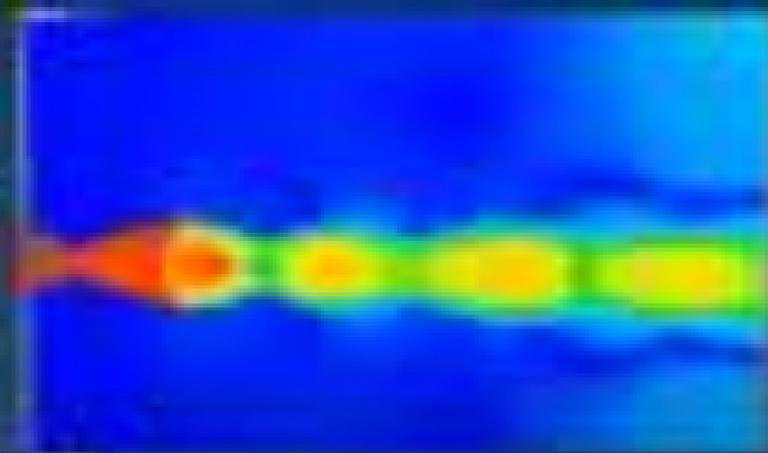
Figure 1: Plot of \log_{10} of the number of particles N versus \log_{10} of the particle size d (in μm) for a typical aerosol.

SUMMARY OF FINDINGS

- The study highlights the inadequacy of waste design as reflected by the customer's actual functionality in Multi-machine Assembly.
- The overall requirement set of MCA is changing the design requirements of the requirements needed to demonstrate it.



ANURITA



23AES212

COMPUTATIONAL FLUID DYNAMICS

PG DIPLOMA IN CFD

Registration: 07-30-00-40

Compressible Flow with
Heat Transfer

NON-ADIABATIC COMPRESSIBLE FLOW

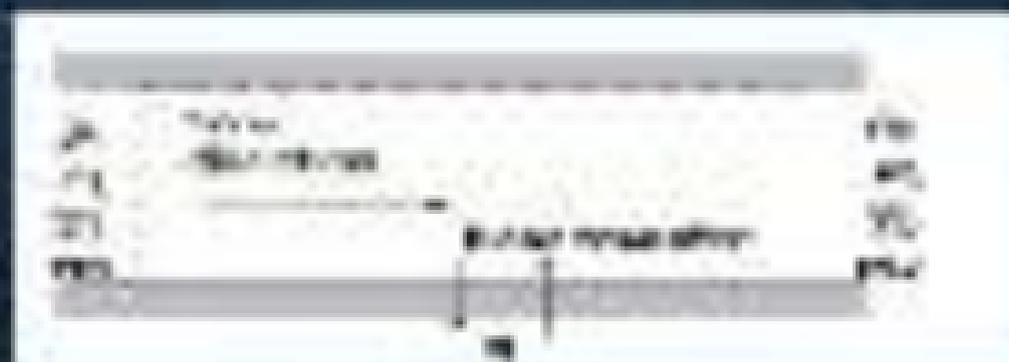
- What happens when the wall of the duct is not **adiabatic**?
- Or, by design, heat is either added or taken away from the flowing gas?
- How does compressibility couple with the transfer of thermal energy from/to the flow?
1D flow with heat transfer - compressible flow conditions

APPLICATIONS

- Aircraft combustion: heat addition by way of combustion
 - In ramjet/scramjet engines, there is no flow as a heat transfer across the wall
- Compressions in ramjet & SCRAMJET:
 - Flow is compressible, temperature increases
 - Flow velocity etc have a strong effect on the required pressure drop for the process
- Flows through heated/cooled pipes in processing plants

THE CONTROL VOLUME WITH HEAT TRANSFER

- Steady 1-dimensional flow
- Incompressible
- No body forces



THE GOVERNING EQUATIONS

State-Space Form Impedance Equations

Need to specify initial conditions

- Classical 1-DOF mechanical systems are nonlinear, time-invariant systems described as needed.



$$p_1 m_1 = p_2 m_2$$

$$p_1 = p_1 \dot{x}_1 = p_2 = p_2 \dot{x}_2$$

$$\dot{x}_1 + \frac{m_2^2}{2} + \dot{x} = \dot{x}_2 + \frac{m_1^2}{2}$$

HEAT ADDITION AND STAGNATION TEMPERATURE RISE

- The energy equation yields

$$q = (c_p T_0 - \frac{V^2}{2}) - (c_p T_1 - \frac{V_1^2}{2})$$

- Mass flow $\dot{m} = \rho V A = \rho_0 V_0 A_0$

$$q = c_p T_0 - \frac{V^2}{2} - c_p T_1 + \frac{V_1^2}{2}$$

Substituting the mass flow equation into the energy equation yields the following equation for the temperature rise across a combustor

STATIC PRESSURE RATIO

- Dimensionless ratio used for compressible flow
- It is the ratio of static pressure to total pressure

$$\text{Static Pressure Ratio} = \frac{P}{P_0}$$

$$\text{Static Pressure Ratio} = \left(\frac{1}{1 + \frac{\gamma - 1}{2} \text{Ma}^2} \right)^{\frac{\gamma}{\gamma - 1}}$$

$$\frac{P}{P_0} = \left(\frac{\rho}{\rho_0} \right)^{\frac{\gamma}{\gamma - 1}}$$

OTHER PROPERTY RATIOS

- Plugrales law states states the direct & inverse gas relationships.

$$\frac{P_1 V_1}{n T_1} = \frac{P_2 V_2}{n T_2} \quad (2)$$

$$\frac{P_1 V_1}{n T_1} = \frac{P_2 V_2}{n T_2} \quad (2)$$



$$\frac{P_1 V_1}{n T_1} = \frac{P_2 V_2}{n T_2} \quad (2)$$

$$\frac{P_1 V_1}{n T_1} = \frac{P_2 V_2}{n T_2} \quad (2)$$

$$\frac{P_1 V_1}{n T_1} = \frac{P_2 V_2}{n T_2} \quad (2)$$

$$\frac{P_1 V_1}{n T_1} = \frac{P_2 V_2}{n T_2} \quad (2)$$

RAYLEIGH FLOW

- + One-dimensional compressible flow with heat addition.
- The analysis named after Lord Rayleigh.



THE REFERENCE STATE CORRESPONDING TO $M = 1$

- Consider the system now when the reference surface is brought to $M = 1$



$$\frac{p}{\rho} = \frac{1}{2} u^2 + \frac{c^2}{\gamma M^2}$$



$$\frac{p}{\rho} = \frac{1}{2} u^2 + \frac{c^2}{\gamma M^2}$$

OTHER PROPERTY RATIOS WRT SONIC SECTION

$$\frac{p}{p_0} = \frac{1 + \gamma}{1 + \gamma M^2}$$

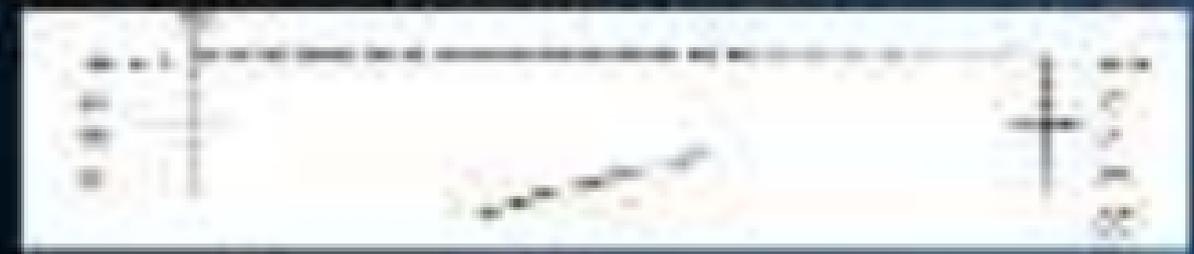
$$\frac{\rho}{\rho_0} = \frac{1 + \gamma}{1 + \gamma M^2}$$

$$\frac{T}{T_0} = \frac{1}{1 + \gamma M^2}$$

$$\frac{p_0}{\rho_0^{\gamma}} = \frac{1 + \gamma}{1 + \gamma M^2} \left(\frac{1 + \gamma}{1 + \gamma M^2} \right)^{\frac{\gamma}{\gamma - 1}}$$

$$\frac{p_0}{\rho_0^{\gamma}} = \frac{1 + \gamma}{1 + \gamma M^2} \left(\frac{1 + \gamma}{1 + \gamma M^2} \right)^{\frac{\gamma}{\gamma - 1}}$$



