25SC810 Topological Quantum Computation 3-1-0-4

Objective:

To explore the mathematical and physical foundations of topological quantum computation through the study of anyons, braiding, knot theory, and category theory, enabling a deep understanding of how topological structures can encode and process quantum information.

Unit – I Foundations of Topological Phases and Anyons

Topological phases of matter: overview and motivation; quasiparticles in 2D systems; anyons: abelian and non-abelian types; braiding statistics and topological degeneracy; concept of braids in 2D particle exchanges; encoding quantum information in topological states.

Unit – II Topological Quantum Computation (TQC)

Non-abelian statistics and their implications; braiding anyons as quantum gate operations; logical gate implementation through braiding; fault tolerance in topological quantum computation; scalability of TQC platforms; quantum information storage in topological media.

Unit – III Algebraic Structures in Topology

Algebra of braids: braid groups and relations; knot theory fundamentals: knots, links, tangles; Reidemeister moves I, II, III; introduction to ribbon and tangle diagrams; RIBBON and TANGLE: definitions and examples; interpreting quantum computation using algebraic topology.

Unit – IV Polynomial Invariants and Skein Theory

Introduction to knot polynomial invariants; Jones polynomial: construction and properties; HOMFLY polynomial and its generalization; Kauffman polynomial: formulation and applications; skein relations and recursive evaluations; applications of knot polynomials to TQC.

Unit – V Category Theory and Diagrammatic Methods

Monoidal categories: structure and examples; braided and ribbon categories: definitions and significance; diagrammatical calculus in category theory; representation theory of RIBBON and TANGLE; categorification: motivation and methods; applications of categorical structures to topological computation.

Textbooks:

1. Mathematics of Topological Quantum Computing by Eric C. Rowell and Zhenghan Wang.
2. A Short Introduction to Topological Quantum Computation by Ville Lahtinen,
3. Jiannis K. Pachos.
4. Non-Abelian anyons and topological quantum computation by Chetan Nayak
5. Majorana Fermions and Non-Abelian Statistics in Three Dimensions by Jeffrey C.Y. Teo and C. L. Kane.

References:

1. Nayak, C., Simon, S. H., Stern, A., Freedman, M., & Das Sarma, S. (2008). Non-Abelian anyons and topological quantum computation. *Reviews of Modern Physics, 80*(3), 1083–1159.
2. Hasan, M. Z., & Kane, C. L. (2010). Colloquium: Topological insulators. *Reviews of Modern Physics, 82*(4), 3045–3067. sss

Outcomes:

CO	Course Outcomes
CO01	Understand the principles of topological phases of matter and the role of anyons in quantum systems.
CO02	Analyze how braiding operations implement quantum gates in topological quantum computation.
CO03	Apply concepts from knot theory, including braids, tangles, and polynomial invariants, to model quantum information processes.
CO04	Interpret and construct algebraic and diagrammatic representations using RIBBON and TANGLE structures.
CO05	Utilize categorical frameworks, including monoidal and ribbon categories, to formalize topological quantum computational models.

Evaluation Pattern:

Category	Marks
Quizzes(2)	10
Assignments(2)	20
Mid-sems(1)	30
End-sems(1)	40