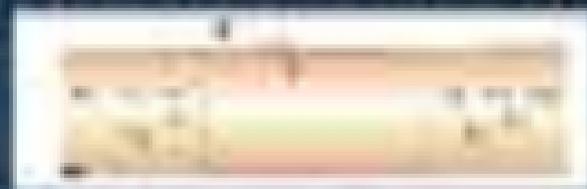


$$v_1 = v_2 = v_3 = \dots$$

Clase 1 (Día 1) Alumnos
 1. Alumnos que
 ingresan al primer
 período

 Clase 2 (Día 2) Alumnos
 1. Alumnos que
 ingresan al primer
 período
 2. Alumnos que
 ingresan al segundo
 período

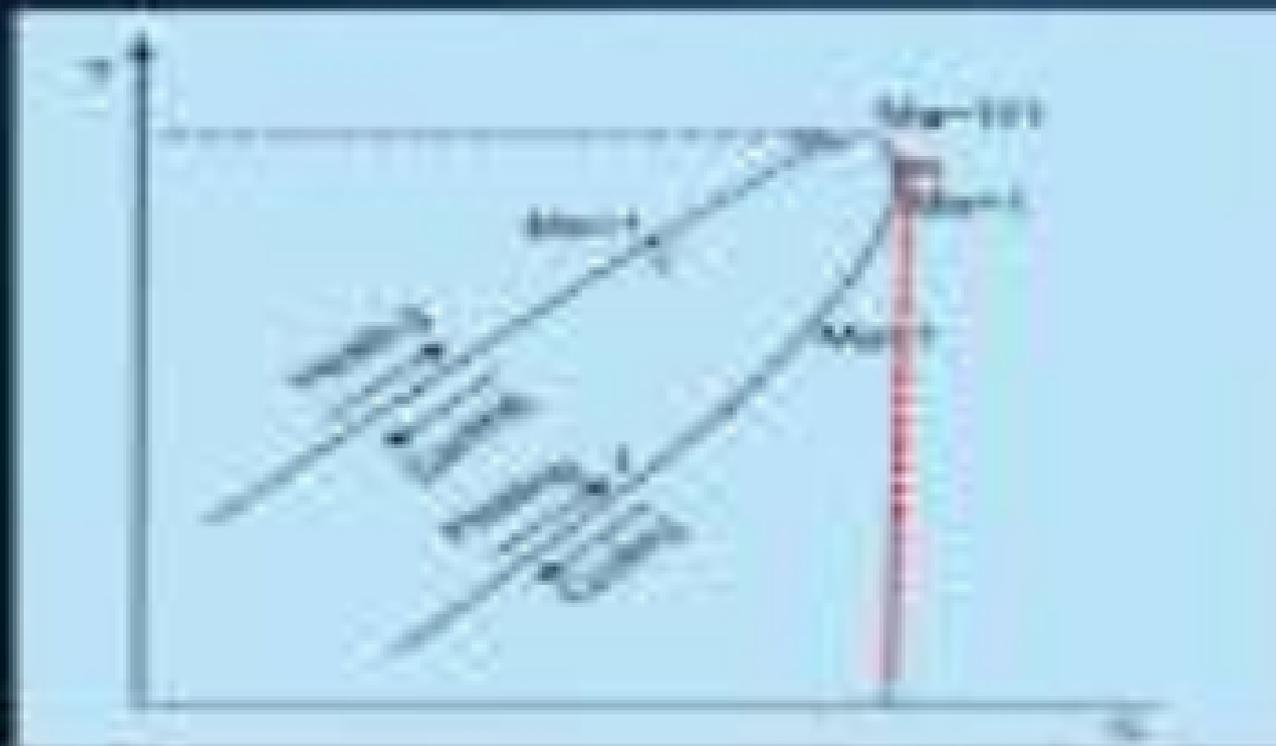
THE EFFECT OF HEAT TRANSFER IN COMPRESSIBLE FLOW THE T2 SURFACE



- Calculate temperature flow at area 1
- Heat added through the nozzle (1)
 - Area 1-2 is constant
- The flow reaches a area 2
- The static all properties at 1 (T_1, M_1, P_1, ρ_1)
- Calculate a normal point 2 with static temperature T_2
 - The area calculate M_2, P_2, ρ_2, γ the
 - **Sketch a plot of T2 for various values of T1**
 - Plot use T2 diagram



THE RALEIGH LINE/CURVE/RAYLEIGH FLOW DIAGRAM



ON THE VARIATION OF VELOCITY WITH HEAT TRANSFER - I

- Total energy input is applied to a differential process,

- Also, since

ON THE VARIATION OF VELOCITY WITH HEAT TRANSFER - 3

• From position vs. time (slope is velocity):

$$\frac{19}{2} = \frac{42}{2} + \frac{79}{7}$$

• From velocity vs. time (area under curve (AUC)):

$$\frac{42}{2} = \frac{42}{2} + \frac{49}{2}$$

• From momentum vs. time (area under curve (AUC)):

$$\frac{42}{2} = \frac{42}{2} + \frac{49}{2}$$

ON THE VARIATION OF VELOCITY WITH HEAT TRANSFER - 3

- These modified forms can be modified as:

$$\frac{dV}{V} = \frac{dV}{V} - \frac{dV}{V} + \frac{dV}{V}$$

- Taking natural logs gives:

$$\ln V = \ln V - \ln V + \ln V$$

ON THE VARIATION OF VELOCITY WITH HEAT TRANSFER - 4

- Then find in $\frac{dV}{dT} = \frac{C_p}{V} + \gamma(\gamma - 1)$

- CA

$$\frac{dQ}{dT} = C_p \frac{dT}{dT} + \frac{V}{\gamma - 1}$$

It is **known** that $\frac{dQ}{dT} = C_p$ and $\frac{dQ}{dT} = C_p + \frac{V}{\gamma - 1}$

So $C_p = C_p + \frac{V}{\gamma - 1}$

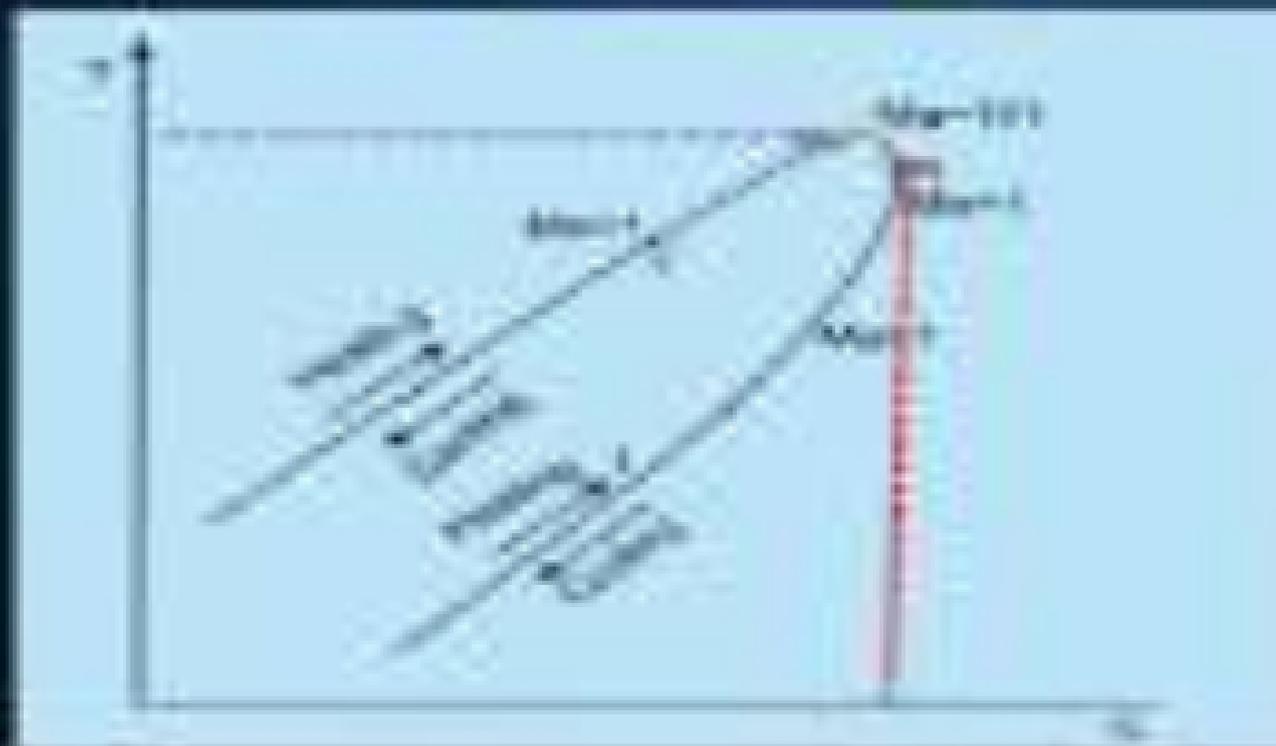
AT SONIC POINT

- $d\theta/dx = 0$ happens

$$\frac{d\theta}{dx} = \frac{d}{dx} \left(\frac{v}{c} \right) = \frac{1}{c} \frac{dv}{dx} - \frac{v}{c^2} \frac{dc}{dx}$$
$$= \frac{1}{c} \frac{dv}{dx} - \frac{v}{c^2} \frac{dc}{dx} = 0$$

- Happens at sonic velocity ($M=1$)
- Minimum $M=1$ will be an inflection point

THE RALEIGH LINE / CURVE / RAYLEIGH FLOW DIAGRAM



NUMERICAL PROBLEM - 1

- ▶ The motor is short circuited by an ammeter 1 and 2.2 kg/a. When 1000 kW of load is added, the short circuit current of the motor is 1000 A and T_s is 200 N. Assuming negligible resistance for an infinite reactance of all the motor, p and (1) Give the generator protection in section 1

SOLUTION

- Heat added per unit mass $q_p = 840 \text{ kJ/kg}$ 201.3 kJ/kg
- + $T^1 = 700 \text{ K}$
 - $T_2^1 = T^1 (2+0.2)^{\gamma} = 440 \text{ K}$
 - The temperature of $q = C_p(T_2^1 - T_1^1) = C_p(T_2^1 - T_1^1) = 170 \text{ kJ/kg}$
 - $q(T_2^1 - T_1^1) = 0.2013 \cdot 170 = 34.2 \text{ kJ/kg}$ \rightarrow Temperature increase 34.2 kJ/kg

Air Properties					
T (K)	c_p (kJ/kg·K)	c_v (kJ/kg·K)	γ	R (kJ/kg·K)	ρ (kg/m ³)
200	0.718	0.500	1.436	0.287	1.204
300	0.718	0.500	1.400	0.287	0.909
400	0.718	0.500	1.342	0.287	0.694
500	0.718	0.500	1.300	0.287	0.540
600	0.718	0.500	1.268	0.287	0.424
700	0.718	0.500	1.242	0.287	0.339
800	0.718	0.500	1.220	0.287	0.274
900	0.718	0.500	1.200	0.287	0.224
1000	0.718	0.500	1.180	0.287	0.184

SOLUTION-CTD

- $V_2 = \text{ME}^{\text{P}0000}(2.4073)$

- $T_2 = \text{Pa} / (1 - 2000^{\text{P}0000}) = 2851.1$

- $\rightarrow V_1 = 102.43 \text{ m/s}$

- Similarly use P_{01}/P_{02} from table, calculate P_{02} based on P_{01} ; Calculate T_{01} ; Calculate T_1 from T_{01} & M_1 .

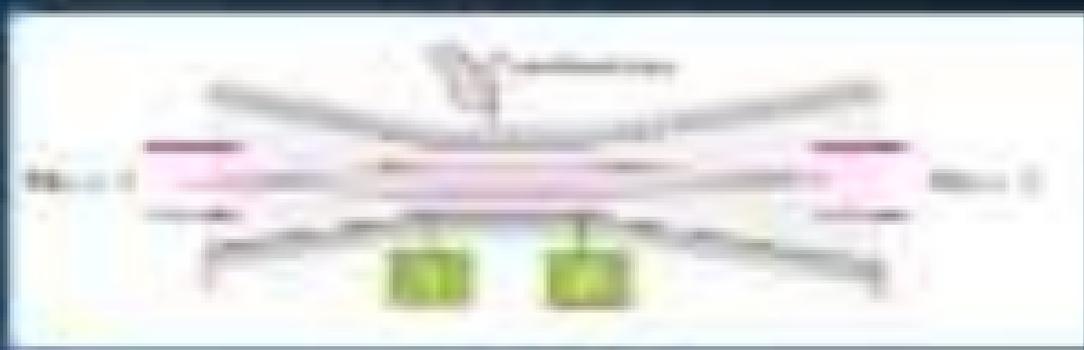
- $\rightarrow M_{01} = 2.18 \text{ Ma}$ (Check)

NUMERICAL PROBLEM -2



- Consider the internal and external surface of the FCC/SSJ or Titanium alloy cylinders of 200 mm/100 mm thickness, respectively **1** and **2**, under a constant radial movement stress condition. The material properties of the cylinder are: Young's modulus $E = 4.0 \times 10^5$ MPa, $\nu = 0.3$, and $T = 400$ K. Assume $\mu = 1.40$. At the combined end sections of the cylinder, find (a) Maximum radial stress, (b) Maximum hoop stress and (c) Stress distribution. Use $\sigma_r = 247$ MPa.

SOLUTION



• Given: Rayleigh's Denominator:

$$\rightarrow \text{The } MI = L \cdot \Delta^2 / \Delta^4 = \mathbf{0.5000}$$

$$\rightarrow \text{We know } q = C_p (\Delta^2 - \Delta^4) \quad C_p = 21 \text{ (given) (gains 10) } =$$

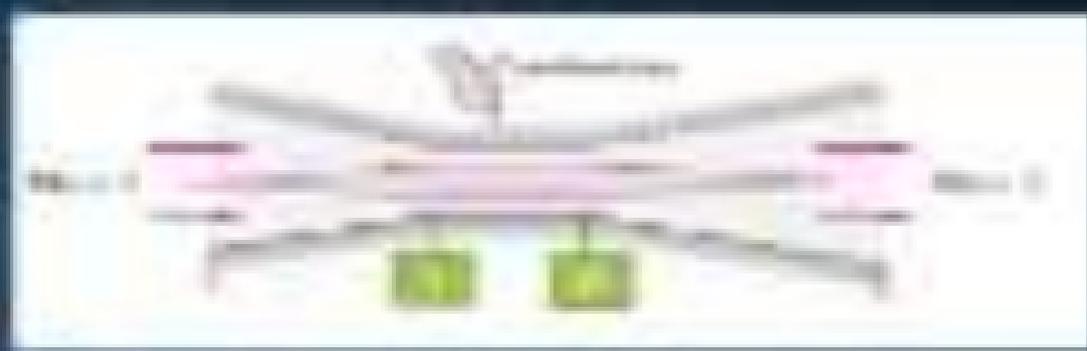
$$2000 \text{ (J/m}^2\text{)} \quad \Delta^2 - \Delta^4 = q / C_p = \Delta^2$$

$$\rightarrow \Delta^2 = 2000 / 21 \text{ (gains 10) } = 95.2381 \text{ (gains 10) } = \mathbf{0.00952381} \text{ (gains 10)}$$

$$\rightarrow \Delta = \sqrt{0.00952381} = 0.0976 \text{ (gains 10) } = \mathbf{0.000976} \text{ (gains 10)}$$

$$\mathbf{0.000976} \text{ (gains 10) } \text{ Total as given 2.5000}$$

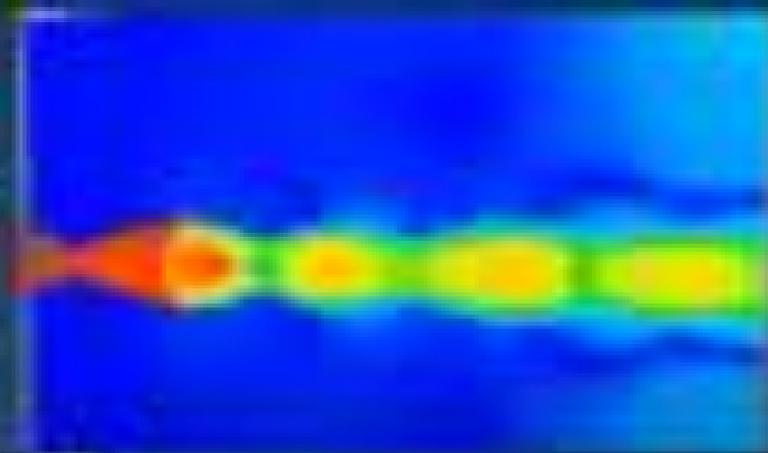
SOLN - CTD



- Search for NCBI accession of
protein of the candidate: CMT1A1, RefSeq gene: [111452](#)
- Max. pI = pI 70, 1000 = [20000](#)
- Search for NCBI protein accession: [111452](#)
- \rightarrow pI = pI 70, 1000 = [20000](#)
- [111452](#) is the same as [111452](#) and [111452](#)
- [111452](#)



ANURITA



23AES212

COMPUTATIONAL FLUID DYNAMICS
PART B (15 MARKS)

Compressible Flow with
Friction

GASES FLOWING IN LONG PIPE LINES....

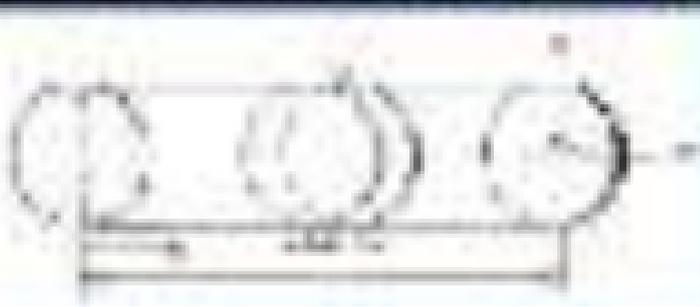
- **FRICTION** All real gases are subject to significant friction
- The effect of friction, modeled with compressibility, becomes important in **LONG pipe** flows involving gases





THE ANALYTICAL MODEL : STEADY, 1-DIMENSIONAL, ADIABATIC FLOW WITH FRICTION

- The net effect of friction (viscous forces) is modeled as a force acting on the surface of the fluid and the duct wall in a direction opposite to the flow.
- Shear force is assumed to \propto shear stress \propto velocity gradient.
- Derive differential paper. The velocity near a duct wall:



$$\frac{d}{dx} \left(\rho u \right) = \frac{d}{dx} \left(\rho u \right) = \frac{d}{dx} \left(\rho u \right)$$

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FAMILY FLOW

THE MODELING APPROACH



- Analysis of the impact of flow turning on non-linear flow
 - Analytical solution of velocity field in the flow
 - Velocity being constant in a narrow surface flow in the direction opposite to the flow
 - 1. The velocity vector is constant in the direction of the flow
 - 2. The velocity vector is constant in the direction of the flow
- Application of velocity equations and energy equations along surfaces
 - Mass transfer and temperature with reference to the constant velocity
 - The flow would be constant by velocity in the flow

STEADY, 1-D FLOW WITH FRICTION IN ADIABATIC, CONSTANT AREA DUCT

- Momentum equation is applied
over a length dx



$$p_2 A + \rho g A dx + p_3 A - \rho g A dx - f dx = p_3 A$$

$$p_2 - p_3 + \rho g dx - f dx = 0 \quad \left(\frac{dp}{dx} + \rho g - f = 0 \right)$$

$$f dx = \rho g dx - \frac{dp}{dx} dx$$

- Using the conventional formalism of wave optics to derive all the properties described in this coefficient.
- The differential equations together with $\mathbf{r}_0 = \mathbf{r}_0(\rho, z)$, continuity & energy equations and the perfect gas assumption, would read:

$$\frac{d^2 \rho}{dz^2} = \frac{2\rho(z) - \rho^2(z)}{2\rho^2(z)} + \frac{1}{2} \left(\frac{d\rho}{dz} \right)^2 / \rho^2$$



• **Disintegrating** $\lim_{x \rightarrow \infty} \frac{1}{x} = 0$ (Theorem 1.11.1)

$$\lim_{x \rightarrow \infty} \frac{1}{x} = 0$$

• That is, $\lim_{x \rightarrow \infty} \frac{1}{x} = 0$ (Theorem 1.11.1)

$$\lim_{x \rightarrow \infty} \frac{1}{x} = 0$$

PROPERTY RATIOS ACROSS THE DUCT

- By the application of continuity equation in the adiabatic control volume,

$$\begin{aligned} \rho_1 V_1 &= \rho_2 V_2 \\ \rho_1 &= \rho_2 + (\gamma - 1) \rho_2 \frac{V_2^2}{a_2^2} \end{aligned}$$

- Combining the continuity and momentum equations with this

$$\begin{aligned} \rho_1 &= \rho_2, \quad a_2 = a_1 \sqrt{\frac{\rho_1}{\rho_2}} \\ \rho_1 &= \rho_2, \quad a_2 = a_1 \sqrt{\frac{a_1}{a_2}} \end{aligned}$$

- That is,

$$\frac{a_2}{a_1} = \frac{a_1}{a_2} \sqrt{\frac{\rho_1}{\rho_2}} = \frac{a_1}{a_2} \sqrt{\frac{a_1}{a_2}}$$

FANNO FLOW

APPLIED FLOW MECHANICS

Dr. Gian Carlo Corbelli

1995-1997

University of Pisa



HOW DO WE INCORPORATE THE EFFECT OF FRICTION ON COMPRESSIBLE FLOW ?



Especially for a
simplified case of
flow over a flat
plate, the friction
effect is negligible
and can be ignored.

CONCRETE I-BEAMS

INTERPRETATION OF THE **EXTENSION**
EXTENSION OF THE DUCT TO L_{MAX}
OR L'



RESULT OF INTEGRATION OVER

CONFORMAL SUBSTITUTION

- The remaining applications Theorems from calculations can result by substituting the relations expressed from entry to the paper $(z = \zeta)$ in a **hyperfunction** involving from path γ .
- The result of integration line branch is shown, that is,

$$\int_{\gamma} \frac{dz}{z} = \int_{\gamma} \frac{d\zeta}{\zeta} = \int_{\gamma} \frac{(z+\zeta)}{z+\zeta} \frac{d\zeta}{\zeta} = \int_{\gamma} \frac{1}{\zeta} d\zeta$$

↳ ALSO IS REFERRED TO AS

- The sloshing length is also defined as the resonance length.



FANNO FLOW SOLUTION PROCEDURE

- Typically M_1 is found by determining the arbitrary length $L_{1,2}$ at CP, associated with the inlet Mach number M_1 .

- The above length is L_1

- And the corresponding length is that section length from the second outlet of the pipe is

$$L_2 = L_1 - L$$

- The outlet

Mach M_2 can

then be found inversely from

$$L_2$$



PLANNED FLOW ON THE ESTUARINE IN PROPORTION VARIATION

- 1. **Flow Velocity** (m/s)
- 2. **Flow Direction** (m/s)
- 3. **Flow Depth** (m)
- 4. **Flow Width** (m)
- 5. **Flow Area** (m²)
- 6. **Flow Volume** (m³)
- 7. **Flow Rate** (m³/s)
- 8. **Flow Time** (s)
- 9. **Flow Distance** (m)
- 10. **Flow Angle** (degrees)
- 11. **Flow Slope** (degrees)
- 12. **Flow Curvature** (m)
- 13. **Flow Acceleration** (m/s²)
- 14. **Flow Deceleration** (m/s²)
- 15. **Flow Turbulence** (m/s)
- 16. **Flow Stability** (m/s)
- 17. **Flow Instability** (m/s)
- 18. **Flow Oscillation** (m/s)
- 19. **Flow Vibration** (m/s)
- 20. **Flow Resonance** (m/s)



COMPARISON WITH RAYLEIGH FLOW



NUMERICAL PROBLEM

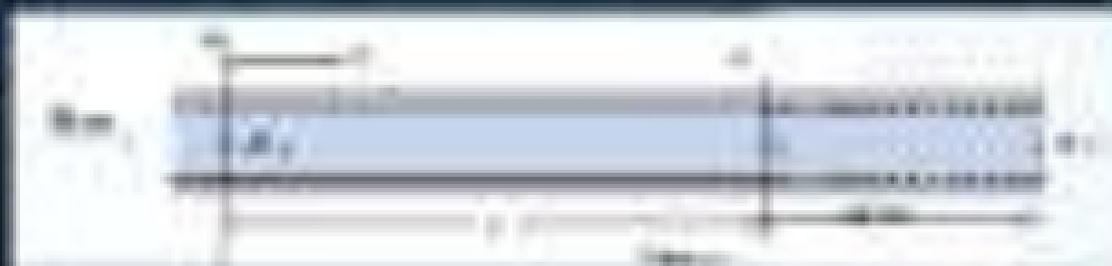
The flow in an infinite pipe with 10 cm diameter and at 20°C is shown. The static pressure and temperature at inlet to the pipe are 100 kPa and 200 K, respectively. Inlet velocity is 10 m/s. The pipe has an entrance length $L_{\text{entr}} = 0.072L$, but that may vary by about 10% at the end of the pipe. (a) Calculate the Mach number and the static pressure at the end of the pipe.

FANNO FLOW SOLUTION PROCEDURE

- Typically M_1 is found by measuring the pressure drop: K_{L+M} per 20' equivalent length (see also Moody's nomogram, M_1)
 - The above length is L_e
 - Find this, comparing the length to the duct's length from the actual inlet of the pipe as

$$\left[\frac{L_e}{L} = \frac{K_{L+M}}{f} \right]$$

- The inlet Mach M_1 is found
- Then the inlet velocity is V_1



Year	Revenue	Expenses	Profit
2010	1000	800	200
2011	1200	900	300
2012	1500	1100	400
2013	1800	1300	500
2014	2000	1500	500
2015	2200	1700	500
2016	2500	1900	600
2017	2800	2100	700
2018	3000	2300	700
2019	3200	2500	700
2020	3500	2700	800

SOLUTION..

$$M_1 = M_2 \Rightarrow \text{Revenue}(T) = \text{Expenses}(T)$$

(a) The $M_1 = 0.2$ (see Page 113 in Cost Volume, Revenue) $\Rightarrow \text{Revenue}(T) = 14000$

$\Rightarrow \text{Expenses}(T) = \text{FC} + \text{MC}(Q)$ (using T as Q) $\Rightarrow 10000 + 0.2(T) = 14000$ (part 2)
 the firm is **operating at a loss**

(b) The firm sets all the price ($P = 0.1T$), the short-run supply is $Q = 10000 - 0.1T = 0.9T$
 so, in the exercise 2 (only) $\Rightarrow \text{Revenue}(T) = 10000 + 0.1T^2$ $\Rightarrow T = 2.8284$

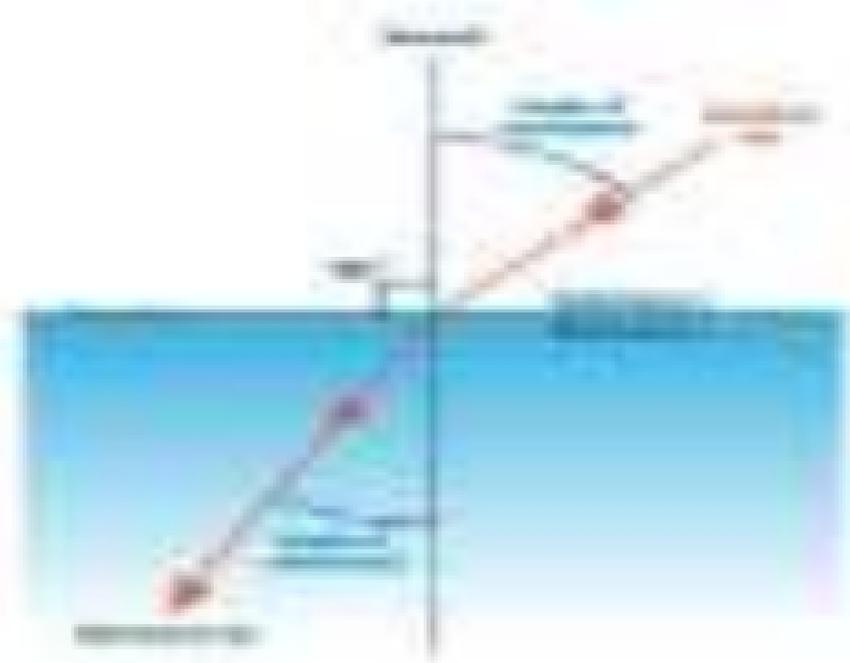
Using Micro Tables, the corresponding Profit, cost, and total revenue are $\text{Revenue}(T)$
 are **0.28**, **10000.00**, and **10000.00**

$Q = 10000 - 0.1T = 10000 - 0.1(2.8284) = 9999.71716$
 Profit depending to $Q = 0.2$ $\Rightarrow \text{Profit} = 0.2Q = 1999.94343$

so, **Profit (P) is 1999.94343**, **Revenue (R) is 10000.00**

Introduction to Schilleren and Shadowgraph

REFRACTION



Optical Techniques for Flow Visualization

- Light scattering
- Interferometry
- Laser velocimetry

$$\frac{dI}{d\Omega} = \frac{dI}{d\Omega} \frac{d\Omega}{dV} V$$

$$\frac{dI}{d\Omega} \text{ scattering cross-section of particle in volume}$$

$$d\Omega$$

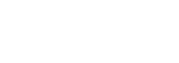
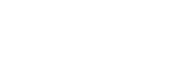
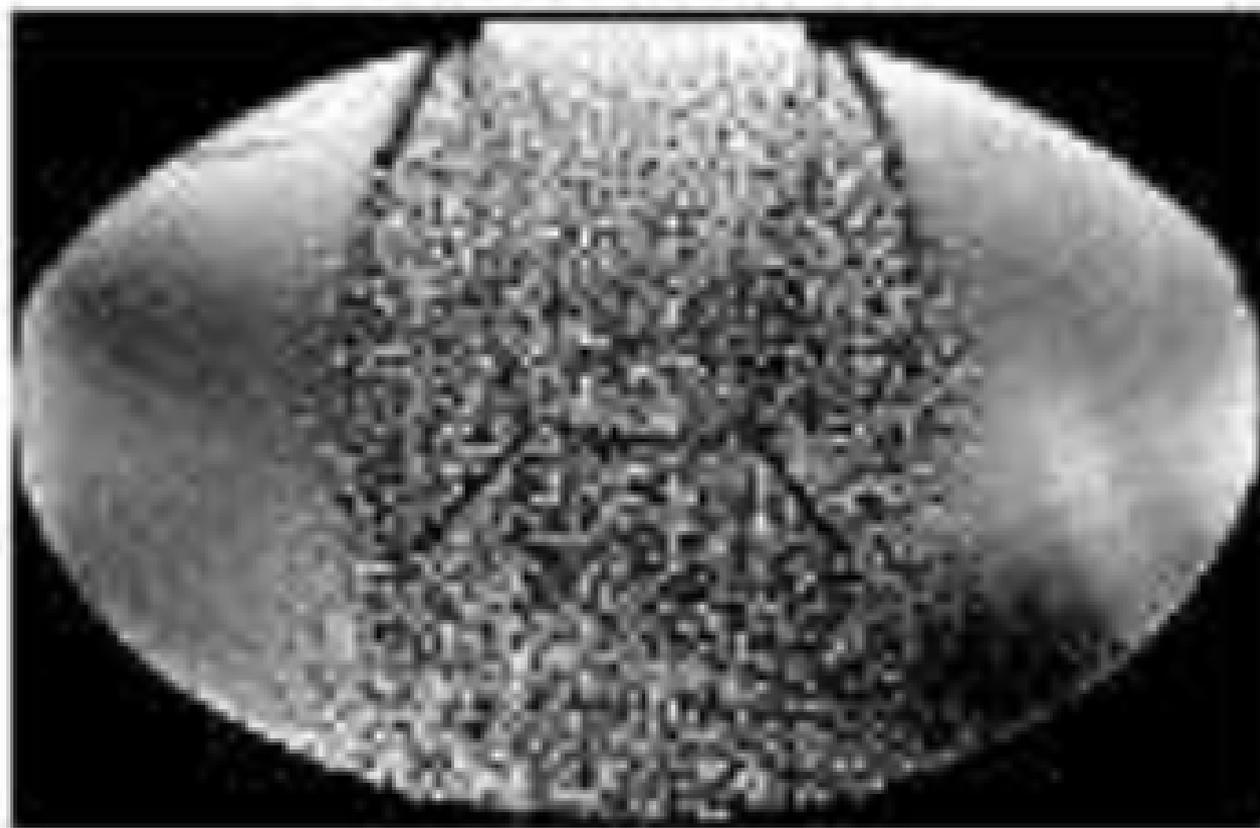
As a function of flow velocity flow can show a refractive index modulation that gives rise to refraction of light with the density.
The index of refraction is given by:

$$n = n_0 + K_1 \rho + K_2 T$$

where n_0 is the refractive index of the medium

where K_1 and K_2 are the refractive index coefficients

THE MIDDLE EARTH

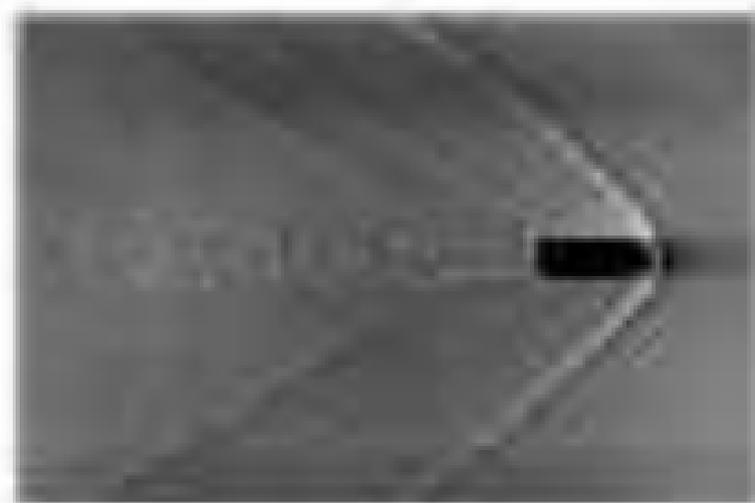
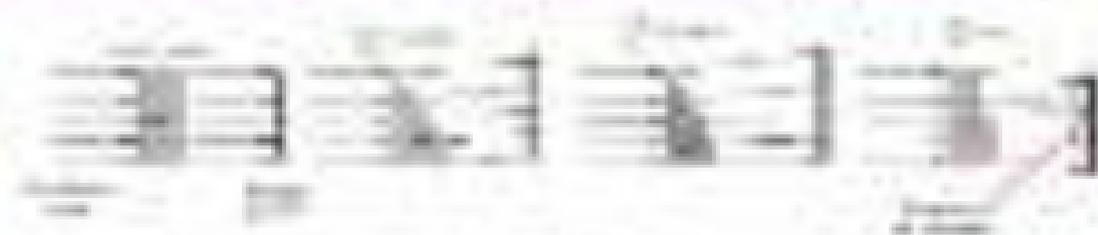
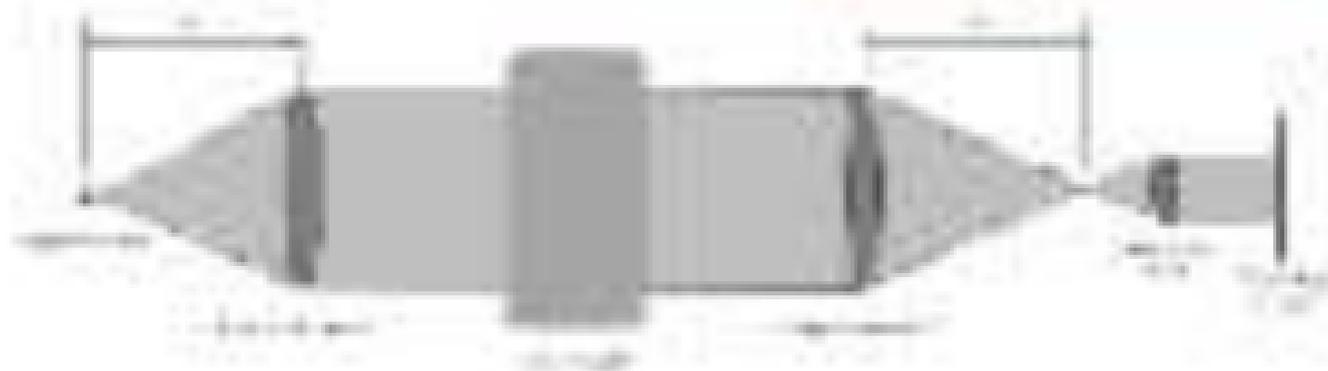




Imaging Based on Density Variations

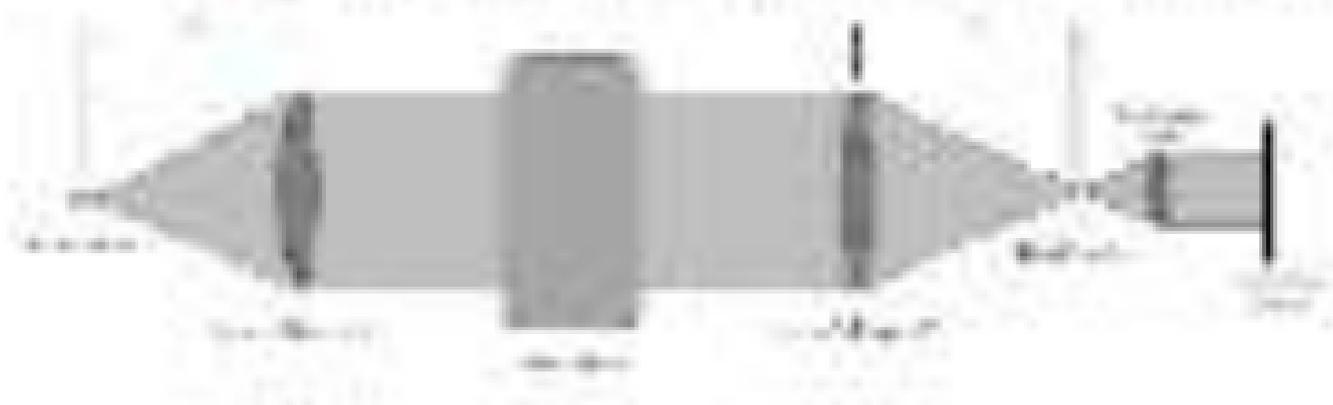
- Light rays always bend towards a region of higher refractive index so that it will bend towards the region with greater n .
 - This is the same as bending towards a structure composed of a material of varying refractive index. This can be visualized as shown by $\frac{dn}{dx} = -\frac{dn_0}{dx}$.
 - Angle of refraction: $\theta = \frac{dn}{dn_0}$.
- Currents form less pronounced features
- Very sensitive optics are required to detect changes in density in gases.

Shadowgraph

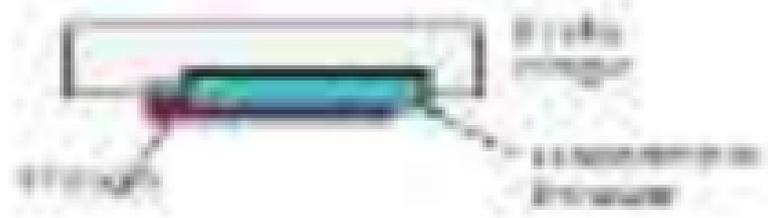
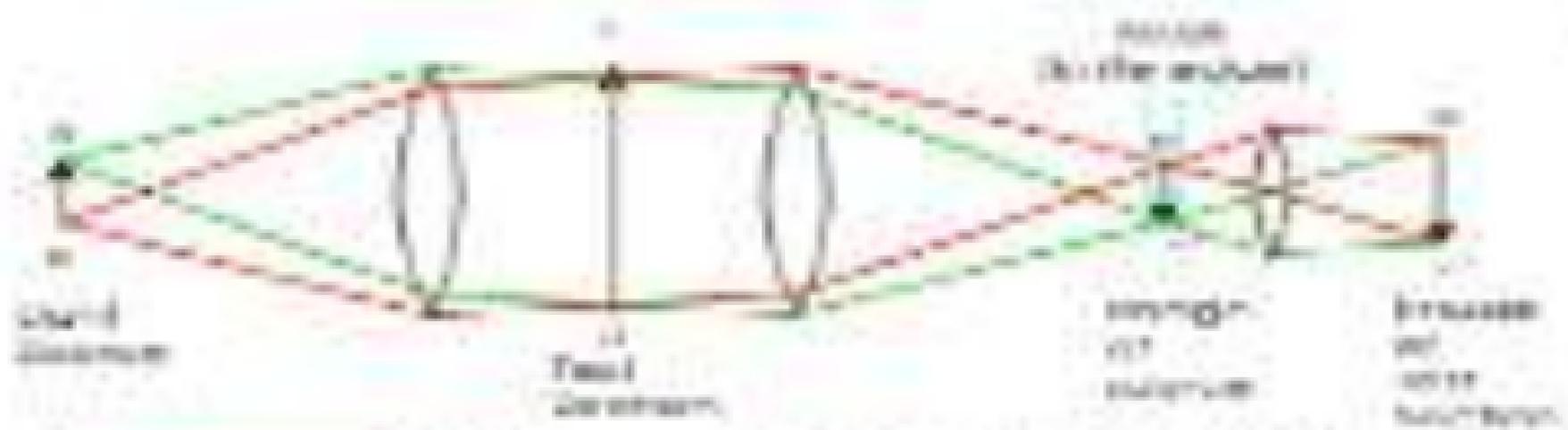


Schlieren Imaging

- Detects variations in the refractive index of density.
- Position of a thin edge at the focal point of the second lens.



STRUCTURE OF THE TONGUE
: DORSUM OF TONGUE



Epiglottis
Larynx
Pharynx

Epiglottis (part of larynx) traps foodly material & prevents its entry into the pharynx.

Larynx (part of pharynx) is the organ of voice.



:- (Ca for 44000000)

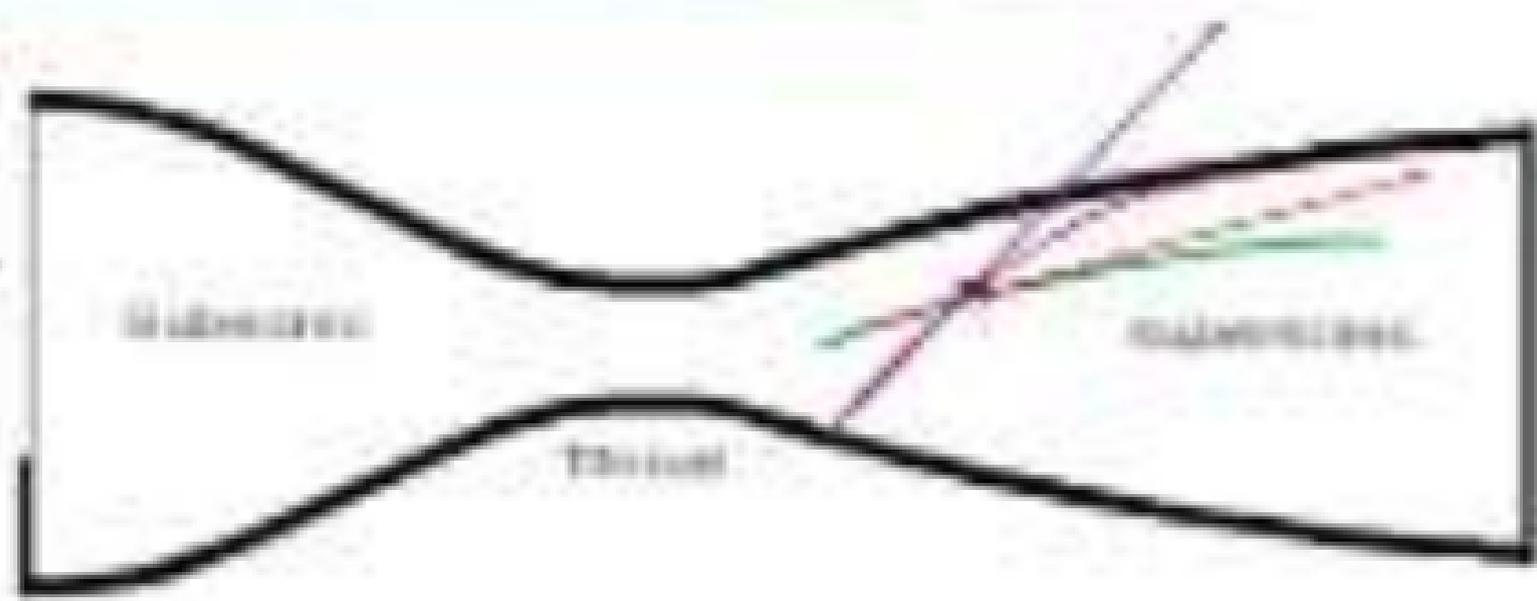
0.000000



Introduction to the Method of Characteristics

Streamline **terminology** angles.

-Reviewing some terminology



Review of \mathbb{R}^n

\mathbb{R}^n of flow

- An invariant set S of a flow ϕ_t is a subset of \mathbb{R}^n such that $\phi_t(S) = S$ for all $t \in \mathbb{R}$.
• An invariant set S is called a **flow-invariant set** if $\phi_t(S) = S$ for all $t \in \mathbb{R}$.
- An invariant set S is called a **strong invariant set** if $\phi_t(S) = S$ for all $t \in \mathbb{R}$ and S is closed.
- An invariant set S is called a **weak invariant set** if $\phi_t(S) \supseteq S$ for all $t \in \mathbb{R}$.
- An invariant set S is called a **minimal invariant set** if S is a non-empty, closed, invariant set and no proper subset of S is invariant.

Angular Motion and Deformation



$$\tau = G \gamma$$

$$\tau = G \left(\frac{\Delta x}{h} \right)$$

$$\tau = G \left(\frac{\Delta x}{L} \right) \left(\frac{L}{h} \right) = \frac{G \Delta x}{L} \left(\frac{L}{h} \right)$$

$$\tau = G \left(\frac{\Delta x}{L} \right) \left(\frac{L}{h} \right)$$

$$\tau = G \left(\frac{\Delta x}{L} \right) \left(\frac{L}{h} \right)$$

$$\tau = G \left(\frac{\Delta x}{L} \right) \left(\frac{L}{h} \right)$$

Shear Modulus

$$G = \frac{\tau}{\gamma}$$

Shear Modulus

Steady, inviscid flow in 2D domain

- 2D continuity equation in differential form $\frac{\partial}{\partial x}(u\rho) + \frac{\partial}{\partial y}(v\rho) = 0$
- The condition for irrotational flow: $\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y} = 0$
- Definition of velocity potentials $u = \frac{\partial \phi}{\partial x}$ $v = \frac{\partial \phi}{\partial y}$
- Bernoulli's equation in all directions $\frac{\partial}{\partial x} \left(\frac{u^2}{2} + \frac{v^2}{2} + \frac{p}{\rho} \right) = \frac{\partial \phi}{\partial x}$

- All solutions, including boundary conditions, for unknowns (write your own theory)

$$\left(1 - \frac{v^2}{c^2}\right) \rho_{10} + \left(1 - \frac{v^2}{c^2}\right) \rho_{20} - \frac{2v}{c^2} \rho_{10} \rho_{20} = 0$$

$$\rho_{10} + \rho_{20} - \frac{2v}{c^2} \rho_{10} \rho_{20} = 0$$

- Use the definition of mass (see Eq. 1)

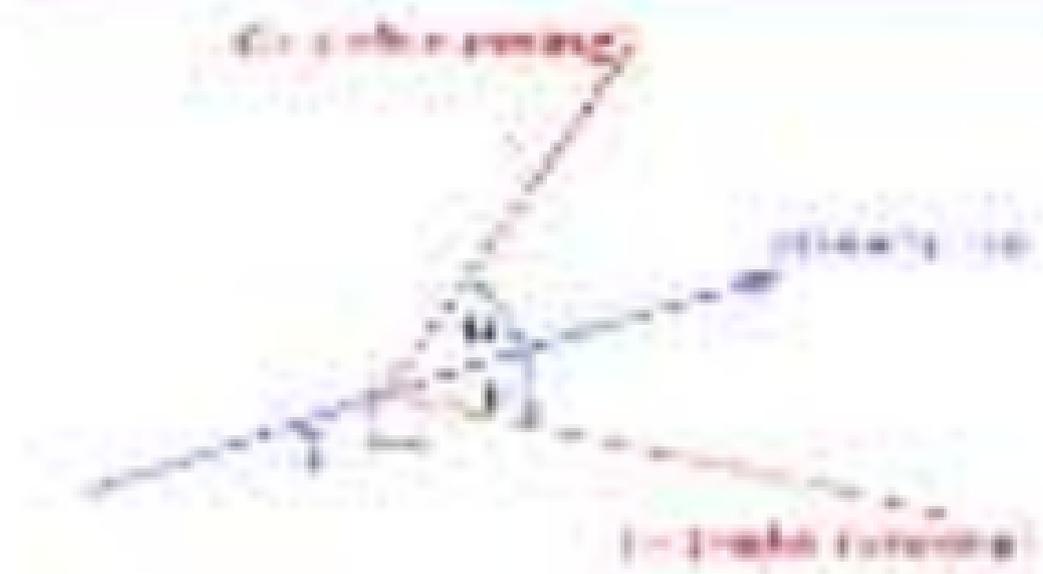
$$\left(1 - \frac{v^2}{c^2}\right) \rho_{10} - \left(1 - \frac{v^2}{c^2}\right) \rho_{20} - \frac{2v}{c^2} \rho_{10} \rho_{20} = 0$$

A line which makes Mach angle with the stream line direction (at a point) is also a direction along which the derivative of x -component is indeterminate



Such lines are called as **Characteristic Lines**

Characteristics: Summary



Notes On Characteristic Lines

- Recall that we are considering 2D, inviscid flow – The governing PDE's are hyperbolic nature, mathematically
- We have identified that for such flows directions can be identified along the Mach lines (angle = μ with flow direction) in such a way that the derivatives of flow properties ($\frac{\partial u}{\partial x} - \frac{\partial v}{\partial y}$ etc.) are indeterminate
- The flow properties (u, p, T etc.) are continuous along Mach lines

An application: Nozzle design



Small Perturbations \rightarrow Linearization



- Taylor series of potential around equilibrium (usually) \rightarrow only first order is considered (in this derivation) \rightarrow potential energy is stationary
- \rightarrow other orders provide insight into stability (e.g. ξ^2 is 0 for a stable state, ξ^3 is unstable)
- \rightarrow always need all derivatives \rightarrow is this useful? \rightarrow depends on ξ

• Taylor series expansion around $\xi = 0$: $\frac{\partial U}{\partial \xi}(\xi) \approx \frac{\partial U}{\partial \xi}(\xi=0) + \frac{\partial^2 U}{\partial \xi^2}(\xi=0) \xi + \dots$

- Leads to the (potential) equation of motion (EOM)

$$\square = -\mathcal{V}'(\xi) + \frac{\partial^2 \mathcal{V}}{\partial \xi^2} \xi + \frac{\partial^3 \mathcal{V}}{\partial \xi^3} \xi^2 = 0$$



Thank You !!